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A Structural Estimation and Interpretation of the New Keynesian Macro Model

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ABSTRACT

We formulate and solve a Rational Expectations New Keynesian macro model that implies non-linear cross-equation restrictions on the dynamics of inflation, the output gap and the Federal funds rate. Our maximum likelihood estimation procedure fully imposes these restrictions and yields asymptotic and small sample distributions of the structural parameters. We show how the structural parameters shape the responses of the macro variables to the structural shocks. While the point estimates imply that the Fed has been stabilizing inflation fluctuations since 1980, our econometric analysis suggests considerable uncertainty regarding the stance of the Fed against inflation.

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1 Introduction

One of the central issues in macroeconomic analysis is the impact of structural shocks on the dynamics of inflation, the output gap and interest rates. Much of the empirical analysis employs large vector autoregressive (VAR) systems which often have a difficult economic interpretation. In this paper we examine the interaction of the exogenous structural shocks with the behavior of both the monetary authority and private agents in the context of a New Keynesian macroeconomic model. We estimate the complete structural model and analyze the small sample properties of its structural parameters. We then show how the structural parameters govern the dynamic responses to structural shocks.

Our structural model comprises an aggregate supply (AS) equation based on a contracting specification, an IS or demand equation based on representative agent utility maximization with habit persistence, and a forward looking monetary policy rule. The Rational Expectations solution of the model implies a set of identifying assumptions which translate into cross-equation restrictions on the time series dynamics of the macro variables. This allows us to recover the effects of structural shocks and interpret the dynamics of the macro variables in response to those shocks.

Our approach makes three main contributions to the analysis of the effects of structural shocks on macro variables. First, we estimate the complete structural model jointly, whereas many of the previous studies focused on equation by equation estimation. The estimation of the full system has the advantage of allowing for the interaction among the different economic agents: Consumers, firms and the Central Bank.

Second, we establish a tight link between structural equation parameters and the responses of the macro variables to economic shocks. This is possible because of the time series restrictions implied by the theoretical model. Our analysis can then address questions such as: “By how much does the output gap fall after a supply shock, when the Central Bank pursues a more stabilizing policy on inflation?”, “What is the relation between the Phillips curve parameter and the price puzzle?”, “Does a higher intertemporal elasticity of substitution imply a bigger impact of the monetary policy shock on the real economy?”, or “What is the contribution of the systematic part of the Central Bank reaction function to the propagation of monetary policy shocks?”

Third, at a technical level, we present a complete procedure for solving Rational Expectations models within the class of bubble-free solutions. In the case of multiple

solutions, we propose an alternative criterion that selects a solution which is bubble-free and consistent with expectations of the structural model.

We estimate the model by full information maximum likelihood (FIML).¹ The sensitivity analysis reveals that the more aggressively the Federal Reserve reacts to expected inflation and the output gap, the smaller impact the monetary policy shock has on the output gap and inflation. In addition, when the Federal Reserve greatly smooths interest rates, the effect of the monetary policy shock is amplified. We find that the more private agents smooth consumption across periods, the less effective the monetary policy shock is on real activity. The effect of the monetary surprise on inflation is very modest: The small estimate of the Phillips Curve parameter in the AS equation, which governs the direction and the size of the inflation response to the monetary policy shock in our baseline model, implies a high degree of inflation rigidity. The AS shock in turn moves the output gap and inflation in opposite directions. We show that the output gap decrease is exacerbated by a more than one to one reaction of the Central Bank to expected inflation. However, while our point estimate of the Fed's response to expected inflation during the 80's and 90's is above unity, it is not significantly so. Finally, a more stabilizing response of the Fed to deviations of the macro variables from their targets unambiguously dampens the impact of the IS shock on inflation and the output gap.

In order to conduct more precise inference about the implications of the structural model, we perform a bootstrap exercise which yields the empirical probability distribution of the structural parameters. Two main empirical facts emerge from this small sample analysis. First, it confirms that the estimate of the Fed's reaction to expected inflation is not significantly above one and reveals that it is upwardly biased. Second, the empirical distributions of both the Phillips curve parameter and the coefficient relating the output gap and the real interest rate in the IS equation are very different from their asymptotic distributions. This finding indicates that frequently used measures of output gap, such as linearly or quadratically detrended output, contain considerable measurement error.

Even though the original model is strongly rejected using the likelihood ratio (LR) test, our analysis shows that when the error terms of the model are allowed to be serially correlated, the model is only marginally rejected at the 5% level using the small sample distribution of the LR test statistic.

¹In order to avoid the potential problem of parameter instability, we select a sample period, 1980:4Q-2000:1Q, which does not include the most likely structural break in all the reduced form parameters of the model. This choice is based on the sup-Wald statistic derived by Bai, Lumsdaine, and Stock (1998).

The paper is organized as follows. Section 2 discusses the related literature. Section 3 lays out the complete but parsimonious model of the macroeconomy. In section 4 we discuss the Rational Expectations solution of the model and describe our new methodology to handle the potential multiplicity of solutions. Section 5 describes our estimation procedure. Section 6 discusses the data and the selection of the sample period based on the sup-Wald break date test statistic. In Section 7 we present our results. First we show the estimates of the structural model and implied dynamics. Then we perform a small sample study which allows us to develop our sensitivity analysis. Finally we carry out model diagnostics using the asymptotic and small sample LR tests. Section 8 concludes.

2 Related Literature

A popular strategy to identify structural policy shocks is to estimate empirical VAR systems. VAR studies recover the implied dynamics of the macro variables following structural shocks by placing a sufficient number of exclusion restrictions. Blanchard and Quah (1989), Leeper, Sims, and Zha (1996) or Christiano, Eichenbaum, and Evans (1999) are examples of this approach. In our study, by imposing a structural model, we do not need to impose these zero restrictions and all the variables are contemporaneously related. Even though the fit to the data cannot be as accurate as in large VAR systems, our Rational Expectations model solution provides a natural structural interpretation of the macro dynamics.

Our structural model is a linearized Rational Expectations model consisting of AS, IS and monetary policy rule equations with endogenous persistence. The AS equation is a modified version of the real wage contracting equation in Fuhrer and Moore (1995). We derive the IS equation through representative agent optimization with external habit persistence, as in Fuhrer (2000). The monetary policy rule in our model is the forward looking Taylor rule proposed by Clarida, Galí, and Gertler (2000). Our model, though parsimonious, is rich enough to capture the macro dynamics implied by recently developed New Keynesian models so that the dynamic paths of inflation, the output gap and the interest rate can be clearly explained in terms of the structural parameters.

Our estimation approach has several advantages with respect to previous studies focused on structural New Keynesian systems. First, we estimate the structural parameters, whereas McCallum (2001), who analyzes a similar model, calibrates them. This is an important difference, since the asymptotic and empirical probability distributions of

the structural parameters obtained through estimation provide a more reliable analysis of the model dynamics. Second, our FIML estimation is more efficient than instrumental variables techniques such as Generalized Method of Moments (GMM). The implications of our policy rule estimates differ from those of Clarida, Galí, and Gertler (2000), who estimate the policy reaction function by GMM. Third, we estimate the complete macro model jointly, whereas McCallum and Nelson (1998), Galí and Gertler (1999) and Rudebusch (2002) estimate the structural equations separately. The joint estimation has the advantage that it accounts for the simultaneous effect of all the structural shocks on each of the variables in estimation.²

A closely related paper is Ireland (2001), who estimates a New Keynesian model based on explicit microfoundations by maximum likelihood. His analysis contains, however, two main differences with respect to our study. First, the equations of our model display endogenous persistence, whereas in his case the persistence is imposed exogenously. Second, and more importantly, while he focuses on studying the instability of the model parameters, our main interest is to interpret the macroeconomic dynamics following the structural shocks in terms of the structural parameters of the model. In order to draw a sharper inference, we perform a small sample study of the structural parameters which accounts for parameter uncertainty.

Finally, we construct an alternative procedure to solve Rational Expectations systems that selects the economically relevant solution in the case of multiplicity of stationary solutions. It differs from the minimal state variable criterion developed by McCallum (1983) in that it yields a bubble-free solution by solving the model forward recursively.

3 The Model

We characterize the set of macroeconomic relations through the following system of 3 equations: The AS or Supply equation, the IS or Demand equation and the forward

²Rotemberg and Woodford (1998), Amato and Laubach (1999), Christiano, Eichenbaum, and Evans (2001), Smets and Wouters (2001) and Boivin and Giannoni (2003) estimate structural New Keynesian models grounded in optimizing behavior. Their estimation approach differs from FIML in that they minimize a measure of distance between empirical VARs and their models. Fuhrer (2000) uses an alternative FIML procedure to estimate a structural model of consumption. We depart from his method in the way we deal with the expectations terms in the maximum likelihood estimation. He approximates the expectation terms with a number of lagged variables included in the agents' information set. Our methodology provides the exact solution to the Rational Expectations system so that the structural errors can be identified.

looking monetary policy equation. Each of them exhibits endogenous persistence, which allows for more realistic dynamics in the macroeconomy, and a forward looking part. We assume that there is no informational difference between the private sector (firms and households) and the Central Bank.

3.1 AS Equation

The AS equation or “New Phillips Curve” describes the short run inflation dynamics as a result of the wage setting process between firms and workers. We generalize the AS equation rationalized by Fuhrer and Moore (1995), who present a model of overlapping wage contracts in which agents care about relative real wages:

$$\pi_t = \alpha_{AS} + \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda(y_t + y_{t-1}) + \epsilon_{AS_t} \quad (1)$$

α_{AS} is a constant. π_t and y_t stand for inflation and the output gap between $t - 1$ and t , respectively, and ϵ_{AS_t} is the aggregate supply structural shock, assumed to be independently and identically distributed with homoskedastic variance σ_{AS}^2 . E_t is the Rational Expectations operator conditional on the information set at time t , which comprises π_t , y_t , r_t (the nominal interest rate at time t) and all the lags of these variables. As can be seen in equation (1) inflation depends not only on expected future inflation but also on lagged inflation with weights δ and $1 - \delta$, respectively.³ ϵ_{AS_t} can be interpreted as a cost push shock that deviates real wages from their equilibrium value or simply as a pricing error. One advantage of this specification is that it captures the inflation persistence that is present in the data. Galí and Gertler (1999) also impart persistence to the inflation rate by letting a fraction of firms use a backward looking rule of thumb to set prices.

3.2 IS Equation

The IS equation describes the demand side of the economy. It is derived by a representative agent model in Fuhrer (2000), with the utility function given by

$$U(C_t) = \frac{\left(\frac{C_t}{H_t}\right)^{1-\sigma} - 1}{1 - \sigma} \quad (2)$$

³For $\alpha_{AS} = 0$ and $\delta = 0.5$ we recover the original Fuhrer and Moore (1995) specification.

where C_t is the consumption level, H_t is the external level of habit and σ is the inverse of the elasticity of substitution. The habit level is external in the sense that the consumer does not consider it as an argument to maximize his utility function. We assume that $H_t = C_{t-1}^h$ ex post, where $h (> 0)$ measures how strong the habit level is. It is the habit specification that will introduce persistence in the IS equation. The budget constraint that the agent faces is

$$C_t + B_t \leq \frac{P_{t-1}}{P_t} B_{t-1} R_t + W_t \quad (3)$$

This constraint implies that agents' consumption at time t , C_t , plus the value of his asset holdings, B_t , cannot exceed his endowment each period, which comes from labor income, W_t , and the real value of the asset holdings that he had at the beginning of the period, $\frac{P_{t-1}}{P_t} B_{t-1}$, multiplied by the nominal gross return on those assets, R_t .

The agent is infinitely lived and maximizes his lifetime stream of utility, subject to the dynamic budget constraint in (3). The Euler equation is given by

$$1 = E_t \left[\psi \frac{U'(C_{t+1})}{U'(C_t)} \frac{P_t}{P_{t+1}} R_t \right] \quad (4)$$

where ψ is the time discount factor and P_t is the price level at time t . By assuming joint lognormality of consumption and inflation, the following expression can be derived:

$$c_t = \alpha_C + \mu E_t c_{t+1} + (1 - \mu) c_{t-1} - \phi(r_t - E_t \pi_{t+1}) \quad (5)$$

where $\alpha_C = \frac{-\ln(\psi) - \frac{1}{2} V_t(\sigma c_{t+1} + \pi_{t+1})}{\sigma(1+h)-h}$, c_t is the log of consumption at time t , V_t is the conditional variance operator at time t , $\mu = \frac{\sigma}{\sigma(1+h)-h}$ and $\phi = \frac{1}{\sigma(1+h)-h}$. As can be seen in equation (5), the monetary transmission mechanism is a function of the inverse of the elasticity of substitution of consumption across periods, σ , and the habit persistence parameter, h .

From the market clearing condition, $Y_t^* = C_t + G_t$, where Y_t^* is the aggregate supply and G_t denotes the remaining demand components: investment, government expenditures and net exports. Taking logs, $c_t = y_t^* + z_t$, where y_t^* is the log of GDP at time t and $z_t = \log\left(\frac{Y_t^* - G_t}{Y_t^*}\right)$. Let $y_t^* = y_t^T + y_t$, where y_t^T denotes the potential output or trend component of y_t^* and y_t is the output gap. Then, equation (5) can be rewritten as:

$$y_t = \alpha_C + \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi(r_t - E_t \pi_{t+1}) + g_t \quad (6)$$

where $g_t = -(z_t + y_t^T) + \mu E_t(z_{t+1} + y_{t+1}^T) + (1 - \mu)(z_{t-1} + y_{t-1}^T)$. Note that y_t rises with G_t . Finally, define $\alpha_g = E g_t$, where E is the unconditional expectation operator, in order to rewrite the IS equation as

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu)y_{t-1} - \phi(r_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (7)$$

where $\alpha_{IS} = \alpha_C + \alpha_g$ and $\epsilon_{IS,t} = g_t - \alpha_g$.⁴ We will interpret $\epsilon_{IS,t}$ as an exogenous shock to aggregate demand throughout the paper.⁵ Since we do not model G_t explicitly, $\epsilon_{IS,t}$ is assumed to be independent and identically distributed with homoskedastic variance σ_{IS}^2 .

3.3 Monetary Policy Equation

The instrument of the monetary authority, the Federal funds rate,⁶ is set according to the following reaction function proposed by Clarida, Galí, and Gertler (2000):

$$r_t = \rho r_{t-1} + (1 - \rho)r_t^* + \epsilon_{MP,t} \quad (8)$$

$$r_t^* = \bar{r}^* + \beta(E_t \pi_{t+1} - \bar{\pi}) + \gamma y_t \quad (9)$$

$\bar{\pi}$ is the long run equilibrium level of inflation and \bar{r}^* is the desired nominal interest rate. There are two parts to the equation. The lagged interest rate captures the well known tendency of the Federal Reserve towards smoothing interest rates, whereas r_t^* , represents the ‘‘Taylor rule’’ whereby the monetary authority reacts to deviations of expected inflation from the long run equilibrium level of inflation and to the current output gap. Hence, the monetary policy equation becomes:

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{MP,t} \quad (10)$$

⁴Equation (7) can also be expressed as $y_t = \mu E_t y_{t+1} + (1 - \mu)y_{t-1} - \phi(r_t - E_t \pi_{t+1} - \bar{r}) + \epsilon_{IS,t}$, where $\bar{r} = \frac{\alpha_{IS}}{\phi}$. \bar{r} represents the long run equilibrium real rate of interest.

⁵Woodford (2001) points out that $\epsilon_{IS,t}$ cannot be interpreted in general as a demand shock, since it includes shocks to the trend component of output which could be driven, for instance, by technological innovations. However, under linear output detrending, the innovations to the output trend vanish from $\epsilon_{IS,t}$, so that $\epsilon_{IS,t}$ could be interpreted as a demand shock. Under quadratic output detrending, even though a deterministic trend component remains in $\epsilon_{IS,t}$, its size is negligible so that our empirical results are virtually unaffected. The interpretation of $\epsilon_{IS,t}$ as a demand shock under other filters, such as the Congressional Budget Office measure of potential output, is clearly more problematic.

⁶We assume that the Federal funds rate is equal to the short term interest rate.

$\alpha_{MP} = (1 - \rho)(\bar{r}^* - \beta\bar{\pi})$ and ϵ_{MP_t} is the monetary policy shock, that we are trying to identify. It is assumed to be independently and identically distributed with homoskedastic variance σ_{MP}^2 .

4 Rational Expectations Solution

4.1 Model Solution and Implications

In this section we derive the Rational Expectations solution of the model and analyze its properties. Our macroeconomic system of equations (1), (7) and (10) can be expressed in matrix form as follows:

$$\begin{bmatrix} 1 & -\lambda & 0 \\ 0 & 1 & \phi \\ 0 & -(1-\rho)\gamma & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \alpha_{AS} \\ \alpha_{IS} \\ \alpha_{MP} \end{bmatrix} + \begin{bmatrix} \delta & 0 & 0 \\ \phi & \mu & 0 \\ (1-\rho)\beta & 0 & 0 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ r_{t+1} \end{bmatrix} + \begin{bmatrix} 1-\delta & \lambda & 0 \\ 0 & 1-\mu & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{AS_t} \\ \epsilon_{IS_t} \\ \epsilon_{MP_t} \end{bmatrix}$$

In more compact notation:

$$B_{11}X_t = \alpha + A_{11}E_tX_{t+1} + B_{12}X_{t-1} + \epsilon_t, \quad \epsilon_t \sim (0, D) \quad (11)$$

where $X_t = (\pi_t \ y_t \ r_t)'$, B_{11} , A_{11} and B_{12} are the coefficient matrices of structural parameters, and α is a vector of constants. ϵ_t is the vector of structural errors, D is the diagonal variance matrix and 0 denotes a 3×1 vector of zeros.⁷ By assuming Rational Expectations and no asymmetric information between the economic agents and the monetary policy authority, we can write:

$$X_{t+1} = E_tX_{t+1} + v_{t+1} \quad (12)$$

where v_{t+1} is the vector of Rational Expectations errors. Following a standard Undetermined Coefficients approach, a bubble-free solution to the system in (11) can be written as the following reduced form:

$$X_{t+1} = c + \Omega X_t + \Gamma \epsilon_{t+1} \quad (13)$$

⁷In what follows, 0 will denote a matrix, vector or scalar of the appropriate dimension.

where c is a 3×1 vector of constants and Ω and Γ are 3×3 matrices. To see this, substitute equation (13) into equation (11) and rearrange by applying Rational Expectations. Then:

$$(B_{11} - A_{11}\Omega)X_t = \alpha + A_{11}c + B_{12}X_{t-1} + \epsilon_t \quad (14)$$

Linear independence among the 3 structural equations implies nonsingularity of $(B_{11} - A_{11}\Omega)$. Thus we assume that $(B_{11} - A_{11}\Omega)$ is nonsingular in what follows. Then, pre-multiply by $(B_{11} - A_{11}\Omega)^{-1}$ on both sides in equation (14) and match the coefficient matrices of X_{t-1} and ϵ_t , to obtain:

$$\Omega = (B_{11} - A_{11}\Omega)^{-1}B_{12} \quad (15)$$

$$\Gamma = (B_{11} - A_{11}\Omega)^{-1} \quad (16)$$

$$c = (B_{11} - A_{11}\Omega - A_{11})^{-1}\alpha \quad (17)$$

Therefore, equation (13) with Ω , Γ and c satisfying equations (15), (16) and (17) is a solution to equation (11). Once we solve for Ω as a function of A_{11} , B_{11} and B_{12} , Γ and c can be easily calculated. A detailed solution method will be given in the following subsection. Notice that the implied reduced form of our structural model is simply a VAR of order 1 with highly nonlinear parameter restrictions. Furthermore, there is a simple linear relation between Ω and Γ through B_{12} , which captures the dependence of the system on the lagged predetermined variables.

$$\Omega = \Gamma B_{12} \quad (18)$$

Note also that there is a linear relation between the structural errors, ϵ_t and the reduced form errors (Rational Expectations errors), v_t , through Γ ,

$$v_t = \Gamma \epsilon_t \quad (19)$$

A pure forward looking system implies $B_{12} = 0$ in our system, and the implied reduced form is simply $X_{t+1} = c + \Gamma \epsilon_{t+1}$ where $\Omega = 0$ and $\Gamma = B_{11}^{-1}$. Since in this case there are no dynamics, forward looking models such as Roberts (1995) and McCallum (2001) fail to explain the empirical persistence in inflation, the output gap and the interest rate. Consequently, in the literature, structural errors are often assumed to be serially correlated to fit the data. Alternatively, when a non-structural VAR approach is used,

statistical selection methods such as the Schwarz and Akaike criteria often lead monetary policy analysis to choose higher order VARs.⁸ Although such models fit the data better, as Clarida, Galí, and Gertler (1999) point out, it is a daunting task to justify macroeconomic models which include more than one lag. The model we are working with in this paper lies between these two approaches: The three equations we consider have a theoretical justification and they also feature persistence of the variables.

4.2 Characterization of the Rational Expectations Solution

Rewrite equation (15) as:

$$A_{11}\Omega^2 - B_{11}\Omega + B_{12} = 0 \quad (20)$$

Once Ω is solved for, Γ and c , the remaining unknown matrices in equation (13), follow directly from equations (16) and (17). For Ω satisfying (20) to be admissible as a solution, it must be real-valued and exhibit stationary dynamics. Because Ω is a nonlinear function of the structural parameters in B_{11} , A_{11} and B_{12} , there could potentially be multiple stationary solutions or no stationary solutions at all. Additionally, the existence of a complex valued solution cannot be ruled out. The singularity of the matrix A_{11} is another difficulty in solving (20).

We employ two different methods to solve equation (20). First, we utilize the generalized Schur (QZ) Decomposition in solving Rational Expectations models. We will follow Uhlig (1997), as he proposes an approach which is closely related to the QZ method.⁹ The QZ method is particularly useful when the matrix A_{11} is singular, which is the case in our model, and it allows us to determine whether there exists a stationary, real-valued bubble-free solution. Specifically, define the $2n \times 2n$ matrices $A = \begin{bmatrix} A_{11} & 0 \\ 0 & I \end{bmatrix}$

and $B = \begin{bmatrix} B_{11} & -B_{12} \\ I & 0 \end{bmatrix}$ where n is the number of endogenous variables. The set of all matrices of the form $B - \lambda A$ with $\lambda \in \mathcal{C}$ is said to be a matrix pencil and λ is called a generalized eigenvalue of the pencil. Define Λ be the diagonal matrix whose diagonal elements are the eigenvalues and S be the eigenmatrix with each column corresponding

⁸Rudebusch and Svensson (1999), for instance, use a backward looking model.

⁹Even though Uhlig's formula only requires to compute the generalized eigenvalues and eigenvectors, these are key concepts in the QZ decomposition. In this sense, we will call this approach the QZ method henceforth. Additionally, McCallum (1999) provides a formula that leads to the same solution as Uhlig's by applying the QZ decomposition.

to its eigenvalue such that $BS = AS\Lambda$. Then:

$$\Omega = S_{21}\Lambda_{11}S_{21}^{-1} \quad (21)$$

satisfies equation (20) where S_{ij} and Λ_{ij} are the $n \times n$ ij -th submatrices of S and Λ , respectively. We can characterize the stationarity, uniqueness and real-valuedness as follows: If all the eigenvalues of Λ_{11} are less than unity in absolute value, then Ω is stationary. If the number of stable generalized eigenvalues is the same as that of the predetermined variables (3 in our model, the lagged endogenous variables), then there exists a unique solution. If there are more than 3 stable generalized eigenvalues, then we have multiple solutions. Conversely, if there are less than 3 stable eigenvalues, there is no stable solution. Finally, Ω is real-valued if (a) every eigenvalue in Λ_{11} is real-valued, or (b) for every complex eigenvalue in Λ_{11} , the complex conjugate is also an eigenvalue in Λ_{11} .

Unfortunately, in the case of multiple stationary solutions, there seems to be no agreement about the selection of a solution among all the candidates.¹⁰ In this case, by solving the model forward recursively, we propose an alternative simple selection criterion that is bubble-free and real-valued by construction. The idea is to construct sequences of convergent matrices, $\{C_k, \Omega_k, \Gamma_k, k = 1, 2, 3, \dots\}$ such that:

$$\bar{X}_t = C_k E_t \bar{X}_{t+k+1} + \Omega_k \bar{X}_{t-1} + \Gamma_k \epsilon_t \quad (22)$$

where $\bar{X}_t = X_t - EX_t$. We characterize the solution that is fully recursive as follows. We check first whether $\Omega^* \equiv \lim_{k \rightarrow \infty} \Omega_k$ and $\Gamma^* \equiv \lim_{k \rightarrow \infty} \Gamma_k$ exist, and Ω^* is the same as one of the solutions obtained through the QZ method. For the limit to equation (22) to be a bubble-free solution, $\lim_{k \rightarrow \infty} C_k E_t \bar{X}_{t+k+1}$ must be a zero vector. Then the solution must be of the form:

$$\bar{X}_t = \Omega^* \bar{X}_{t-1} + \Gamma^* \epsilon_t \quad (23)$$

Finally, we check whether $\lim_{k \rightarrow \infty} C_k E_t \bar{X}_{t+k+1} = \lim_{k \rightarrow \infty} C_k \Omega^{*k} = 0$ using equation (23). We will call this the recursive method. The complete procedure is detailed in the Appendix.

While the QZ method can determine whether there exists a stationary real-valued

¹⁰Blanchard and Kahn (1980) suggest the choice of the 3 smallest eigenvalues and McCallum (1999) suggests the choice that would yield $\Omega = 0$ if it were the case that $B_{12} = 0$. Uhlig (1997) points out that McCallum's criterion is difficult to implement but it often coincides with Blanchard and Kahn's criterion.

solution within the class of no bubble solutions, it does not give any information about which solution should be chosen in the case of multiple solutions. In this case we select the solution from the recursive method. On the other hand, while the recursive method produces a stationary solution if it exists, one does not know whether the solution is unique or not. Hence these two methods are complementary so that they can be used jointly as a criterion to select the economically relevant solution.

5 Full Information Maximum Likelihood Estimation

We estimate the structural parameters using FIML by assuming normality of the structural errors. Our FIML estimation procedure allows us to obtain the structural parameters and the VAR reduced form in one stage, affording a higher efficiency than two-stage instrumental variables techniques such as GMM. Galí and Gertler (1999) and Clarida, Galí, and Gertler (2000) estimate separately by GMM some of the equations of the model that we study. It seems adequate to estimate the whole model jointly, given the simultaneity between the private sector and the Central Bank behavior, as explained by Leeper and Zha (2000).

The log likelihood function can be written as:

$$\ln L(\theta|\bar{X}_T, \bar{X}_{T-1}, \dots, \bar{X}_1) = \sum_{t=2}^T \left[-\frac{3}{2} \ln 2\pi - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} (\bar{X}_t - \Omega \bar{X}_{t-1})' \Sigma^{-1} (\bar{X}_t - \Omega \bar{X}_{t-1}) \right] \quad (24)$$

where $\theta = (\delta, \lambda, \mu, \phi, \rho, \beta, \gamma, \sigma_{AS}^2, \sigma_{IS}^2, \sigma_{MP}^2)$, the vector of the structural parameters and $\Sigma = \Gamma D \Gamma'$. Ω and Γ can be calculated by the QZ method or our recursive method. Note that we maximize the likelihood function with respect to the structural parameters, not the reduced form ones. Therefore, given the structural parameters, the matrices Ω and Γ must be calculated at each iteration. This requires to check whether there is a unique, real-valued stationary solution at each iteration. Whenever there are multiple solutions at the i -th iteration, we apply the recursive method to select one solution. We choose the initial parameters from the values used in the literature. In order to check for robustness of our estimates we set up different initial conditions, randomizing around the obtained parameter estimates five times. In all of the cases convergence to the same parameter estimates was attained. We also found that the estimates obtained through our recursive method converge to the c , Ω and Γ matrices obtained through the QZ method.

6 Data description and sample selection

We estimate the model with U.S. quarterly data from 1980:4Q to 2000:1Q. Implicit GDP deflator data is used for inflation. The inflation rate is computed as the log difference of the GDP deflator between the end and the beginning of each quarter. The Federal funds rate is the monetary policy instrument: We use the average of the Federal funds rate over the previous quarter. Our results are by and large robust to the use of the Consumer Price Index (CPI) for inflation and the 3 Month T-Bill rate for the short term interest rate. We use three different measures for the output gap: Output detrended with the Congressional Budget Office (CBO) Measure of Potential GDP, linearly and quadratically detrended real GDP.¹¹ The data is annualized and in percentages. Federal funds rate data was collected from the Board of Governors of the Federal Reserve website. Real GDP and the GDP deflator were obtained from the National Income and Product Accounts (NIPA).

Clarida, Galí, and Gertler (1999), Boivin and Watson (1999) and others have shown evidence of parameter instability across sample periods. We select our sample period based on the sup-Wald statistic for parameter instability, derived by Bai, Lumsdaine, and Stock (1998). This statistic detects the most likely date for a break in all the parameters of a reduced form VAR. We run the sup-Wald statistic for unconstrained VARs of order 1 to 3. As shown in Table 1 and in Figure 1, the beginning of the 4th quarter of 1980, one year after Paul Volcker's beginning of his tenure as Federal Reserve chairman, is clearly identified as the most likely break date for the parameters of the reduced form relation. In all three cases, the value of the Sup-Wald statistic is significant at the 1% level¹² and the 90% confidence interval is very tight, including only three quarters. The break date test is also robust across output gap measures. This date coincides with the biggest increase, between two quarters, in the average Federal funds rate during the whole sample: From 9.83% in the 3rd quarter of 1980 to 15.85% in the 4th. This severe contraction engineered by the Federal Reserve lies at the root of the

¹¹The Hodrick Prescott filter, linear filter, quadratic filter and the CBO Measure of Potential GDP have been used extensively in the literature. There seems to be no consensus about the choice of filter to generate the output gap, since all of them seem to contain some measurement error. Galí and Gertler (1999) introduce a real marginal cost measure in a Calvo-type AS equation which yields better results than any of the filtered variables above. However, since our AS specification comes from a contracting model, and we estimate the complete macro model, we do not use their measure.

¹²The associated asymptotic critical values can be found in Bekaert, Harvey, and Lumsdaine (2002).

early 80's disinflation.¹³ We start the sample right after the break date occurs.¹⁴

[Insert Table 1 Here]

[Insert Figure 1 Here]

7 Empirical Results

In this section we present our empirical findings. First, we report the structural parameter estimates and their statistical properties. Then we provide the parameters' small sample distributions based on a bootstrap exercise. The second part of this section is devoted to the analysis of the impulse response functions of the variables to the monetary policy shock as well as the other structural shocks. In the following subsection, we present our main empirical results: We show how changes in the structural parameters around the estimated values affect the propagation mechanism of structural shocks through a sensitivity analysis. Finally, we perform specification tests of the structural model based on the asymptotic and small sample LR test statistic.

7.1 Parameter estimates

7.1.1 Structural parameters

FIML estimates are shown in Table 2.¹⁵ Asymptotic standard errors are obtained as the inverse of the Hessian Matrix. We present three sets of estimates in columns (1), (2) and (3): The first one is obtained using linearly detrended output, the second one uses quadratically detrended output and the third one uses output detrended with the CBO measure of potential output. As is clear from Table 2, the estimates are reasonably robust across output gap specifications.

[Insert Table 2 Here]

The parameter estimates are by and large consistent with previous findings in the literature. In the AS equation, δ is significantly greater than 0.5, implying that agents place

¹³Right after Volcker's arrival, the Federal Reserve also increased the Federal funds rate sharply, but it was decreased shortly thereafter. Feldstein (1994) dubs this episode the unsuccessful disinflation.

¹⁴Empirical results are similar if we start the sample outside the 90% confidence interval.

¹⁵Even though we estimate the model constants, we do not report the estimates, since we are interested in the system dynamics, where the constants are irrelevant.

a larger weight on expected inflation than on past inflation. Galí and Gertler (1999) found similar estimates. The Philips Curve parameter, λ , has the right sign in two of the three specifications, but it is not statistically different from 0 in any of the three cases. Fuhrer and Moore (1995) obtained estimates of similar magnitude using the same pricing specification. In the IS equation, μ is statistically indistinguishable from 0.5, implying that agents place similar weights on expected and past output gap. The implied habit persistence parameter, h , is around 1.07 for the three output gap specifications and statistically different from 0 at the 5% level.¹⁶ Fuhrer (2000) reports 0.80 for the habit persistence parameter in his model. The estimates of the implied inverse of the elasticity of substitution, σ , range from 73 (when output is detrended with the CBO measure of potential output) to 110 (when output is detrended linearly).¹⁷ However it is not significantly different from 0 in any of the three specifications. This value is considerably larger than the ones usually employed in calibration (see McCallum (2001)), but similar to the ones found in estimation of the linearized IS equation.¹⁸ In the monetary policy equation, the smoothing parameter, ρ , is around 0.85, reflecting the well known persistence in the short term interest rate. β , the coefficient on expected inflation, is larger than 1, but only significantly above unity at the 5% level when the output gap is detrended with the CBO measure of potential output. γ , the coefficient on output gap, is also positive and only significantly different from 0 in the specification which uses the CBO measure of potential output. While these estimates are similar to the ones found by Clarida, Galí, and Gertler (1999) for the same monetary policy rule, our standard errors are considerably larger.

7.1.2 Model solution

For the first two specifications the sets of FIML estimates imply a unique stationary solution, as we describe in the Appendix. For the remainder of our discussion we will focus on the parameter estimates obtained when output is linearly detrended since their signs are fully in agreement with the theoretical model. Additionally, the linear detrending method for output allows us to interpret ϵ_{IS_t} as a pure demand shock, as explained in footnote 5. The estimates of the implied reduced form matrices, Ω and Γ , which drive

¹⁶Recall that $\mu = \frac{\sigma}{\sigma(1+h)-h}$ and $\phi = \frac{1}{\sigma(1+h)-h}$. Thus $\sigma = \frac{\mu}{\phi}$ and $h = \frac{1-\mu}{\mu-\phi}$.

¹⁷The statistical significance of σ in our model is difficult to interpret. Since $\sigma = \frac{\mu}{\phi}$, σ is not normally distributed. Additionally, since ϕ is not statistically different from zero, inference based on first order approximation is not reliable.

¹⁸See Smets (2000) and Estrella and Fuhrer (1999).

the dynamics of the model, are¹⁹:

$$\begin{bmatrix} \pi_t \\ y_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0.670 \\ 0.258 \\ 0.579 \end{bmatrix} + \begin{bmatrix} 0.782^* & 0.056 & -0.011 \\ -0.002 & 0.961^* & -0.031 \\ 0.154^* & 0.114^* & 0.838^* \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} 1.772^* & 0.106 & -0.013 \\ -0.004 & 1.870^* & -0.037 \\ 0.350^* & 0.221^* & 0.991^* \end{bmatrix} \begin{bmatrix} \epsilon_{AS_t} \\ \epsilon_{IS_t} \\ \epsilon_{MP_t} \end{bmatrix} \quad (25)$$

Panel A and B of Table 3 show the autocorrelation and cross correlation patterns exhibited by the structural errors, respectively. Panel C and D report some diagnostic tests of the residuals. The diagnostic tests give mixed results. Even though the Jarque-Bera test cannot reject the hypothesis of normality for the AS and IS residuals, the Ljung-Box Q-statistic rejects the hypothesis that their first five autocorrelations are zero.²⁰ Under the null of the model, there should not be significant autocorrelations or cross-correlations, but this is a very difficult test to pass given our parsimonious VAR(1) specification. The cross correlations of the error terms reveal nonzero contemporaneous correlations among the structural shocks.

The top row of Figure 2 compares the one step ahead predicted values of the model with the actual values of inflation, the output gap and the interest rate. The predicted values generated by the model track the real values very closely. The bottom row of Figure 2 graphs the structural errors of the model. It shows that there were not major AS shocks during the sample. The IS shocks exhibit some persistence, as reported in Panel A of Table 3. Finally, it can be seen that the monetary policy shocks were of very small magnitude after 1983. This corroborates the analysis in Taylor (1999) and Leeper and Zha (2000) showing that monetary policy shocks during the 90's were small.

[Insert Table 3 Here]

[Insert Figure 2 Here]

7.1.3 Small Sample Distributions of the Structural Parameters

Because our sample is relatively short, inference based on asymptotic distribution may be misleading. In order to draw a more precise inference on the validity of the structural

¹⁹The stars denote the parameters that are significantly different from zero at the 5% level. The standard errors can be calculated using delta method. Even though Ω and Γ cannot be expressed analytically in terms of structural parameters, we can derive numerical derivatives of Ω and Γ with respect to the structural parameters.

²⁰In the last subsection of the Empirical Results we allow the structural errors to be serially correlated.

parameters, we perform a bootstrap analysis. We bootstrap 1,000 samples under the null and re-estimate the structural model to obtain an empirical probability distribution of the structural parameters. The Appendix details the bootstrap procedure. In Figure 3 we present the small sample distributions of the structural parameters. In the last two columns of Table 2 we report the small sample means of the parameters and their associated 95% confidence intervals, respectively.

[Insert Figure 3 Here]

The empirical distributions of δ and ρ are mildly positively and negatively skewed, respectively. This bias is related to the well known small sample downward bias of the first order autocorrelation coefficients, as reported in Bekaert, Hodrick, and Marshall (1997). The most severe small sample problem is the strong positive skewness exhibited by the empirical distribution of the Phillips curve parameter, λ , and that of the coefficient on the real interest rate in the IS equation, ϕ . These two parameters were not significantly different from zero in the FIML estimation. This bias present in λ and ϕ seems to be related to the measurement error contained in the detrended output measure. In the sensitivity analysis, we show the implications of different values of λ and ϕ on the macro dynamics. The coefficient on expected inflation in the monetary policy rule, β , appears significantly upwardly biased, and its small sample 95% confidence interval is clearly wider than its asymptotic counterpart. In the sensitivity analysis below, we show that alternative values of β give rise to qualitatively and quantitatively different dynamics in response to the macroeconomic shocks. Finally, the averages of the empirical distributions of γ and those of the three structural shocks standard deviations are very similar to the FIML parameter estimates.

7.2 Impulse Response Analysis to Structural Shocks

Figure 4 shows the structural impulse response functions of the variables to a one standard deviation of each shock, and the associated 95% confidence intervals.²¹ The units for the responses of the variables are in percentage deviations from their steady state values. For ease of exposition, based on equation (25), we denote $\Omega_{i,j}$ as the ij -th element of Ω where $i, j = \pi, y, r$, and $\Gamma_{i,j}$ as the ij -th element of Γ where $i = \pi, y, r$ and $j = AS, IS, MP$.

²¹All the impulse responses and the confidence intervals exhibit mean reversion, reflecting the stationarity of the economy.

For instance, $\Omega_{y,r}$ is the coefficient on the lagged interest rate in the IS equation and $\Gamma_{\pi,MP}$ is the initial response of inflation to the monetary policy shock.

[Insert Figure 4 Here]

A contractionary monetary policy shock decreases inflation, but the impact is very small and not significantly different from zero for the whole time span. As we show in the sensitivity analysis, this is due to the sign and magnitude of the Phillips curve parameter, λ : A positive λ ensures that prices decrease immediately after the contractionary policy shock. This precludes the appearance of the price puzzle, often obtained in empirical systems, whereby prices initially rise after an unexpected increase in the interest rate. The initial effect of the monetary policy shock on the variables can be easily identified by observing the coefficient matrix of the structural shocks, Γ . In the case of the initial effect on inflation, $\Gamma_{\pi,MP} = -0.013$. Subsequently, the dynamic path of inflation is governed by the first row of Ω .

The monetary policy shock has real effects on the economy: When the Fed surprises the economy by increasing the interest rate by 0.73%, the output gap decreases, reaching its trough of -0.1% with respect to its steady state value after 10 quarters. $\Gamma_{y,MP}$ is -0.037, so that the output gap decreases immediately after the shock. Finally, the monetary policy shock has a persistent effect on the interest rate, given the smoothing behavior of the Fed.

Since our supply curve is derived from a real wage contract and we do not model technology explicitly, the AS shock can be interpreted as a sudden increase in wages and thus the price level. The AS shock increases prices during 10 quarters. Interestingly, the Federal funds rate responds strongly (and significantly) to the AS shock ($\Gamma_{r,AS}=0.35$). This makes the output gap decrease for a long period of time, reaching its trough, -0.8% , after 15 quarters. As we will show in the next section, this depressing effect of the AS shock is triggered by the aggressive reaction of the Fed to the inflationary pressure ($\beta > 1$).

Following a positive IS or demand shock, inflation, the output gap and the interest rate exhibit a positive co-movement. This is directed by the coefficients $\Gamma_{\pi,IS}$, $\Gamma_{y,IS}$ and $\Gamma_{r,IS}$, respectively, all of them larger than zero. As in the case of $\Gamma_{\pi,MP}$, the sign of $\Gamma_{\pi,IS}$ is governed by the Phillips curve parameter, λ .

7.3 Sensitivity Analysis

Each element of Ω and Γ is a complicated function of the structural parameters. Therefore when there is a change in the systematic part of the economy, identified as a change in a structural parameter in A_{11} , B_{11} or B_{12} , all of the reduced form parameters change simultaneously and thus affects the propagation mechanism of any structural shock. Specifically, we vary the parameters describing the systematic part of the monetary policy rule, β , γ and ρ , one at a time. We also vary some of the structural parameters in the AS and IS equations and see how the impulse responses react to these changes. This experiment amounts to an analysis of the partial derivatives of Ω , Γ , and of all the impulse response functions, with respect to the estimated structural parameter vector θ . The impact of the Lucas Critique is minimized in our setting, since the restricted reduced form varies with changes in the structural parameters.

In order to account for the uncertainty regarding the structural parameters, the range of values for this analysis is chosen from the confidence interval of the empirical distribution of the structural parameters, shown in column (6) of Table 2. Recall that the estimates provide a unique stationary solution. In some cases, a change in a parameter value within the confidence interval may result in a nonstationary solution or multiple solutions. Thus we choose parameter values so as to guarantee a stationary solution. In case of multiplicity of solutions, we select the one obtained through our recursive method.²² We believe that this exercise provides more information than a simple calibration exercise, where lack of knowledge about the estimated value of the parameters can make the impulse responses highly misleading.

7.3.1 Changes in Systematic Monetary Policy

Clarida, Galí, and Gertler (1999) and others have found that the systematic reaction to expected inflation in the monetary policy rule, β , is greater than one after 1979, implying that the Fed has been stabilizing since then. Our FIML estimate of β is also bigger than 1, but it is not significantly above unity at the 5% level. Panel A of Figure 5 shows the impulse responses when $\beta = 0.5$, 1 and 3 (which are all in the 95% confidence interval of the empirical distribution). As β increases, the Fed responds more aggressively to demand and supply shocks. A higher β also makes the private sector's responses to the

²²For instance, when β is less than one, there are multiple solutions, since there are 4 eigenvalues less than 1 in moduli.

monetary policy shock less pronounced. This is due to the fact that the contractionary policy shock lowers expected inflation below the steady state in the future. Larger values of β partially offset the impact of the monetary policy shock, since a stronger reaction from the Fed to lower expected inflation moves the interest rate in the opposite direction to the one implied by the shock. Conversely, if the Fed is not very responsive ($\beta = 0.5$), the impact of the policy shock is magnified. These results confirm the findings of Boivin and Giannoni (2003), who, with a different methodology, conclude that the more reactive Fed of the 80's and 90's has been the main source of the decreased the impact of the monetary policy shock.

The magnitude of β plays a pivotal rule in the output gap response to the AS shock. When β is larger than one, the output gap decreases for a long time. Therefore, a monetary policy that is very responsive to inflationary pressures may result, under an AS shock, in costly recessionary effects. This result is consistent with Clarida, Galí, and Gertler (1999), who find that a β larger than one, which is required for monetary policy optimality, makes the AS shock move inflation and the output gap in opposite directions. On the other hand, a higher β dampens the effects of the IS shock on inflation and the output gap, since it produces a larger interest rate increase through the more aggressive policy reaction to the higher future expected inflation generated by the IS shock.

[Insert Figure 5 Here]

Panel B of Figure 5 performs an analogous exercise with γ ranging from 0 to 1.5. A higher γ has a stabilizing effect on inflation and the output gap after both IS and monetary policy shocks. It also reduces the recessionary effects of the AS shock at the expense of a very small increase in the variability of inflation. The small magnitude of this latter effect is due to the high degree of inflation rigidity implied by the small estimate of λ . Overall, given our parameter estimates of the U.S. economy, larger values of γ result in a reduced variability of inflation and the output gap.

In Panel C of Figure 5, the interest rate smoothing parameter, ρ , takes the values of 0.68, 0.8 and 0.92. Our experiment clearly shows that, in the presence of a monetary policy shock, too persistent an interest rate depresses the output gap greatly. As ρ grows, the initial shock is preserved through time and the contractionary initial impact is subsequently amplified.

7.3.2 Changes in Private sector behavior

In the literature, researchers have used alternative private sector parameter values in order to analyze macroeconomic models. Our framework allows us to examine the sensitivity of the impulse response functions to varying these structural parameters within an empirically relevant range.²³ In Panel A of Figure 6 we see how the response of inflation following an IS or a monetary policy shock qualitatively changes with a different sign of λ , the Phillips curve parameter in the AS equation. Recall that our estimate of λ is not significantly different from zero, and the estimate is even negative with the CBO measure of the output gap. A positive λ , consistent with the theoretical model, makes inflation decrease immediately after the monetary policy shock. A negative λ makes inflation increase after the monetary policy shock for a long period of time.²⁴ On the other hand, it is interesting to note that a lower value of λ , associated with a higher level of inflation rigidity or a lower speed of price adjustment, increases the real effects of the monetary policy shock. This is due to the fact that under a lower λ , inflation decreases less following the monetary policy shock so that, through the policy rule, the interest rate remains higher for several periods after.

[Insert Figure 6 Here]

Finally, in Panel B of Figure 6 we perform an analogous analysis of the estimated parameter ϕ , with values 0.005, 0.05 and 0.125. A lower ϕ , which may be brought about by a larger σ (smaller elasticity of substitution) or by a higher h (higher habit persistence), yields a smaller reaction of the output gap to the monetary policy shock. In other words, the more agents smooth consumption across time, either by having a smaller elasticity of substitution or by placing a larger weight on past consumption on the utility function, the less effect monetary policy shocks have on output. With h held fixed, these parameter values are equivalent to σ with values 100, 10 and 4 (4 is the smallest possible value within the 99% empirical confidence interval). Panel B of Figure 6 reveals that a more accepted value in the literature for σ , such as 4, gives rise to an exceedingly large impact of the monetary policy shock on the output gap.

²³In a recent paper ? shows that some structural parameters of a model derived through optimization exhibit subsample instability. This finding casts doubt on the assertion that private sector parameters of any structural model derived through optimization are invariant to policy changes.

²⁴When we allow for serially correlated error terms λ does not govern the reaction of inflation to the monetary policy shock, as we show in the following subsection.

7.4 Model Specification

In this subsection, we examine, both asymptotically and at the small sample level, how our estimated model fits the actual U.S. economy for our sample period with respect to an unrestricted model. Since our model is nested in a VAR(1) with highly nonlinear parameter restrictions, we compare the model with an unrestricted VAR(1).²⁵

Panel A of Figure 7 compares the reduced form impulse response functions of our model with the ones of an empirical VAR(1).²⁶ The restricted dynamics are qualitatively similar to the unrestricted ones, except in the case of the inflation response to the interest rate shock. Quantitatively the other restricted impulse response functions track their unrestricted counterparts closely, except for the output gap response to the inflation innovation: In the restricted model the impact is lower than in the VAR(1).

[Insert Figure 7 Here]

Although the New Keynesian model matches most of the impulse responses reasonably well, we reject the model using an LR test: We have 7 parameters in the structural model and 3 variances of structural shocks. The unrestricted VAR(1) has 9 parameters in the coefficient matrix and 6 in the variance covariance matrix of innovations. Therefore, there are 5 over-identification restrictions. The likelihood of our model and the unrestricted VAR are -259.975 and -243.360 , respectively. This implies an LR test statistic of 33.230, rejecting the null that the restricted model comes from the same asymptotic distribution than the unrestricted one.

As shown by Bekaert and Hodrick (2001) in the context of the Expectation Hypothesis, asymptotic tests such as the LR test can be severely biased in small samples. With the data generated by our bootstrap exercise, we re-estimate the structural model and the unconstrained VAR(1). This yields the small sample distribution of the LR test statistic. As we report in the Panel A of Table 4, there is a considerable size distortion in the LR test of our model. For instance, the 5% critical value is 15.48, instead of the 11.07 asymptotic value, and the empirical size is 15.5%. The top Panel of Figure 8 shows that the empirical distribution of the LR test statistic has a higher mean and a

²⁵Even though the optimal number of lags chosen by the Schwarz criterion is 3 among the unrestricted VARs, it seems appropriate to compare our model with the nested VAR(1) for the purpose of our study. The impulse responses of an unrestricted VAR(3) are similar to those of the unrestricted VAR(1). Additionally, the right number of lags can be questionable with a small sample size. For instance, the Akaike information criterion selects 15 lags.

²⁶We do not use orthogonalized shocks of the reduced form because the dynamics can be very different with alternative orderings of the variables in the unrestricted VAR.

fatter tail than the asymptotic distribution. Unfortunately, the structural model is still strongly rejected. We also bootstrap 1,000 samples under the alternative hypothesis of an unrestricted VAR(1) to calculate the empirical power of the LR test. The empirical power measures the probability of rejecting the null hypothesis when the alternative is true in a small sample. It is calculated as the percentage of LR tests obtained, under the alternative hypothesis, that are lower than a given empirical critical value. For a 5% significance level, the power of the test is 91.4%.

[Insert Table 4 Here]

[Insert Figure 8 Here]

Why is the model so severely rejected? If we visually compare the dynamics implied by the model with the ones of the unrestricted VAR(1), (Panel A of Figure 7), one candidate for the source of rejection seems to be the fact that the model does not reproduce the price puzzle. As mentioned earlier, this is caused by the positive λ , which is, however, what economic theory implies. Below we show that the tight link between the sign of λ and the price puzzle can be broken if we allow for serial correlation of the error terms. Suppose that the structural errors follow a VAR(1) process:

$$\epsilon_{t+1} = F\epsilon_t + w_{t+1} \quad (26)$$

where F is a 3×3 stationary matrix that captures the structural shock serial correlation and w_{t+1} is independently and identically distributed with diagonal variance covariance matrix D . The reduced form solution of the model is still given by (13). The same method of undetermined coefficients can be applied to solve for Ω , Γ and c in terms of α , A_{11} , B_{11} , B_{12} and F . It can be shown that the expressions for Ω and c are the same as equations (15) and (17), and therefore the same methodology for solving the matrix quadratic form, equation (20), can be applied. However, Γ now depends on F :

$$\Gamma = (B_{11} - A_{11}\Omega)^{-1}(I + A_{11}\Gamma F) \quad (27)$$

Γ can be solved as $vec(\Gamma) = [I_3 \otimes (B_{11} - A_{11}\Omega) - (F' \otimes A_{11})]^{-1}vec(I_9)$ where I_3 and I_9 denote 3×3 and 9×9 identity matrices. In order to estimate this model, we first express the model solution in terms of w_{t+1} as:

$$X_{t+1} = (I - \Gamma F \Gamma^{-1})c + (\Omega + \Gamma F \Gamma^{-1})X_t - \Gamma F \Gamma^{-1}\Omega X_{t-1} + \Gamma w_{t+1} \quad (28)$$

One implication of the Rational Expectations solution of the model with serial correlation is that now neither λ nor ϕ govern the direction of the inflation and output gap responses to the monetary policy shock, respectively. This can be seen in equation (28): The coefficient matrices of X_t , X_{t-1} and Γ are now functions of F . We first estimate the model by FIML without any restriction on F . Let F_{ij} be ij -th element of F . Then zero restrictions on F_{13} , F_{21} , F_{32} and F_{33} are imposed because they are not significantly different from zero. Since the reduced form solution is VAR(2), a natural alternative is an unrestricted VAR(2). These 4 additional restrictions imply that the model has 9 degrees of freedom in total.

In Panel B of Figure 7 we show the reduced form Impulse Response Functions associated with the structural model with autocorrelation. Now the model can reproduce the price puzzle, and the impulse response functions of the model match their unrestricted counterparts very closely. Even though the asymptotic LR test still rejects the model at the 5% level, the rejection is marginal using the small sample LR test (the p-value is 0.039), as is shown in Panel B of Table 4 and the bottom Panel of Figure 8. However, the empirical power of the test is much lower than the one in the original model: The power associated with empirical size of 5% and 1% are 64.4% and 37.8%, respectively. In contrast, the corresponding powers in the model without serial correlation are 91.4% and 73.1%. This evidence suggests that tests of models which imply restricted higher order VARs may suffer from low power against their unrestricted counterparts.

To summarize, even though the model with serial correlation is only marginally rejected, the original New Keynesian model that we consider remains inconsistent with the data. Our results highlight the need to produce different model specifications, in order to uncover the structural macro relations behind this significant autocorrelation of the residuals.

8 Conclusion

Policy parameters have qualitative and quantitative implications on the relation between macro dynamics and structural shocks. Our econometric policy evaluation exercise shows that when the Fed reacts strongly to deviations of expected inflation from its target, two different effects take place: On the one hand, inflation returns faster to the target in response to AS and IS shocks. On the other hand, the economy enters into a long recession in response to an AS shock. A number of authors have estimated a strong reaction of

the Fed to deviations of expected inflation from the target since 1979. Our maximum likelihood estimation shows, however, that this result is not statistically significant using linearly and quadratically detrended output. Moreover, our small sample study reveals that the coefficient on expected inflation is upwardly biased. One possibility is that the Taylor rule does not describe accurately the way the Fed conducts monetary policy and that the Fed reacts differently to AS and IS shocks. Further work is necessary to determine whether this is the case.

We intend to extend this framework to a finance setting. Instead of using the short rate in the IS equation we could employ the long rate, which can have more relevance in aggregate demand decisions, such as consumption or durable equipment purchases. The long term rate could in turn be introduced into the model through an additional term structure equation. One caveat of our study is that two important parameters such as the Phillips curve parameter or the inverse of the elasticity of substitution are not statistically significant at standard levels. Introducing the long term interest rate could help estimate them more precisely. Finally, another direction for future research will be to allow the policy parameters to change across regimes. Even though this is technically challenging, it would enable us to estimate the system over a longer sample period and to investigate the implications of those switches on the macro dynamics.

Appendix

A Recursive Method

For ease of exposition, we use the mean deviation form of equation (11) in what follows.

$$B_{11}\bar{X}_t = A_{11}E_t\bar{X}_{t+1} + B_{12}\bar{X}_{t-1} + \epsilon_t \quad (29)$$

where $\epsilon_t = F\epsilon_{t-1} + w_t$ and $E_{t-1}w_t = 0$. In this appendix we solve equation (29) forward. We consider a more general case by allowing the structural errors to follow a VAR(1) process. First we show that there exist sequences of matrices, $\{C_k, \Omega_k, \Gamma_k, k = 1, 2, 3, \dots\}$ such that:

$$\bar{X}_t = C_k E_t \bar{X}_{t+k+1} + \Omega_k \bar{X}_{t-1} + \Gamma_k \epsilon_t$$

By investigating the properties of the matrices, we propose a solution to the model.

Claim 1 *Consider equation (29). Suppose $A_{11}, B_{11}, B_{12}, F$ are real-valued and B_{11} is nonsingular. There exist sequences of matrices $\{\Phi_k, \Psi_k, \Xi_k, k = 1, 2, 3, \dots\}$, which are functions of A_{11}, B_{11}, B_{12} and F (thus they are real-valued by construction) such that:*

$$E_t \bar{X}_{t+k} = \Phi_k E_t \bar{X}_{t+k+1} + \Psi_k \bar{X}_t + \Xi_k \epsilon_t \quad (30)$$

where

$$\Phi_{k+1} = [I - \Psi_1 \Phi_k]^{-1} \Phi_1 \quad (31)$$

$$\Psi_{k+1} = [I - \Psi_1 \Phi_k]^{-1} \Psi_1 \Psi_k \quad (32)$$

$$\Xi_{k+1} = [I - \Psi_1 \Phi_k]^{-1} [\Psi_1 \Xi_k + \Xi_1 F^k] \quad (33)$$

if $I - \Psi_1 \Phi_k$ is nonsingular for all k .

Proof. Premultiply B_{11}^{-1} on both sides of equation (29). Then by shifting one period forward and taking expectations:

$$\begin{aligned} E_t \bar{X}_{t+1} &= B_{11}^{-1} A_{11} E_t \bar{X}_{t+2} + B_{11}^{-1} B_{12} \bar{X}_t + B_{11}^{-1} F \epsilon_t \\ &= \Phi_1 E_t \bar{X}_{t+2} + \Psi_1 \bar{X}_t + \Xi_1 \epsilon_t \end{aligned} \quad (34)$$

where $\Phi_1 = B_{11}^{-1} A_{11}$, $\Psi_1 = B_{11}^{-1} B_{12}$ and $\Xi_1 = B_{11}^{-1} F$. (We use the fact $E_t \epsilon_{t+1} = F \epsilon_t$.)

Suppose $E_t \bar{X}_{t+k}$ can be written as follows for some natural number k .

$$E_t \bar{X}_{t+k} = \Phi_k E_t \bar{X}_{t+k+1} + \Psi_k \bar{X}_t + \Xi_k \epsilon_t \quad (35)$$

where Φ_k , Ψ_k and Ξ_k are sequences of matrices of the appropriate dimension. Shift equation (34) k periods forward and take expectations at time t . Then:

$$\begin{aligned} E_t \bar{X}_{t+k+1} &= \Phi_1 E_t \bar{X}_{t+k+2} + \Psi_1 E_t \bar{X}_{t+k} + \Xi_1 E_t \epsilon_{t+k} \\ &= \Phi_1 E_t \bar{X}_{t+k+2} + \Psi_1 [\Phi_k E_t \bar{X}_{t+k+1} + \Psi_k \bar{X}_t + \Xi_k \epsilon_t] + \Xi_1 F^k \epsilon_t \end{aligned} \quad (36)$$

by the law of iterative expectations and from equation (34). Then if $I - \Psi_1 \Phi_k$ is invertible for k :

$$E_t \bar{X}_{t+k+1} = [I - \Psi_1 \Phi_k]^{-1} \Phi_1 E_t \bar{X}_{t+k+2} + [I - \Psi_1 \Phi_k]^{-1} \Psi_1 \Psi_k \bar{X}_t + [I - \Psi_1 \Phi_k]^{-1} [\Psi_1 \Xi_k + \Xi_1 F^k] \epsilon_t \quad (37)$$

Therefore Φ_{k+1} , Ψ_{k+1} , and Ξ_{k+1} are defined as coefficient matrices in this equation, and these are given by equations (31), (32) and (33). ■

Claim 2 Consider equation (29) and $\{\Phi_k, \Psi_k, \Xi_k, k = 1, 2, 3, \dots\}$ defined as above. Then there exist sequences of matrices $\{C_k, \Omega_k, \Gamma_k, k = 1, 2, 3, \dots\}$ such that:

$$\bar{X}_t = C_k E_t \bar{X}_{t+k+1} + \Omega_k \bar{X}_{t-1} + \Gamma_k \epsilon_t \quad (38)$$

Proof. First, expand $E_t \bar{X}_{t+1}$ using equation (30).

$$\begin{aligned} E_t \bar{X}_{t+1} &= \Phi_1 E_t \bar{X}_{t+2} + \Psi_1 \bar{X}_t + \Xi_1 \epsilon_t \\ &= \Phi_1 \Phi_2 E_t \bar{X}_{t+3} + [\Phi_1 \Psi_2 + \Psi_1] \bar{X}_t + [\Phi_1 \Xi_2 + \Xi_1] \epsilon_t \\ &= \Phi_1 \Phi_2 \Phi_3 E_t \bar{X}_{t+4} + [\Phi_1 \Phi_2 \Psi_3 + \Phi_1 \Psi_2 + \Psi_1] \bar{X}_t + [\Phi_1 \Phi_2 \Xi_3 + \Phi_1 \Xi_2 + \Xi_1] \epsilon_t \\ &\quad \dots \\ &= \prod_{j=1}^k \Phi_j E_t \bar{X}_{t+k+1} + \sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Psi_i \bar{X}_t + \sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Xi_i \epsilon_t \end{aligned} \quad (39)$$

where $\Phi_0 = I$. Therefore equation (29) can be written as:

$$\begin{aligned} B_{11}\bar{X}_t &= A_{11}E_t\bar{X}_{t+1} + B_{12}\bar{X}_{t-1} + \epsilon_t \\ &= A_{11}\prod_{j=1}^k \Phi_j E_t \bar{X}_{t+k+1} + A_{11}\sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Psi_i \bar{X}_t + B_{12}\bar{X}_{t-1} + [A_{11}\sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Xi_i + I]\epsilon_t \end{aligned} \quad (40)$$

We can arrange (40) to obtain equation (38), where

$$S_k = \sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Psi_i \quad (41)$$

$$G_k = \sum_{i=1}^k \prod_{j=1}^i \Phi_{j-1} \Xi_i \quad (42)$$

$$C_k = (B_{11} - A_{11}S_k)^{-1} A_{11} \prod_{j=1}^k \Phi_j \quad (43)$$

$$\Omega_k = (B_{11} - A_{11}S_k)^{-1} B_{12} \quad (44)$$

$$\Gamma_k = (B_{11} - A_{11}S_k)^{-1} (I + A_{11}G_k) \quad (45)$$

■

Notice the similarity between (44), (45) and (15), (16) in the case that $F = G_k = 0$. Now we define our solution as follows.

Proposition 1 *Consider equation (29). Suppose that the sequences of $\{S_k, G_k, \Omega_k, \Gamma_k\}$ defined above satisfy the following:*

1. S_k, G_k, Ω_k and Γ_k are convergent sequences of matrices: $S^* \equiv \lim_{k \rightarrow \infty} S_k, G^* \equiv \lim_{k \rightarrow \infty} G_k,$
 $\Omega^* \equiv \lim_{k \rightarrow \infty} \Omega_k, \Gamma^* \equiv \lim_{k \rightarrow \infty} \Gamma_k.$
2. $\Omega^* = S^*, G^* = \Gamma^* F$ and all the eigenvalues of Ω^* are within a unit circle.
3. $\lim_{k \rightarrow \infty} C_k \Omega^{*k} = 0$: No bubble condition.

Then:

$$\bar{X}_t = \Omega^* \bar{X}_{t-1} + \Gamma^* \epsilon_t \quad (46)$$

is a stationary real-valued bubble-free solution to the structural model (29).

Proof. Since $\Omega^* = S^*$ and $G^* = \Gamma^*F$, equations (44) and (45) are the same as (15) and (27), which implies that (46) is a solution to (29). Since all the eigenvalues of Ω^* are within the unit circle, Ω^* is stationary. Since all the sequences are real-valued, their limits are real-valued, too. Finally, $\lim_{k \rightarrow \infty} C_k E_t \bar{X}_{t+k+1} = \lim_{k \rightarrow \infty} C_k \Omega^{*k} \bar{X}_t = 0 \times \bar{X}_t = 0$ implies that the solution (46) is bubble-free.

Remark 1 In practice this recursive solution is the same as that obtained through the QZ method in the case of a unique stationary real-valued solution. In the case of multiple solutions it typically corresponds to the solution with the smallest 3 eigenvalues obtained through the QZ method. There is however a nontrivial case where the QZ method gives multiple stationary real valued solutions but none of them coincides with the recursive solution. Suppose there are 2 stable real-valued eigenvalues and one stable complex conjugate pair arranged in increasing order. The solution associated with the smallest 3 eigenvalues is complex valued. The choice of one of the two real eigenvalues and the complex conjugate pair yields a real-valued solution. Therefore we have 2 real-valued solutions. However, neither of them coincides with the recursive solution. In this case, the conditions for the recursive method are not met.²⁷

■

B Uniqueness of the solution

Table 5 shows the generalized eigenvalues associated with the three FIML estimated sets of parameters. As explained in section 4.2, in the first two specifications (with output linearly and quadratically detrended), we have a unique solution, since there are exactly 3 eigenvalues less than unity, the same number as predetermined state variables in the model. We also verified that the recursive solution coincides with the one obtained through the QZ method.

[Insert Table 5 Here]

For the third specification (with output detrended using the CBO measure), we have multiple solutions, since there are 4 eigenvalues less than 1 in moduli. Our recursive

²⁷Notice that our recursive method is different from that of Binder and Pesaran (1997) in the sense that we construct the sequences of matrices forward. Instead of imposing a terminal condition, we can simply check whether the solution (46) satisfies the no bubble condition.

method converges to the QZ solution with the first 3 eigenvalues. In general, we found that, holding the remaining parameters at their estimated values in column (1) of Table 2, when λ is positive, the solution is unique. For negative values of λ , large in absolute value, there is no real valued solution. For small negative values of λ , as estimated with the CBO measure, there are multiple solutions. The dynamics implied by our recursive solution for this case are shown in the sensitivity analysis with $\lambda = -0.0014$.

C Bootstrap Analysis

Our structural model and the unrestricted VAR(1) can be expressed respectively as:

$$X_t = c + \Omega X_{t-1} + \Gamma \epsilon_t \quad (47)$$

$$X_t = d + \Theta X_{t-1} + u_t \quad (48)$$

where $Var(\Gamma \epsilon_t) = \Gamma D \Gamma'$ and $Var(u_t) = \Upsilon$. If the structural model is true, it should be the case that $\Gamma D \Gamma' = \Upsilon$. We orthogonalize the unrestricted VAR(1) error terms through a Choleski decomposition, so that $Var(u_t) = E(u_t u_t') = \Upsilon = C C'$, where C is lower triangular. Therefore, $u_t = C \zeta_t$, where ζ_t has mean zero and ones in the diagonal of its variance covariance matrix. The unrestricted VAR(1) can then be expressed as:

$$X_t = d + \Theta X_{t-1} + C \zeta_t \quad (49)$$

Under the null of the model $\epsilon_t = \sqrt{D} \xi_t$, where ξ_t has mean zero and ones in the diagonal of its variance covariance matrix. The model can then be expressed as:

$$X_t = c + \Omega X_{t-1} + \Gamma \sqrt{D} \xi_t \quad (50)$$

Therefore, if the model is true it should be the case that $\Gamma \sqrt{D} = C$ and that $Var(\Gamma \sqrt{D} \xi_t) = Var(C \zeta_t)$. We perform a bootstrap analysis under the null of the structural model and under the alternative data generating process, the VAR(1). Under the null we proceed as follows:

1. We bootstrap the unconstrained errors, u_t , with replacement.
2. We reconstruct 1,000 sample data sets of size 578 under the null hypothesis, using the estimated parameter matrices c , Ω and D , and the historical initial values,

along with the ζ_t disturbances, which are obtained by pre-multiplying the u_t errors by C^{-1} . For every sample we discard the first 500 data points and retain the last 78 observations to have the same size as the original data set.

3. We re-estimate both the model and the unrestricted VAR(1) 1,000 times. This yields 1,000 parameter sets and 1,000 LR tests.

With the 1,000 parameter sets, we obtain the small sample distribution of the structural parameters under the null of the model. To compute the empirical critical values of the LR test statistic, we select the corresponding quantiles of the empirical distribution of the LR test statistic. The bootstrap simulations under the alternative hypothesis differ from the ones under the null in that, in step 2, the data sets are constructed conditional on d and Θ , instead of c , Ω and D . The power of the test is calculated as the percentage of LR tests obtained, under the alternative hypothesis, which is lower than a given empirical significance level.

The case of the bootstrap of the model with autocorrelation, $F \neq 0$, is analogous to the one just presented. There are two differences with respect to the baseline case. First, the unconstrained residuals are bootstrapped from a VAR(2) model. Second, under the null hypothesis, equation (28) is used to reconstruct the small sample data sets.

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Table 1: **Sup-Wald Break Date Statistics**

<i>Sample Period</i>	<i>VAR</i>	<i>Sup-Wald</i>	<i>Break Date</i>	<i>90% Confidence Interval</i>
1954:3Q-2000:1Q	1	72.02	1980:4Q	1980:3Q-1981:1Q
1954:3Q-2000:1Q	2	103.33	1980:4Q	1980:3Q-1981:1Q
1954:3Q-2000:1Q	3	116.86	1980:4Q	1980:3Q-1981:1Q

This Table lists the Sup-Wald values of the break date test derived by Bai, Lumsdaine, and Stock (1998). The test detects the most likely break date of a break in all of the parameters of unconstrained VARs of orders 1 to 3. The Table shows the results of the test using the GDP deflator, linearly detrended output gap and the Federal funds rate.

Table 2: **FIML Estimates and Small Sample Distribution of the Structural Parameters of the Model**

<i>Parameters</i>	(1)	(2)	(3)	(4)	(5)	(6)
δ	0.5586 (0.0168)	0.5585 (0.0173)	0.5681 (0.0248)	[0.5256 0.5915]	0.5764	[0.5239 0.6565]
λ	0.0011 (0.0011)	0.0011 (0.0011)	-0.0002 (0.0014)	[-0.0010 0.0032]	0.0028	[-0.0034 0.0131]
μ	0.4859 (0.0339)	0.4810 (0.0358)	0.4801 (0.0376)	[0.4195 0.5523]	0.4826	[0.2386 0.5728]
ϕ	0.0045 (0.0051)	0.0054 (0.0056)	0.0065 (0.0057)	[-0.0055 0.0146]	0.0140	[-0.0065 0.0760]
ρ	0.8458 (0.0519)	0.8419 (0.0415)	0.8767 (0.0404)	[0.7441 0.9475]	0.8148	[0.6629 0.9211]
β	1.6409 (0.7725)	1.6413 (0.4487)	2.1506 (0.5058)	[0.1267 3.1551]	1.9027	[0.3983 5.0267]
γ	0.6038 (0.4145)	0.6126 (0.3163)	1.0079 (0.4648)	[-0.2086 1.4161]	0.6214	[-0.3402 1.7680]
σ_{AS}	0.4585 (0.0396)	0.4588 (0.0389)	0.4661 (0.0429)	[0.3809 0.5361]	0.4635	[0.3956 0.5344]
σ_{IS}	0.3734 (0.0326)	0.3766 (0.0341)	0.3570 (0.0330)	[0.3096 0.4372]	0.3841	[0.2996 0.5553]
σ_{MP}	0.7327 (0.0555)	0.7305 (0.0588)	0.7281 (0.0586)	[0.6239 0.8416]	0.7105	[0.5399 0.8818]

This Table shows the FIML parameter estimates of the structural macro model in equation (11), using the GDP deflator, the output gap and the Federal funds rate. Standard errors are in parentheses below the estimates. The parameter sets in columns (1), (2) and (3) correspond to the estimations with linearly detrended output, quadratically detrended output and output detrended using the CBO measure of potential output, respectively. Column (4) shows the 95% confidence interval of the asymptotic parameter estimates. Column (5) shows the sample means of the 1000 bootstrap parameter estimates. Column (6) shows the 95% interval of the empirical distribution of the parameter estimates. These last three columns are based on the estimates in (1). The sample period is 1980:4Q-2000:1Q.

Table 3: **Residuals Diagnostic Tests**

Panel A: Autocorrelations of the Structural Errors

$Lag=i$	$\epsilon_{AS_t}, \epsilon_{AS_{t-i}}$	$\epsilon_{IS_t}, \epsilon_{IS_{t-i}}$	$\epsilon_{MP_t}, \epsilon_{MP_{t-i}}$
1	-0.3213	0.3555	0.1138
2	-0.1596	0.3798	-0.3057
3	0.1894	0.1860	0.2251
4	0.1356	-0.0029	0.2055

Panel B: Contemporaneous Crosscorrelations of the Structural Errors

$\epsilon_{AS_t}, \epsilon_{IS_t}$	$\epsilon_{AS_t}, \epsilon_{MP_t}$	$\epsilon_{IS_t}, \epsilon_{MP_t}$
0.0736	-0.2306	0.3027

Panel C: Ljung-Box Q-statistics

Lag	$Q(AS_t)$	$pval(AS_t)$	$Q(IS_t)$	$pval(IS_t)$	$Q(MP_t)$	$pval(MP_t)$
1	5.6600	(0.0174)	10.1481	(0.0014)	0.9574	(0.3278)
2	8.6191	(0.0134)	22.1299	(0.0000)	8.1270	(0.0172)
3	11.6441	(0.0087)	24.4316	(0.0000)	11.8207	(0.0080)
4	13.0648	(0.0110)	24.4745	(0.0001)	15.0232	(0.0047)

Panel D: Jarque-Bera Tests

$JB(\epsilon_{AS_t})$	$pval(\epsilon_{AS_t})$	$JB(\epsilon_{IS_t})$	$pval(\epsilon_{IS_t})$	$JB(\epsilon_{MP_t})$	$pval(\epsilon_{MP_t})$
3.6277	(0.1630)	5.0576	(0.0798)	55.5700	(0.0000)

Panel A reports the serial correlation of the AS, IS and monetary policy shocks. Panel B lists the contemporaneous cross-correlations among the structural shocks. Panel C shows the Ljung-Box Q-statistics for autocorrelation of the error terms, with their corresponding probability values. Panel D reports the Jarque-Bera tests for normality of the residuals, with their corresponding probability values.

Table 4: Empirical size and power for the Likelihood Ratio Test

Panel A: Model with uncorrelated residuals								
	Mean	Median	Std. Dev.	90%	95%	99%	Sample LR	Pval
$\chi^2(5)$	5	4.35	3.16	9.24	11.07	15.09	33.23	0.000
MODEL LR	6.83	5.94	4.54	12.43	15.48	22.61		0.000
SIZE(%)				23.0	15.5	5.2		
POWER(%)				95.6	91.4	73.1		
Panel B: Model with correlated residuals								
	Mean	Median	Std. Dev.	90%	95%	99%	Sample LR	Pval
$\chi^2(9)$	9	8.34	4.25	14.68	16.92	21.66	20.60	0.015
MODEL LR	9.88	9.03	4.89	16.26	19.83	25.37		0.039
SIZE(%)				14.1	8.9	3.1		
POWER(%)				79.4	64.4	37.8		

This Table provides summary statistics for the asymptotic and empirical distributions of the likelihood ratio (LR) test statistic. The statistics are the Mean, Median, Standard Deviation (Std. Dev) and the 90%, 95% and 99% quantiles. MODEL LR refers to the empirical distribution of the LR statistic under the null hypothesis (restricted model). The Table also provides empirical sizes and powers from the empirical distributions of the LR test statistic. The empirical size is the percent of the bootstrap experiments generated under the null hypothesis, where the test statistic exceeds a given asymptotic critical value. The empirical power of the test is the percent of the bootstrap experiments generated under the alternative hypothesis (unrestricted VAR), where the test statistic exceeds the given empirical critical value. Panel A and B show the statistics for the model with and without serially correlated structural errors, respectively.

Table 5: **Generalized Eigenvalues**

<i>Gen. Eig.</i>	(1)	(2)	(3)
ξ_1	0.7845	0.7837	0.7608
ξ_2	0.8986-0.0348i	0.8973-0.0385i	0.9110-0.0593i
ξ_3	0.8986+0.0348i	0.8973+0.0385i	0.9110+0.0593i
ξ_4	1.0148	1.0148	0.9970
ξ_5	1.0987	1.1192	1.1419
ξ_6	∞	∞	∞

This Table reports the generalized eigenvalues which determine the stability of the structural macro model. The sets of eigenvalues in columns (1), (2) and (3) correspond to the estimations of the systems with linearly detrended output, quadratically detrended output and output detrended using the CBO measure of potential output, respectively.

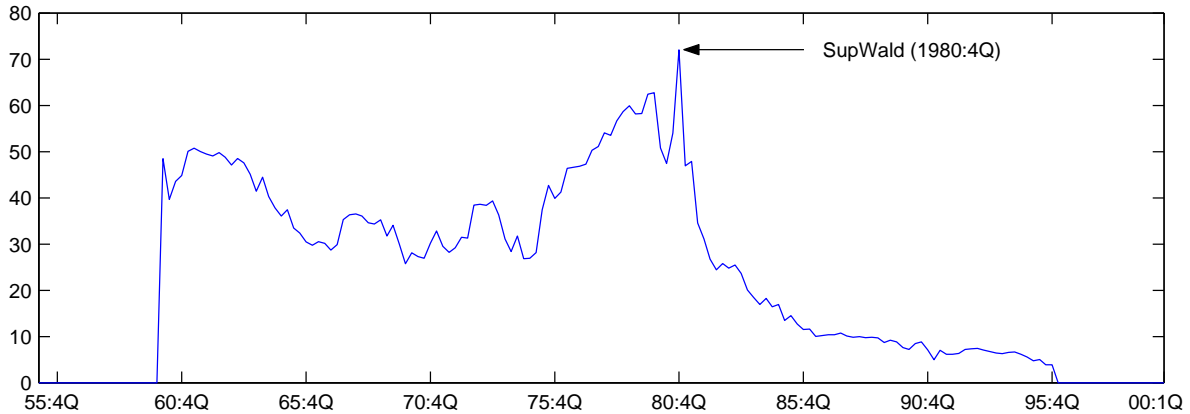


Figure 1: Series of Wald Statistics: All parameters break for a VAR(1)

This Figure shows the values of the time series of the Sup-Wald statistic for a break in all the parameters of an unconstrained VAR(1) developed by Bai, Lumsdaine, and Stock (1998). The variables in the VAR are U.S. inflation, linearly detrended output and Federal funds rate. The sample period is 1954:3Q-2000:1Q.

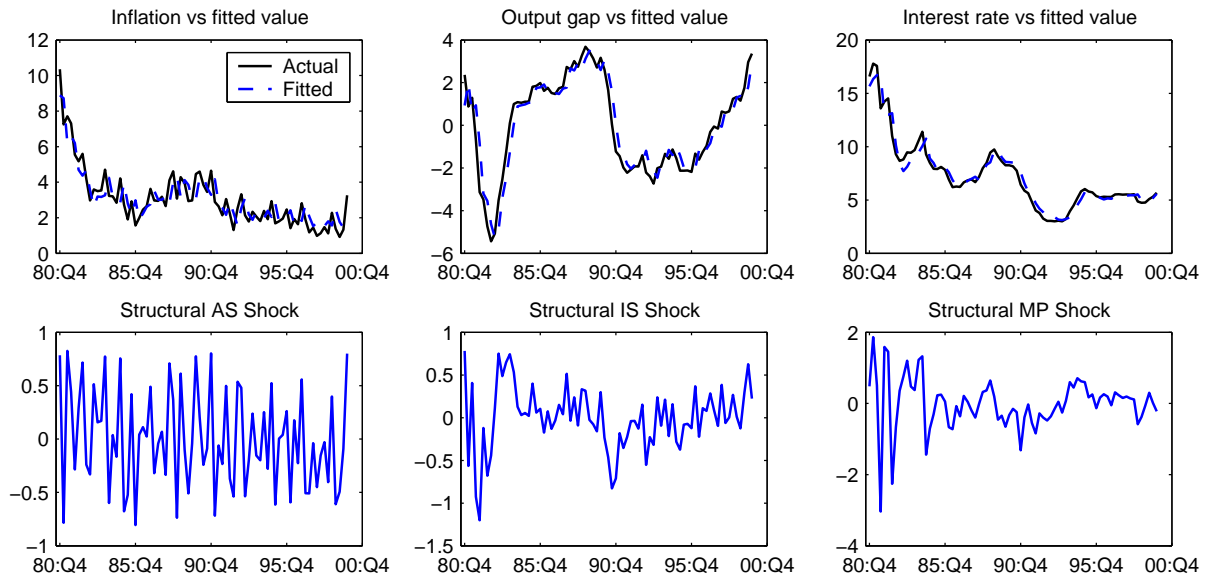


Figure 2: Predicted Values and Structural Shocks : 1980:Q4-2000:Q1

The top row of these Figure shows the predicted (dashed lines) and actual (solid lines) values for inflation, the output gap and the Federal funds rate associated with the FIML estimation of the structural model in equation (13). The bottom row shows the structural errors estimates (ϵ_{AS_t} , ϵ_{IS_t} and ϵ_{MP_t}).

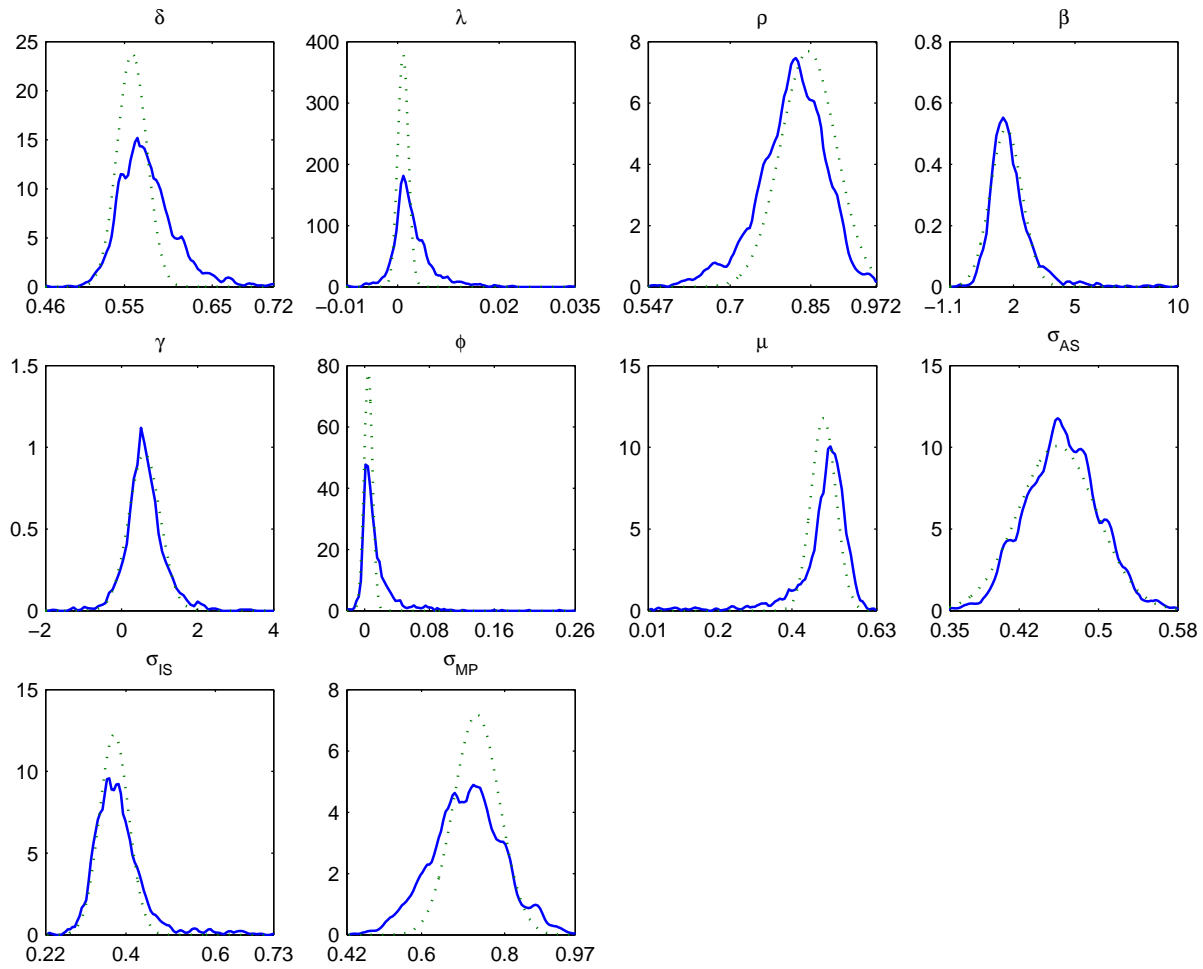


Figure 3: Empirical Distribution of the structural parameters

The empirical probability distribution of the structural parameters is the solid line and is compared to the asymptotic distribution of the FIML estimates, the dotted line. The empirical distribution of the structural parameters is obtained through an Epanechnikov kernel estimation of the values yielded by the bootstrap exercise explained in the Appendix.

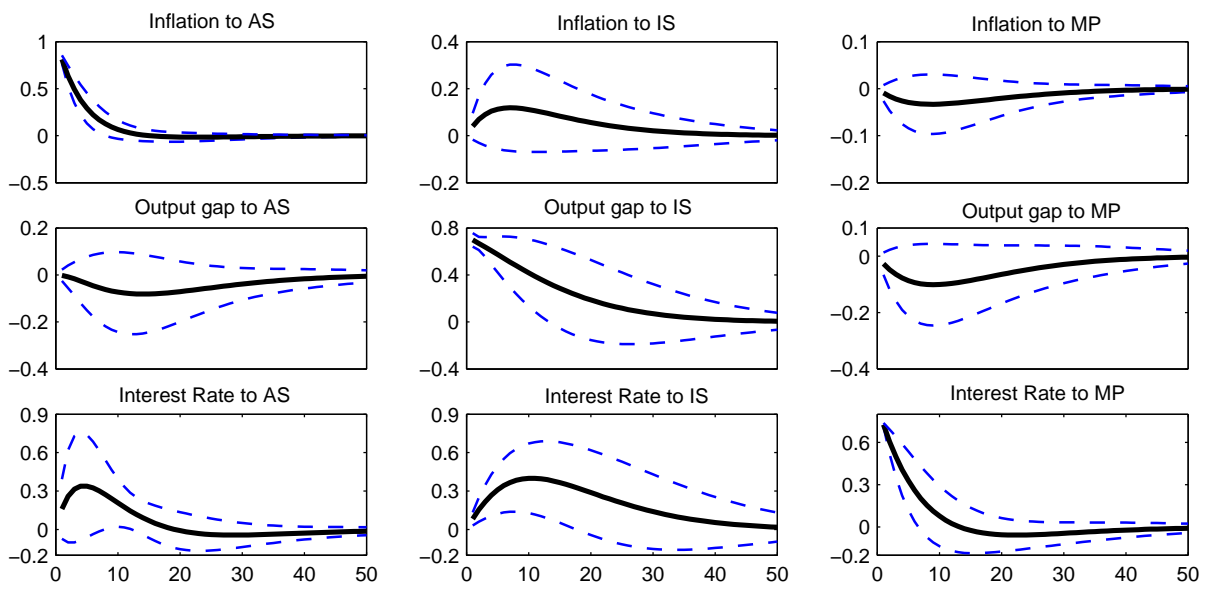
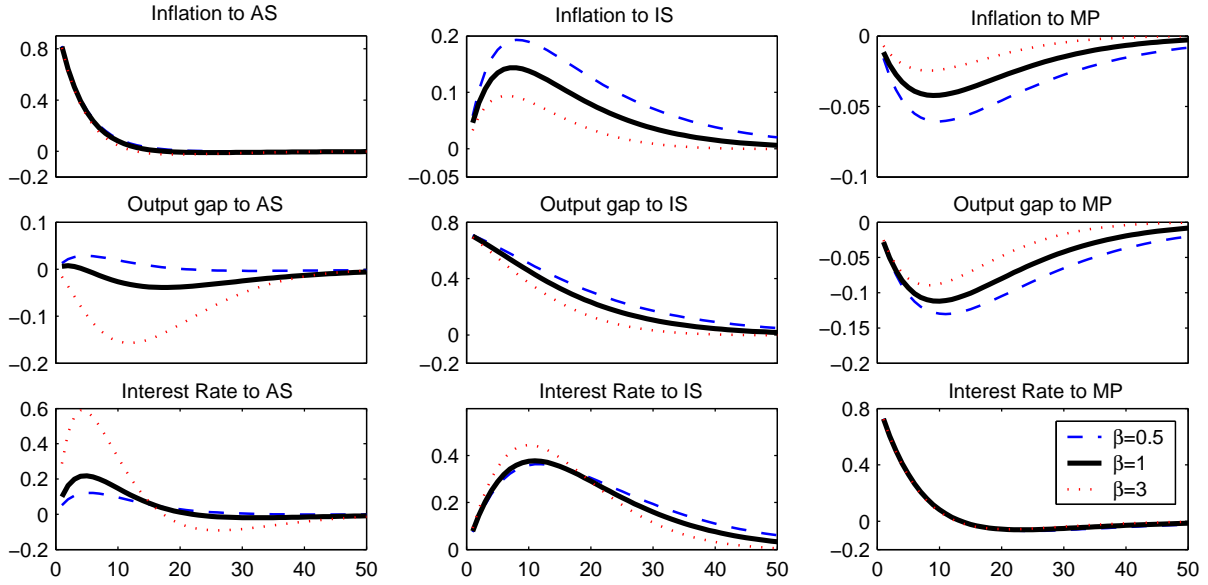


Figure 4: Structural Impulse Response Functions

This Figure graphs the impulse responses of the macro variables to the structural shocks ϵ_{AS_t} , ϵ_{IS_t} and ϵ_{MP_t} . The magnitude of the structural shocks is the estimated standard deviation of each shock. The dashed lines represent the asymptotic 95% confidence intervals around the Impulse Response Functions.

Panel A: Sensitivity analysis for β



Panel B: Sensitivity analysis for γ

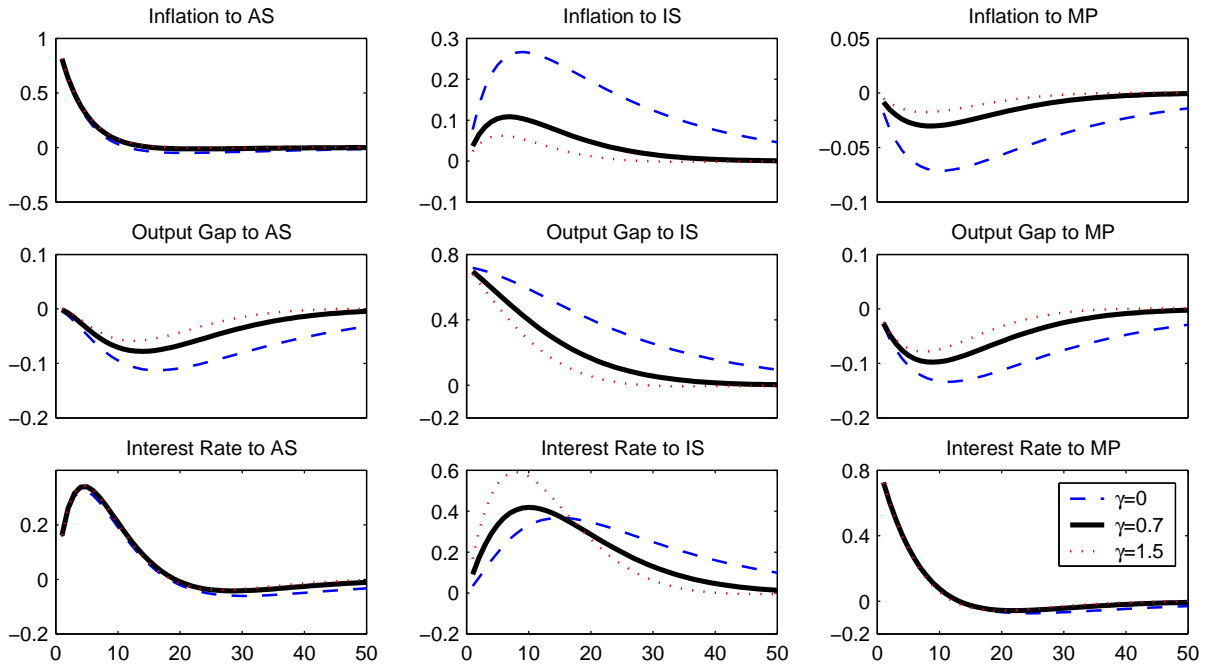


Figure 5: Sensitivity analysis for monetary policy parameters

Panel C: Sensitivity analysis for ρ

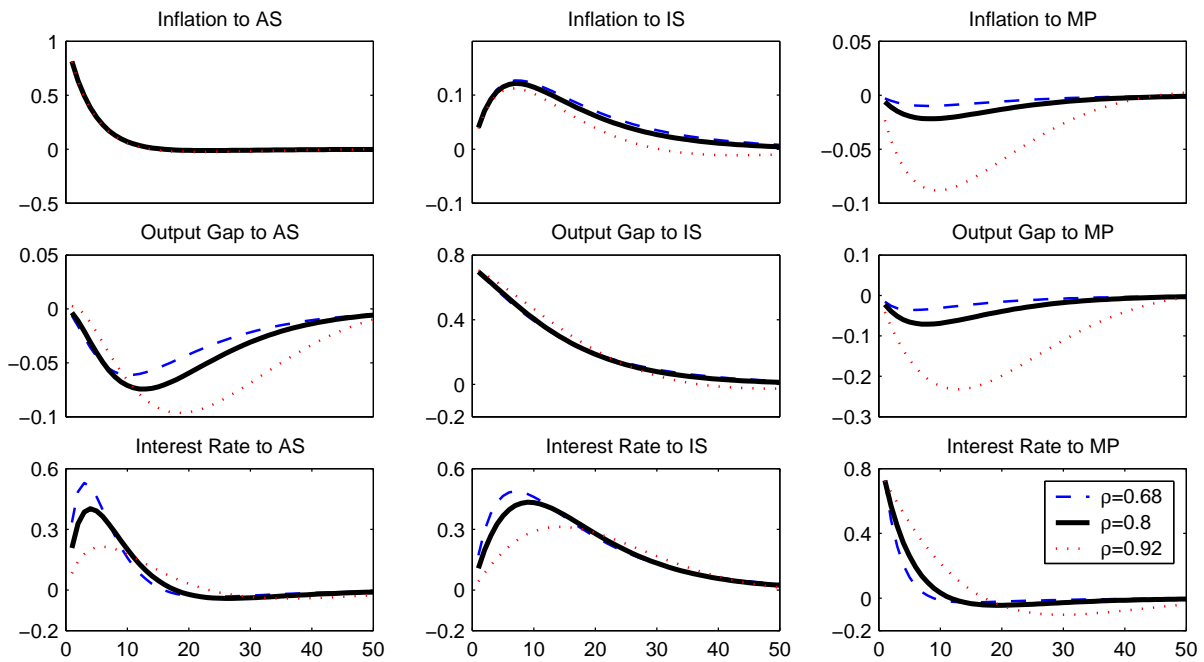
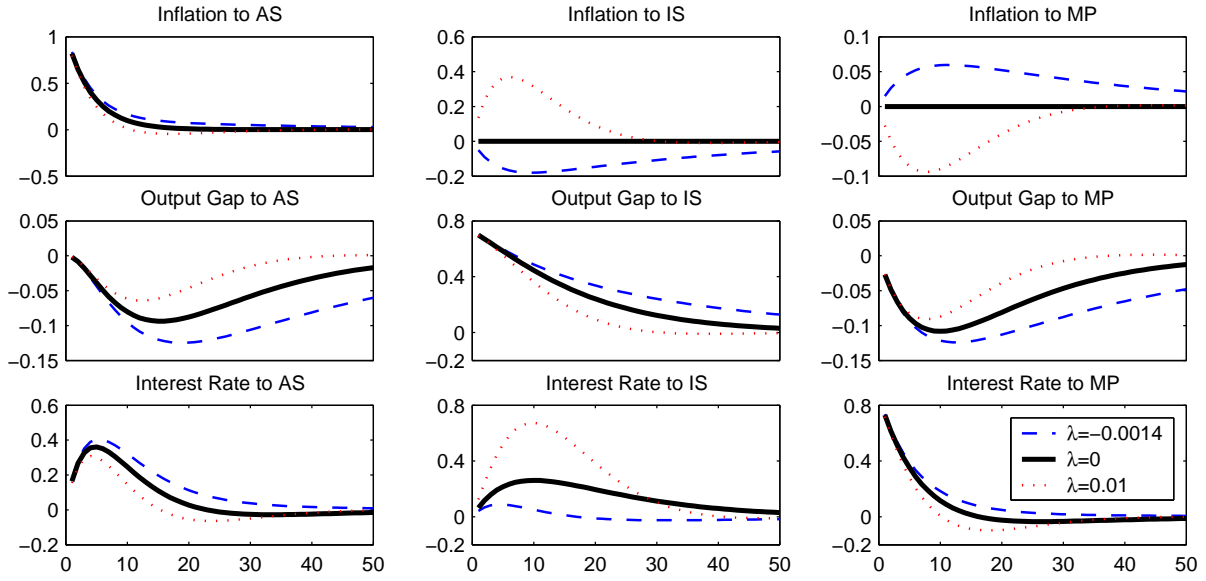


Figure 5: (continued) Sensitivity analysis for monetary policy parameters

This Figure presents the impulse responses which arise under different values of β, γ and ρ chosen from the 95% interval of their empirical distribution. For each analysis, the remaining parameter values are held fixed at their corresponding estimates, as shown in Table 2, column (1).

Panel A: Sensitivity analysis for λ



Panel B: Sensitivity analysis for ϕ

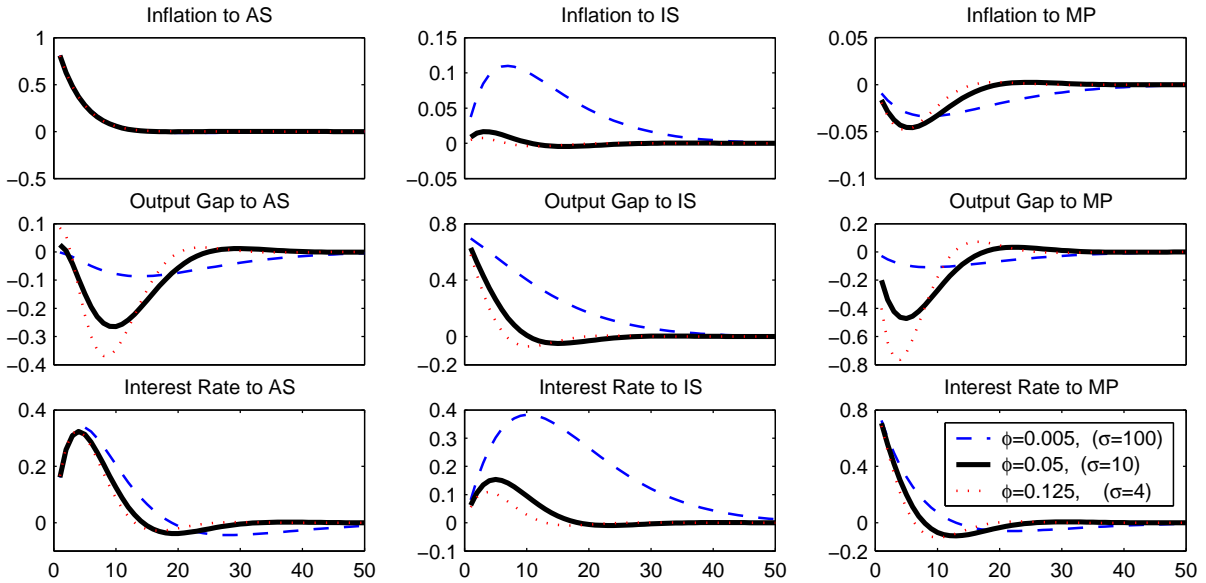
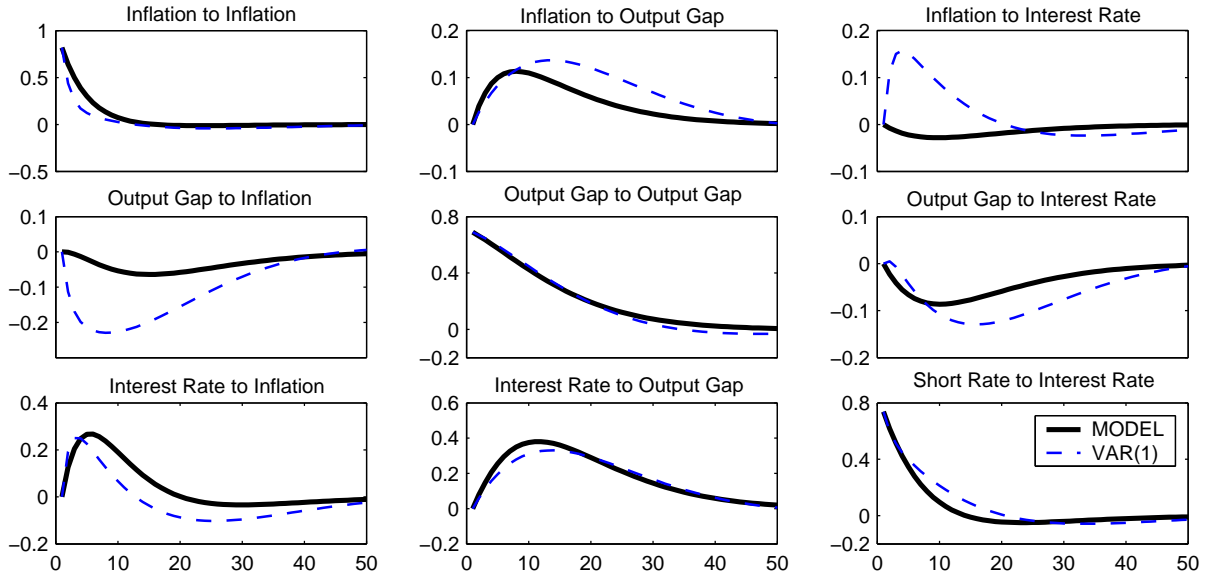


Figure 6: Sensitivity analysis for private sector parameters

This Figure presents the impulse responses which arise under different values of λ and ϕ chosen from the 95% interval of their empirical distribution. For each analysis, the remaining parameter values are held fixed at their corresponding estimates, as shown in Table 2, column (1).

Panel A: Reduced Form Impulse Response Functions (model with $F = 0$)



Panel B: Reduced Form Impulse Response Functions (model with $F \neq 0$)

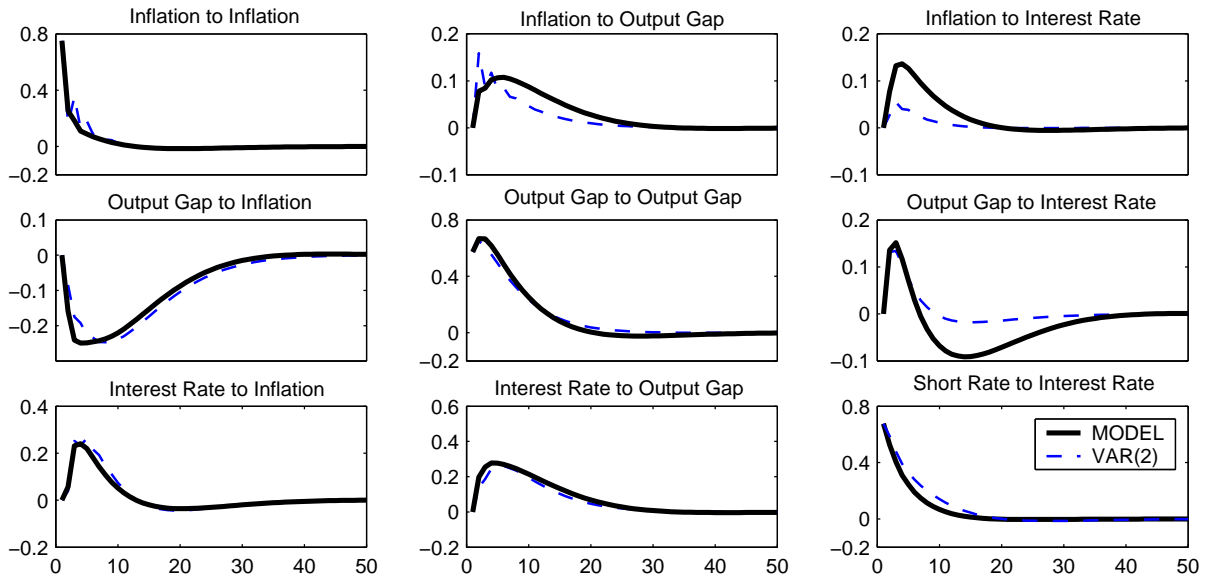


Figure 7: Reduced Form Impulse Response Functions

This Figure shows the impulse responses of the macro variables to a one standard deviation of the reduced form shocks under the model (solid lines) and the corresponding unrestricted VAR (dashed lines). The top and bottom panels display the impulse responses for the models with and without serially correlated structural errors, respectively.

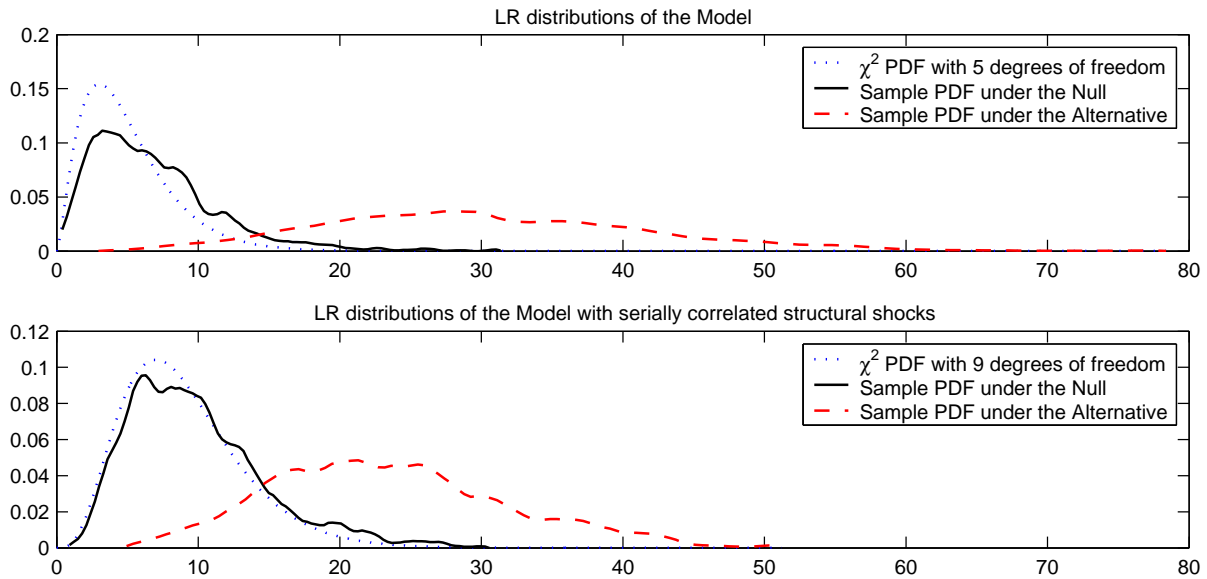


Figure 8: Empirical Distribution of the Likelihood Ratio

This Figure compares the asymptotic probability density function (PDF) of the Likelihood Ratio test (dotted line) with its small sample counterpart (solid line) under the null of the structural model. It also graphs the Likelihood Ratio test under the alternative hypothesis, the unconstrained VAR(1) (dashed line). The PDF of the empirical LR test statistic is estimated using an Epanechnikov kernel. The top and bottom panels display the PDFs for the model with and without serially correlated structural errors, respectively.