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Fractional integration and business cycle features

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ABSTRACT

We show in this article that fractionally integrated univariate models for GDP may lead to a better replication of business cycle characteristics. We firstly show that the business cycle features are clearly affected by the degree of integration as well as by the other short run components of the series. Then, we model the real GDP in France, the UK and the US by means of fractionally ARIMA (ARFIMA) models, and show that the three time series can be specified in terms of this type of models with orders of integration higher than one but smaller than two. Comparing the ARFIMA specifications with those based on ARIMA models, we show via simulations that the former better describes the business cycles features of the data at least for the cases of the UK and the US.

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1. Introduction

With the development of the National Bureau of Economic Research (NBER)'s project of "Measurement without Theory" and the first extensive study of Burns and Mitchell (1947) on the American Economy, business cycles and their features have constituted a direct object of empirical analysis. Numerous studies have tried to describe them and to consider their stability over time. Romer (1986, 1994), Diebold and Rudebush (1992) and Watson (1994) have, for example, explored data to know if fluctuations have been smoother (lower amplitude and longer duration) after the second World War. Also, Neftci (1983), Hamilton (1989), Beaudry and Koop (1993) have created new business cycles features[1] to show that business cycles exhibit an asymmetry in their phases: recessions being deeper and shorter than expansions.

Recently, business cycles features have been used for another purposes. Candelon and Hénin (1995) have built distributions of these features via bootstrapped simulation of simple linear (ARIMA) models for GDP. They could then locate the observed features of the last cycle and conclude that they are rather normal. A step further, Isawa and Hess (1997) used them as benchmarks to gauge the adequacy of macroeconomic stochastic time series models. They replicate via Monte-Carlo simulations different models for GDP. Then, they build for each model the distribution of the business cycles features and compare them to the historical business cycles characteristics. The best model is selected as the one which replicate the best historical feature. Three types of linear models, namely, integrating a stochastic trend (ARIMA), a deterministic trend and a segmented trend (as in Perron, 1989) as well as several non linear ones (SETAR, Markov-Switching and Beaudry and Koop's, 1993, non linearity) are considered. They conclude that complex non-linear or linear models do not better replicate business cycles features than a simple linear ARIMA(1,1,0) with a

[1] These features integrate the third moment of the cycle as the conditional asymmetry in mean.

drift. Such a conclusion appears to be rather destructive for recent attempts, which have tried to better model GDP.

Nevertheless, they do not consider a recent and growing literature, which tries to model GDP and other macroeconomic time series in terms of fractionally integrated processes. Examples are Diebold and Rudebusch (1989); Sowell (1992); Gil-Alana and Robinson (1997); etc. A proper definition of fractional integration will be given in Section 2. We can, however, mention here that the ARIMA model can be viewed as a particular case of a much more general class of models, called fractionally ARIMA (ARFIMA), in which we allow for a fractional degree of differencing in a given raw time series.

In this article, we show that the ARFIMA models can better describe the business cycle characteristics of the GDP in France, the UK and the US, compared with the ARIMA specifications as well as other approaches. The structure of the paper is as follows: Section 2 briefly describes the concepts of fractional integration and business cycles. Section 3 shows with some simulations that the degree of fractional integration of an univariate model affects to the characteristics of the fluctuations. Section 4 uses both ARIMA and ARFIMA models to describe the behaviour of the GDP series. Section 5 compares both types of models in terms of business cycle features while Section 6 concludes.

2. Fractional integration and business cycle characteristics

For the purpose of the present paper, we define an I(0) process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that x_t is I(d) if

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

where L is the lag operator ($Lx_t = x_{t-1}$), and d can be any real number. The macroeconomic literature stresses the cases of d = 0 and d = 1. In the latter case, we say that x_t follows a unit root process or that the model contains a stochastic trend. This model became popular after the paper of Nelson and Plosser (1982), who following the work and ideas of Box and Jenkins (1970), showed that many US macroeconomic series could be specified in terms of unit roots. A huge amount of empirical work has followed this approach (eg. Stock and Watson, 1986; Diebold and Nerlove, 1989; etc.) However, as it was shown by Adenstedt (1974), Taqqu (1975) and subsequent work, d can also be a real number. When d = 0 in (1), $x_t = u_t$, and a weakly autocorrelated x_t is allowed for. However, if d > 0, x_t is said to be long memory, so-called because of the strong association between observations widely separated in time. Note that the polynomial in (1) can be expanded in terms of its Binomial expansion, such that for all real d,

$$(1 - L)^d = \sum_{j=1}^{\infty} \frac{\Gamma(d+1)\,(-1)^j}{\Gamma(d-j+1)\,\Gamma(j+1)} = 1 - d\,L + \frac{d\,(d-1)}{2} - \ldots$$

where Γ(x) means the gamma function. This type of processes was initially proposed by Granger (1980, 1981), Granger and Joyeux (1980), Hosking (1981) and were theoretically justified in terms of aggregation by Robinson (1978), Granger (1980) and more recently, in terms of the duration of shocks by Parke (1999).

There is an interest in the estimation and testing of the fractional differencing parameter. If d ∈ (0, 0.5), x_t in (1) is covariance stationary while d ∈ [0.5, 1) will imply that the series is nonstationary but still mean reverting, with the effect of the shocks dying away in the long run. On the contrary, if d ≥ 1, the process will be nonstationary and non-mean reverting, with the effects of the shocks persisting forever. Thus, for example, if d > 1 and the data are in logs, that means that the growth rates have a long memory component and therefore, the stochastic trend overcome other potential characteristics of the series, In other words, the fractional differencing parameter can be used as an indicator of the degree of persistence of the series and, higher d is, higher will be the degree of persistence, implying that the cycles are less likely to occur.

We now describe a datation rule to date the business cycles and define their characteristics. Numerous methods have been proposed in the literature. They can be based on direct data analysis (Burns and Mitchell, 1946), on expert judgment (NBER) or rely on the most recent econometric methods (Hamilton, 1994)[2]. In this paper, we have decided to consider exclusively classical cycles (directly extracted from the data in levels) in order to avoid statistical problems caused by the extraction of the cyclical component (See Canova, 1994). Besides, we apply the most common rule to date classical business cycles. It is at the basis of the famous program developed by Bry and Boschan (1971) and defines the phases of the business cycles as follows[3]:

a) $y_{t-1} > y_t < y_{t+1} < y_{t+2}$, then there is a trough in t.

b) $y_{t-1} < y_t > y_{t+1} > y_{t+2}$, then there is a peak in t.

c) When several identical turning points are detected consecutively, we retain the optimal one (i.e., the highest peak and the deepest trough).

This rule is very intuitive because it simply considers that a turning point occurs when there is a change in the slope: Conditions a) and b) can be rewritten as $\square_j\, y_{t+-j} > 0$ and $\square_j\, y_{t+-j} < 0$. Such a definition insures that phases of the cycles have a minimum duration of 2 quarters and the completed cycles a minimum length of one year. This definition also presents the advantage to induce an asymmetry in the length of the cycle phase. This is even greater when the generated process is I(d) with $d \geq 1$: As there is no more mean-reversion, activity has a stochastic growth rate and the length of an expansion is longer than the duration of a recession. This property is more difficult to exhibit from growth cycles (extracted from filtered data) but is confirmed in historical data (Moore and Zarnowitz, 1982). On the

[2] See Pagan and Harding (1999) for an exhaustive survey of the procedures for determining turning points.
[3] The same method can also be used on filtered data to exhibit growth cycles (see Canova, 1994).

contrary, we can not expect to detect an asymmetry in the amplitude of the phases, as the conditions on the change in slope are symmetric for troughs and peaks.

This method has suffered a stream of criticisms: For example, it could exhibit not only major but also minor cycles. McNees (1991) and Webb (1991) propose to solve this problem via an increase in the reference period (for example, a peak could be characterized by 3 consecutive periods of growth over a year period). Candelon and Hénin (1995) have also noticed that this method leads to slight differences with the algorithms based on the detection of local optimum in the cases of growth cycles[4]. However, integrating these extensions in our datation algorithm will not alter the links between the degree of fractional integration and the business cycle characteristics. We thus make the choice of simplicity and keep rules a) - c) as our datation algorithm.

(Insert Figure 1)

From this datation, we have built five indicators (see Figure 1): the number of peaks (which corresponds to the number of cycles, as we consider that a cycle begins with a trough), the length of the cycles (period running between two successive troughs), the length and the amplitude of an expansion (period running from a trough to a peak) and the length and the amplitude of a recession (period running from a peak to a trough).

3. A simulation study

We explore in this section the link between the degree of fractional integration and business cycle features via simulations. To this goal, we consider a process $\{y_t\}_{t=1\ldots T}$, with the following DGP: $(1-L)^d\, y_t = u_t$. According to the values taken by d, y_t can be stationary ($d < 0.5$), or non stationary ($d \geq 0.5$). To analyse the effect of d on the business cycle features, we simulate 2500 series of length 100, 300 and 500, for some values of d = {0, 0.25, 0.5, 0.75,

[4] A local optimum is not a turning point for our methodology if it is preceeded and followed by only one quarter of increase or decrease in the activity.

1, 1.5, 2} and then compute the mean and the variance of the five pre-defined features of the cycle (number of cycles, length and amplitude of the phase of the cycles)[5]. The results could indeed be affected by the process followed by u_t. Thus, as in Isawa and Hess (1997), three different linear processes are considered :

1. u_t is a white noise N(0,1). Results are gathered in Table 1.
2. $u_t = \phi u_{t-1} + □t$, (AR1). Results for $\phi = \{0.25, 0.5, 0.75\}$ are gathered in Table 2.
3. $u_t = □_t + \theta □_{t-1}$, (MA1). Results for $\theta = \{0.25, 0.5, 0.75\}$ are gathered in Table 3.

(Insert Figure 2)

To have a more precise view, Figure 2 plots the results for white noise u_t. As expected the average length of expansion is in all the cases greater than the duration of the recession. This asymmetry in the duration is due to the stochastic trend of the generated series. The tables also confirm that the amplitude of the phase is symmetrical: Recession amplitudes seems to be higher than expansion ones, but the variance is such that the symmetry can not be rejected. It also turns out that the relationship between the degree of fractional integration and the business cycle features has the same evolution in all cases. The average number of cycles increases until a value of d around 0.5 and then goes down. The other features dealing with the length and the amplitude of the phases exhibit an opposite evolution. The variance of the features exhibits similar paths. These results can be interpreted in the following way: When the degree of integration increases, the mean reversion is less important. A large part of the dynamic of y_t is then impulsed by the stochastic trend. The variance and the mean of the process are thus higher, leading a smaller number of longer and deeper business cycles. For the extreme case where d tends to infinite, the process is exclusively driven by the trend and no more cycles could be extracted. Figure 2 also shows that the level of inflection is quicker for the amplitude characteristics (d ~ 0.25 for white noise and d ~ 0.5 for AR and

[5] The distribution of these features could be computed as in Isawa and Hess (2000). However, as all processes

MA u_t) and a little bit longer for the duration ones ($d \sim 0.75$ for white noise). It also appears that if $d > 1$, the path is explosive (for 300 observations, we only find a mean value of 4 cycles for a white noise u_t and $d = 2$, whereas when $d = 1$, 24 peaks can be observed on average)

(Insert Tables 1 – 3 about here)

When the AR or the MA coefficients become high (Tables 2 and 3), the evolution of the features with respect to the degree of fractional integration becomes linear, with a negative slope for the number of cycles and a positive one for the other criterion. When ϕ is closed to one, the process possesses a near-unit-root, removing the mean reversion and increasing the variance. The average number of cycles is thus smaller, whereas their length and amplitude become higher, ϕ thus playing a similar role as the degree of integration.

4. The empirical application

The time series data analysed in this section correspond to the logarithmic transformation of the real Gross Domestic Product (GDP) in France, United Kingdom and United States, quarterly, (seasonally adjusted), for the time period 1961:1-2000:1 and are extracted from the IMF-IFS database. We have performed our datation algorithm and compared its results with reference studies. For the United States, it is referred to the National Bureau of Economic Research (NBER) business cycle datation. It turns out in Table 4 that our algorithm leads to a nearly identical[6] datation except for the cycle (80:3-81:1), which is considered as minor in the official datation. For the European countries, as it does not exist official datation, we refer to the paper of Artis and al. (1997). Our results are similar but not identical. Nevertheless, it is worth noticing that Artis and al. (1997) consider Industrial Production for a different period, define cycles period running between two peaks, and use a

are linear, the distribution will not give more information than the mean and the variance.

more complex datation algorithm. So, the small differences could be justified and do not lead to a rejection of our datation algorithm.

(Insert Tables 4 and 5 about here)

Table 5 gathers the business cycle characteristics of the three time series. We notice that in each country, 5 major cycles occurred during the last 40 years. It also turns out that the expansions are longer and deeper than recessions. This stylised fact is generally acknowledged for classical cycles.

We now start with the empirical application. Let's assume that u_t in (1) is a stationary ARMA(p, q) process of form:

$$\phi_p(L)\, u_t \;=\; \theta_q(L)\, \varepsilon_t, \qquad t \;=\; 1, 2, ..., \tag{2}$$

with white noise ε_t. Substituting (2) in (1), the general time series model becomes

$$\phi_p(L)\,(1-L)^d\, x_t \;=\; \theta_q(L)\, \varepsilon_t, \qquad t \;=\; 1, 2, ..., \tag{3}$$

which is usually called an ARFIMA(p, d, q) model. Sowell (1992) estimated the parameters in (3) using a procedure that allows quick evaluation of the likelihood function in the time domain, which is given by:

$$(2\pi)^{-T/2}\,|\Sigma|^{-1/2}\,\exp\left\{-\frac{1}{2} X_T^{'}\,\Sigma^{-1}\,X_T\right\},$$

with $X_T = (x_1, \ldots, x_T)' \sim N(0, \Sigma)$. An Ox-programme of this procedure (see, Doornik and Ooms, 1999) will be employed in the empirical application below.

We estimate for the three time series different ARFIMA models like (3), taking values of p and q smaller than or equal to 3. Following standard practise, the models were estimated in first differences and then converted back to level by adding 1 to the estimated

[6] As NBER datation is performed for monthly data and our for quartely observations, sometimes our datation differs from a quarter.

value of d. Across the sixteen potential models, we choose the best one according to the Bayesian Information Criteria (BIC). The results are given in Table 6.[7]

(Insert Table 6 about here)

We see that the best model specifications are an ARFIMA(0, 1.47, 2) for France; an ARFIMA(1, 1.38, 2) for the UK; and an ARFIMA(0, 1.36, 0) for the US. Thus, the orders of integration are in all cases higher than one but smaller than two, and the t-statistics based on the nulls d = 1 and d = 2 reject both hypotheses for the three series. (Note that the estimates are based on maximum likelihood and thus, standard tests based on the statistics (d-1)/SE(d) and (d-2)/SE(d) are applicable in these cases). As a validation control for each of the selected models, we use a very simple version of a testing procedure due to Robinson (1994). He proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d \; = \; d_o, \tag{4}$$

in (1) for any real value d_o. Specifically, the test statistic is given by:

$$\hat{r} \; = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{5}$$

where $\hat{a} \; = \dfrac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j);$ $\qquad \hat{\sigma}^2 \; = \dfrac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j);$

$$\hat{A} \; = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 \; - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\psi(\lambda_j) = \; \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\varepsilon}(\lambda_j) \; = \; \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \quad \lambda_j \; = \; \frac{2\pi\, j}{T}.$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$ where $\hat{u}_t \; = \; (1-L)^{d_o}\, y_t$ and g above is a known function coming from the spectral density of u_t: $f(\lambda;\tau) = \dfrac{\sigma^2}{2\pi} g(\lambda;\tau),$ evaluated at

[7] The models were estimated with no intercept based on the assumption that the first differenced series have zero mean. Note that the inclusion of an intercept in first differences would imply that a linear trend in the

$\hat{\tau} = \arg\min \sigma^2(\tau)$. Based on H_o (4), Robinson (1994) showed that under certain regularity conditions,

$$\hat{r} \rightarrow_d N(0,1) \quad as \quad T \rightarrow \infty. \qquad (6)$$

Thus, an approximate 100α% level test of (4) will reject H_o against the alternative: H_a: $d > d_o$ ($d < d_o$) if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α. He also showed that the tests are efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, with $\delta \neq 0$, $\hat{r}$ has a limit distrtibution which is normal with variance 1 and mean that cannot (when u_t is Gaussian) be exceeded in absolute value by that of any other rival regular statistic. Empirical applications based on this version of Robinson's (1994) tests can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000) and, other versions of his tests based on seasonal (quarterly and monthly) and cyclical models are respectively Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001).

We report, in the last column of Table 6, the results of $\hat{r}$ in (5) in a model given by (1), testing H_o (4) for values $d_o = 1$, d^* and 2, where d^* is the chosen value according to the previous estimation procedure.[8] Note that the non-rejections of H_o (4) in these cases will imply that the series follow respectively an I(1), an ARFIMA, and an I(2) process. We see that when testing with $d_o = 1$, H_o (4) is rejected against alternatives with $d > 1$, and similarly, if $d_o = 2$, the null is rejected this time against alternatives with $d < 2$, implying that the order of integration of the series might fluctuate between these two extreme cases. Furthermore, we also observe that H_o (4) cannot be rejected in any series when d_o is chosen as the estimated value with the previous model selection criterion, indicating that the models can be correctly specified.

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original series for $t > 1$ only for the unit root case but not for I(d) processes.

[8] The null hypothesis H_o (4) in the tests of Robinson (1994) considers d_o as any given real value and thus, we can test H_o: $d = d^*$, taking d^* as a given value rather than as an estimated one.

(Insert Table 7 about here)

Table 7 firstly reports the results of ARIMA models imposing d = 1 in the GDP series. The best model specifications appear to be an ARIMA(1, 1, 2) for France and the UK, and an ARIMA(1, 1, 1) for USA. However, we observe that in all these cases, the AR parameter is very close to the unit root case (0.99 for France and 0.95 for UK and USA). Thus we also report the results assuming that d = 2. In this context, the best model specifications, according to the BIC, are an ARIMA(0, 2, 2) for France; an ARIMA(0, 2, 1) for UK, and an ARIMA(1, 2, 1) for USA, and in the three cases, the roots of the MA part seem to indicate now that there is a common unit root in the process. In view of these results, it becomes apparent that the ARFIMA models presented in Table 6 may better describe the long run behaviour of the three series since it does not restrict themselves to the integer differencing of the series. To show this, we describe in the following section simulated business cycle characteristics based on both, the ARFIMA models described in Table 6 and the ARI(2)MA models of Table 7.

5. A simulated comparison between ARIMA and ARFIMA models

Once the coefficients of the ARI(2)MA and ARFIMA models have been estimated, our objective now consists of selecting the best model, with respect to its ability to reproduce the business cycles features. So, we simulate 2500 ARI(2)MA and ARFIMA models for each country and compute their business cycles characteristics. Their empirical mean and variance are indicated in Table 8[9].

(Insert Table 8 about here)

The selection of the best model stems out from the comparison with the observed features in Table 5. It is first noticeable that Table 8 confirms the results exhibited in Section 3: As the

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[9] These two moments are sufficient to resume the complete distribution as we only consider linear models.

degree of fractional integration is always lower than 2, the number of peaks is lower in the cases of ARI(2)MA models, whereas the contrary results hold for the lengths and the amplitudes.

In the case of the UK, the ARFIMA model leads to a better replication of the features: the number of peaks corresponds to what is observed (5 cycles) and the amplitudes (of both phase of the cycle) are closer to the historical observations. Both length features are also bettered but not too significantly. For the US, the ARFIMA model overestimates the number of cycles (9) whereas the ARI(2)MA underestimates it (3). Nevertheless, we notice that the length features and the mean amplitude of the recession are more in line with the observed features when the fractional model is entertained.. Only the mean amplitude of expansion is underestimated by the ARFIMA model. This result is probably due to the linearity of the models: ARI(2)MA models exhibit mean amplitude features corresponding to the observed expansion amplitude (0.18) and so overestimate recession amplitude, whereas ARFIMA models replicate amplitude of the recession (0.02) and so overestimate the expansion amplitude. However, the ARFIMA model appears here slightly better than the ARI(2)MA one. In the case of France, the results appear to be more in favour of the ARI(2)MA model, which outperforms the ARFIMA for nearly all the features except the length of the recession. In conclusion, it appears that the ARFIMA models perform better than the ARIMAs , at least for the cases of the US and the UK.

6. Conclusions

We have tried in this article to analyse how fractionally integrated models can modify the reproduction of business cycle features. From a theoretical point of view, several Monte Carlo experiments conducted via simulations showed that the business cycle features can be seriously affected by the degree of integration of the series as well as by the short run

(ARMA) components associated to it. We built up five indicators for the business cycle characteristics, namely, the number of peaks, and the length and amplitude of the recessions and the expansions. It turns out that the average number of cycles increases until d ~ 0.5, and then sharply decreases. The other features share a symmetric paths. The importance of the stochastic trend part of the process (when d > 0.5) justifies this result. Next, we modelled the real GDP series in France, the UK and the US by means of fractionally ARIMA (ARFIMA) models. We used the Sowell's (1992) procedure of estimating by maximum likelihood in the time domain. The results indicate that the three series can be specified in terms of ARFIMA models, with orders of integration higher than one but smaller than two. This is also corroborated by the tests of Robinson (1994). When imposing an integer order of differencing, the series appear to be I(2), and comparing the ARFIMA models with the ARIMA ones, the former models seem to better describe the business cycle characteristics of the data, at least for the cases of the UK and the US. Isawa and Hess (2000) showed that the ARIMA models better replicate the business cycle features of many historical data compared with other approaches and, in that respect, we have shown in this article that the ARFIMA specification can do it even better than the ARIMA model.

References

Adenstedt, R.K., 1974, On large sample estimation for the mean of a stationary random sequence, Annals of Statistics 2, 1095-1107.

Artis, M., Osborn D. and Z. Kontolemnis., 1997, Classical business cycles for G7 and European countries, Journal of Business 70, 249-281.

Beaudry P. and G. Koop, 1993, Do recessions permanently change output?, Journal of Monetary Economics 31, 149-163.

Box, G.E.P. and G.M. Jenkins, 1970, Time series analysis, forecasting and control, San Francisco, Holden Day.

Burns, A and W. Mitchel, 1946, Measuring Business Cycles, NBER, New-York.

Bry, G and C. Boschan, 1971, Cyclical analysis of time series: Selected procedures and computer programs, New-York, NBER.

Canova, F., 1994, Detrending and turning points, European Economic Review 38, 614-623.

Candelon, B. and P.Y. Hénin, 1995, La récession des années quatre-vingt dix a-t-elle été exceptionnelle?, Economie et Prévision 120, 51-71.

Diebold, F.X. and M. Nerlove, 1989, Unit roots in economic time series: A selective survey, Advanced in Econometrics 8, 3-69.

Diebold, F.X. and G.D. Rudebusch, 1989, Long memory and persistence in aggregate output, Journal of Monetary Economics 24, 189-209.

Diebold, F.X. and G.D. Rudebusch, 1992, Have the postwar economic fluctuation been stabilized?, American Economic Review 82, 993-1005.

Doornik, J.A. and M. Ooms, 1999, A package for estimating, forecasting and simulating ARFIMA models: Arfima package 1.0 for Ox. Oxford: www.nuff.ox.ac.uk/Users/Doornik.

Gil-Alana, L.A., 1999, Testing fractional integration with monthly data, Economic Modelling 16, 613-629.

Gil-Alana, L.A., 2000, Mean reversion in the real exchange rates, Economics Letters 69, 285-288.

Gil-Alana, L.A., 2001, Testing stochastic cycles in macroeconomic time series, forthcoming in Journal of Time Series Analysis.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson, 2001, Testing seasonal fractional integration in the UK and Japanese consumption and income, Journal of Applied Econometrics 16. 1-23.

Granger, C.W.J., 1980, Long memory relationships and the aggregation of dynamic models, Journal of Econometrics 14, 227-238.

Granger, C.W.J., 1981, Some properties of time series data and their use in econometric model specification, Journal of Econometrics 16, 121-130.

Granger, C.W.J. and R. Joyeux, 1980, An introduction to long memory time series and fractional differencing, Journal of Time Series Analysis 1, 15-29.

Hamilton, J.D., 1989, A new approach to the economic analysis of non-stationary time series and the business cycle, Econometrica 57, 357-384.

Hosking, J.R.M., 1981, Modelling persistence in hydrological time series using fractional differencing, Water Resources Research 20, 1898-1908.

Isawa, S. and G.D. Hess, 1997, Measuring business cycle features, Journal of Business Economics and Statistics 15, 432-444.

McNess, 1991, “Forecasting cyclical turning points. The lessons of the last three recessions“ in K. Lahiri and G.H. Moore editors, Leading Economic Indicators CUP, 151-168.

Moore, G. and V. Zarnowitz, 1982, Sequential signals of recession and recovery, Journal of Business 55, 57-85.

Neftçi, S.N., 1984, Are economic time series asymmetric over the business cycle?, Journal of Political Economy 92, 307-328.

Nelson, C.R. and C.I. Plosser, 1982, Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139-162.

Pagan, A.R. and Harding, D., 1999, Dissecting the cycle: A methodological investigation, Melbourne Institute Working Paper 137, University of Melbourne.

Parke, W.R., 1999, What is fractional integration?, Review of Economics and Statistics 81, 632-638.

Perron, P., 1989, The great crash, the oil hock and the unit root hypothesis, Econometrica 77, 1361-1401.

Robinson, P.M., 1978, Statistical inference for a random coefficient autoregressive model, Scandinavian Journal of Statistics 5, 163-168.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Romer, C..D., 1986, Is stabilization of the post-war economy a figment of the data?, American Economic Review 76, 314-334.

Romer, C..D., 1994, Remeasuring business cycles, Journal of Economic History 54, 573-609.

Sowell, F., 1992, Modelling long run behaviour with the fractional ARIMA model, Journal of Monetary Economics 29, 277-302.

Stock, J.H. and M.W. Watson, 1986, Does GNP have a unit root?, Economics Letters 22, 147-151.

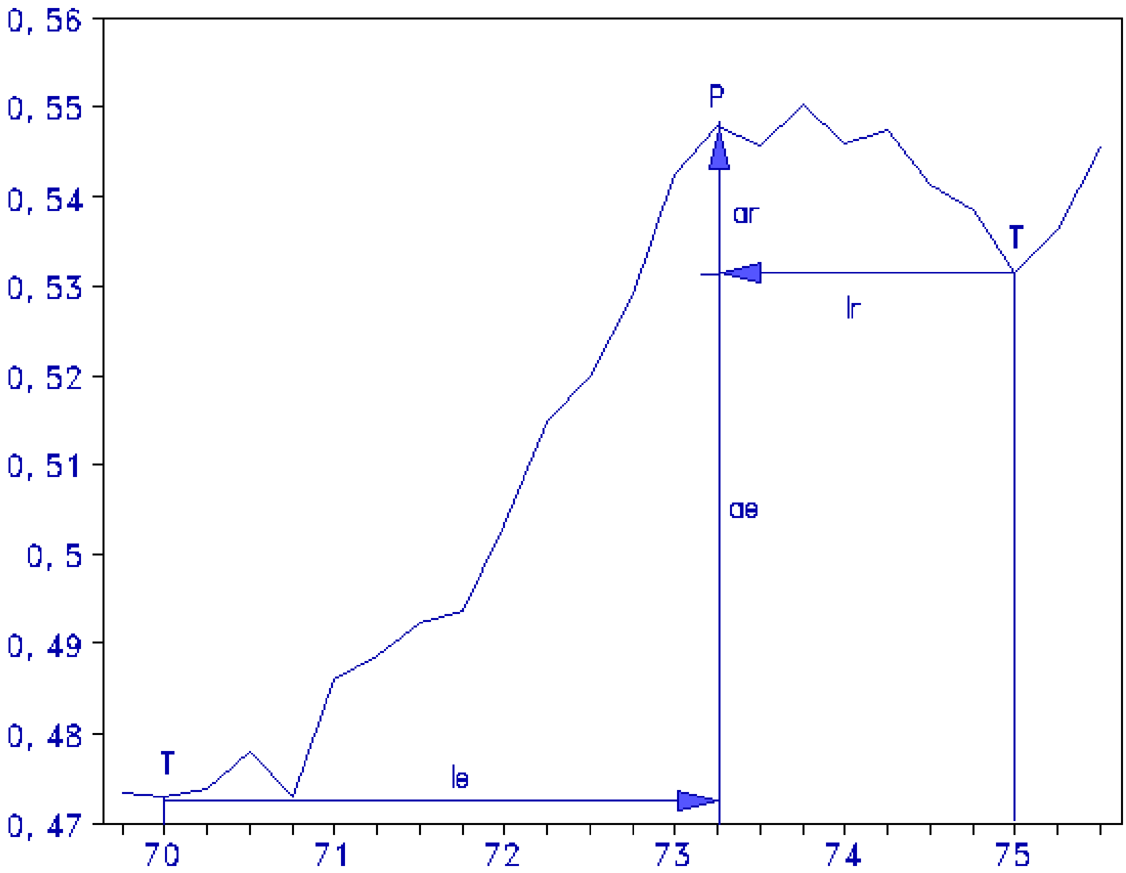
Taqqu, M.S., 1975, Weak convergence to fractional Brownian motion and to the Rosenblatt process, Z. Wahrscheinlichkeitstheorie verw. Geb. 31, 287-302.

Watson, M. W., 1994, Business-cycle durations and postwar stabilization of the U.S. economy, American Economic Review 84, 24-46.

Webb, R., 1991, On predicting stages of business cycles in : K. Lahiri and G. Moore, eds, Leading economic indicators : New approaches and forecasting record, Cambridge University Press, 129-140.

FIGURE 1

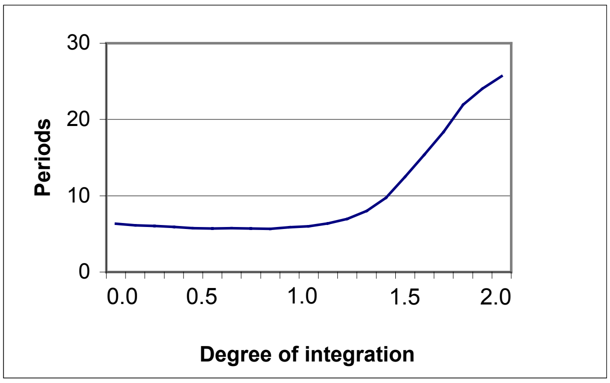
Business cycle features



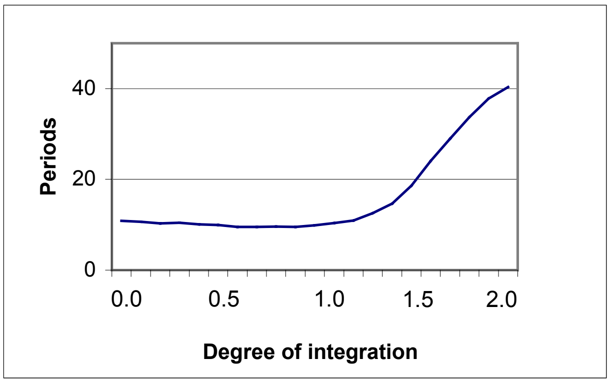
Note: This figure represents the first cycle in US data. T stands for Trough, P for Peak, le for length of expansion, lr for length of recession, ae for amplitude of expansion and ar for amplitude of recession. The length of the cycle is the sum of the two lengths.

FIGURE 2

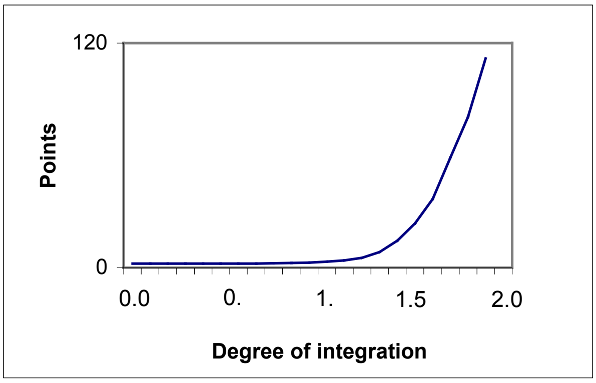
Mean length of recession



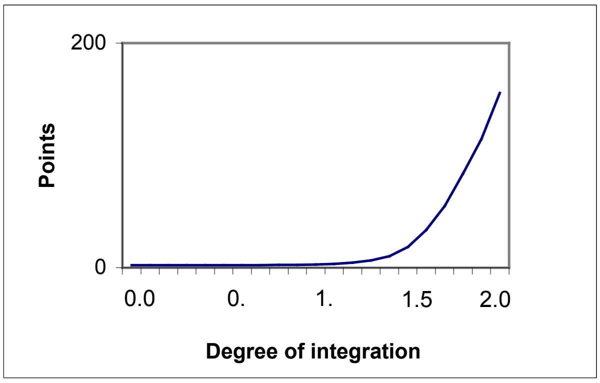
Mean length of expansion



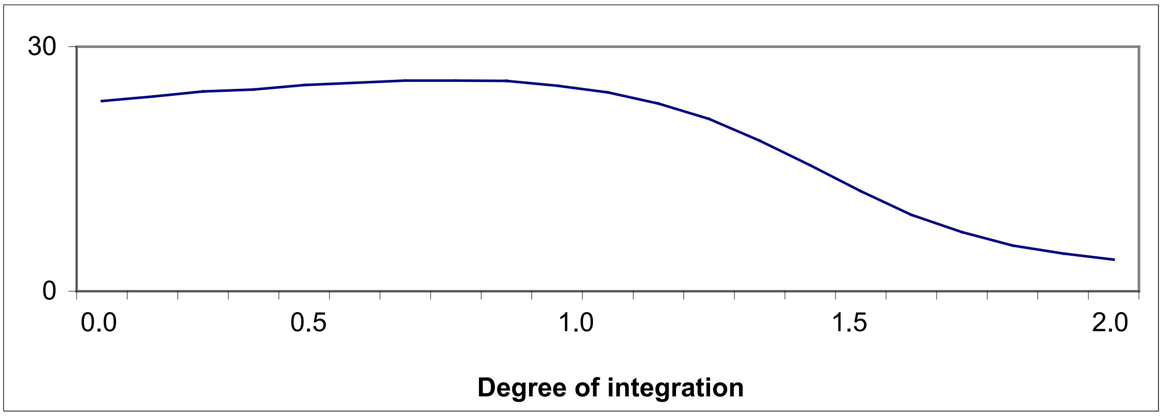
Mean amplitude of recession



Mean amplitude of expansion



Average number of peaks



Note: we perform 2500 replications of process 1 (u_t is a white noise). The sample length is 300 observations.

TABLE 1

Business cycle characteristics for fractional processes with white noise disturbances

Sample size	Values of d	Aver. number of cycles	Mean length of recession	Mean length of expansion	Mean amplitude of recession	Mean amplitude of expansion
T = 100	0.00	7.37 (0.03)	6.29 (1.64)	11.01 (1.97)	2.24 (0.33)	2.24 (0.33)
	0.25	7.76 (0.03)	5.94 (1.48)	10.46 (1.75)	2.16 (0.34)	2.16 (0.35)
	0.50	8.07 (0.06)	5.73 (1.35)	9.83 (1.54)	2.15 (0.37)	2.15 (0.39)
	0.75	8.17 (0.06)	5.68 (1.30)	9.48 (1.45)	2.27 (0.45)	2.36 (0.48)
	1.00	7.69 (0.06)	6.15 (1.45)	9.63 (1.50)	2.79 (0.68)	3.00 (0.73)
	1.50	4.43 (0.04)	10.41 (2.90)	15.11 (2.02)	10.37 (3.67)	12.02 (3.61)
	2.00	2.66 (1.92)	13.00 (1.92)	19.39 (2.02)	29.12 (5.48)	36.84 (5.86)
T = 300	0.00	23.30 (0.18)	6.22 (0.99)	11.09 (1.21)	2.24 (0.19)	2.24 (0.19)
	0.25	24.64 (0.19)	5.92 (0.89)	10.37 (1.06)	2.16 (0.19)	2.16 (0.20)
	0.50	25.55 (0.20)	5.72 (0.81)	9.91 (0.95)	2.16 (0.21)	2.17 (0.23)
	0.75	25.79 (0.20)	5.70 (0.78)	9.52 (0.85)	2.29 (0.26)	2.35 (0.28)
	1.00	24.30 (0.19)	6.10 (0.86)	9.64 (0.90)	2.77 (0.40)	2.95 (0.43)
	1.50	12.35 (0.11)	12.21 (3.85)	18.86 (4.39)	13.73 (6.22)	19.13 (7.53)
	2.00	3.82 (0.04)	26.04 (8.01)	44.10 (9.66)	114.77 (44.12)	177.19 (51.54)
T = 500	0.00	39.33 (0.15)	6.24 (0.78)	11.08 (0.94)	2.25 (0.15)	2.25 (0.15)
	0.25	41.38 (0.33)	5.97 (0.70)	10.22 (0.82)	2.17 (0.15)	2.17 (0.15)
	0.50	43.11 (0.34)	5.73 (0.63)	9.76 (0.72)	2.16 (0.16)	2.18 (0.17)
	0.75	43.68 (0.34)	5.64 (0.59)	9.51 (0.67)	2.28 (0.20)	2.35 (0.21)
	1.00	41.07 (0.32)	6.07 (0.66)	9.67 (0.70)	2.76 (0.31)	2.95 (0.33)
	1.50	19.92 (0.18)	13.07 (3.70)	19.47 (4.06)	15.81 (6.64)	20.77 (7.62)
	2.00	4.63 (0.05)	33.54 (12.24)	61.70 (16.84)	196.67 (85.03)	336.16 (122.33)

Note : We perform 2500 replications. Standard errors in parenthesis.

TABLE 2

Business cycle characteristics for fractional processes with AR(1) disturbances and T = 300

Sample size	Values of d	Aver. number of cycles	Mean length of recession	Mean length of expansion	Mean amplitude of recession	Mean amplitude of expansion
ϕ = 0.25	0.00	26.12 (0.20)	5.66 (0.82)	9.62 (0.94)	2.28 (0.19)	2.29 (0.20)
	0.25	27.28 (0.21)	5.34 (0.72)	9.48 (0.85)	2.26 (0.21)	2.27 (0.22)
	0.50	27.92 (0.22)	5.25 (0.66)	9.02 (0.75)	2.33 (0.24)	2.37 (0.25)
	0.75	27.10 (0.21)	5.38 (0.67)	9.20 (0.75)	2.60 (0.31)	2.72 (0.34)
	1.00	24.51 (0.19)	5.95 (0.82)	9.91 (0.90)	3.35 (0.53)	3.73 (0.58)
	1.50	11.53 (0.11)	12.91 (4.21)	21.61 (5.07)	19.35 (8.89)	28.93 (11.06)
	2.00	3.75 (0.04)	27.38 (7.90)	44.55 (9.91)	157.92 (56.34)	223.80 (68.04)
ϕ = 0.50	0.00	27.88 (0.22)	5.25 (0.68)	9.17 (0.80)	2.34 (0.21)	2.34 (0.22)
	0.25	28.36 (0.22)	5.18 (0.63)	8.91 (0.71)	2.42 (0.24)	2.45 (0.25)
	0.50	27.78 (0.22)	5.22 (0.62)	9.15 (0.71)	2.64 (0.30)	2.73 (0.32)
	0.75	25.87 0.20)	5.63 (0.71)	9.57 (0.79)	3.22 (0.45)	3.46 (0.49)
	1.00	22.28 (0.17)	6.62 (1.00)	10.82 (1.08)	4.71 (0.90)	5.37 (0.98)
	1.50	9.54 (0.09)	15.82 (5.42)	26.83 (6.52)	35.61 (16.41)	51.23 (19.83)
	2.00	3.32 (9.03)	30.27 (7.93)	50.28 (9.22)	264.89 (83.44)	363.07 (93.09)
ϕ = 0.75	0.00	27.41 (0.21)	5.32 (0.66)	9.23 (0.76)	2.44 (0.25)	2.47 (0.27)
	0.25	26.76 (0.21)	5.40 (0.66)	9.57 (0.77)	2.72 (0.33)	2.82 (0.35)
	0.50	24.79 (0.19)	5.84 (0.75)	10.18 (0.86)	3.39 (0.50)	3.66 (0.54)
	0.75	21.39 (0.17)	6.82 (1.03)	11.62 (1.17)	4.91 (0.96)	5.60 (1.03)
	1.00	16.83 (0.13)	8.60 (1.67)	14.85 (1.96)	8.84 (2.38)	11.11 (2.67)
	1.50	6.42 (0.06)	22.15 (8.06)	40.25 (1.96)	88.09 (41.56)	136.14 (51.08)
	2.00	2.91 (0.03)	33.41 (6.43)	60.58 (8.20)	531.91 (124.68)	842.48 (153.26)

Note: We perform 2500 replications. Standard errors in parenthesis.

TABLE 3						
Business cycle characteristics for fractional processes with MA(1) disturbances and T = 300						
Sample size	Values of d	Aver. number of peaks	Mean length of recession	Mean length of expansion	Mean amplitude of recession	Mean amplitude of expansion
θ = 0.25	0.00	27.38 (0.21)	5.34 (0.74)	9.36 (0.88)	2.33 (0.19)	2.33 (0.19)
	0.25	28.52 (0.22)	5.09 (0.66)	9.05 (0.78)	2.29 (0.20)	2.30 (0.21)
	0.50	29.16 (0.23)	4.97 (0.60)	8.83 (0.71)	2.33 (0.22)	2.36 (0.24)
	0.75	28.39 (0.22)	5.13 (0.62)	8.82 (0.70)	2.54 (0.29)	2.65 (0.31)
	1.00	25.83 (0.20)	5.72 (0.76)	9.31 (0.81)	3.21 (0.48)	3.49 (0.52)
	1.50	12.13 (0.11)	12.64 (4.24)	20.25 (4.80)	18.46 (8.73)	25.75 (10.15)
	2.00	3.84 (0.04)	24.79 (7.32)	42.11 (9.01)	130.87 (47.07)	197.12 (57.26)
θ = 0.50	0.00	32.33 (0.25)	4.52 (0.53)	8.02 (0.61)	2.48 (0.19)	2.48 (0.19)
	0.25	33.01 (0.26)	4.42 (0.48)	7.82 (0.55)	2.47 (0.20)	2.47 (0.22)
	0.50	32.65 (0.25)	4.46 (0.48)	7.88 (0.54)	2.55 (0.24)	2.60 (0.26)
	0.75	30.95 (0.24)	4.72 (0.52)	8.21 (0.58)	2.86 (0.32)	2.99 (0.35)
	1.00	27.32 (0.21)	5.35 (0.69)	9.13 (0.77)	3.68 (0.56)	4.11 (0.62)
	1.50	12.37 (0.11)	12.74 (4.31)	20.70 (4.96)	22.77 (10.65)	32.14 (12.71)
	2.00	3.83 (0.04)	25.55 (7.55)	44.90 (10.15)	169.58 (60.35)	273.02 (81.86)
θ = 0.75	0.00	36.14 (0.28)	4.04 (0.41)	7.25 (0.47)	2.69 (0.20)	2.69 (0.20)
	0.25	36.13 (0.28)	4.07 (0.40)	7.05 (0.44)	2.71 (0.22)	2.72 (0.23)
	0.50	34.94 (0.27)	4.18 (0.41)	7.44 (0.46)	2.85 (0.26)	2.90 (0.26)
	0.75	32.52 (0.25)	4.53 (0.48)	7.76 (0.53)	3.22 (0.36)	3.39 (0.39)
	1.00	28.32 (0.22)	5.18 (0.66)	8.88 (0.73)	4.18 (0.64)	4.71 (0.70)
	1.50	12.46 (0.11)	12.23 (4.15)	20.54 (4.92)	25.05 (11.84)	36.50 (14.54)
	2.00	3.79 (0.04)	25.51 (7.23)	47.07 (10.31)	197.69 (69.00)	328.39 (97.74)

Note: We perform 2500 replications. Standard errors in parenthesis.

TABLE 4				
Business cycle datation				
Country	Our datation		Reference datation*	
	Peak	Trough	Peak	Trough
FRANCE	80:1 84:1 90:3 92:1 95:3	75:1 80:4 85:1 91:1 93:1	79:4 82:1 84:1 92:1	75:1 81:1 82:4 85:1
UNITED KINGDOM	64:4 73:1 78:4 84:1 90:2	65:1 74:1 81:1 84:3 91:3	79:2 83:4 90:1	74:1 81:1 84:2 92:1
UNITED STATES	73:2 80:1 81:1 90:2 92:4	70:1 75:1 80:3 82:1 91:1	73:4 80:1 90:3	70:4 75:1 82:4 91:1

* Reference datation corresponds to the NBER datation for the United States and to the datation proposed by Artis and al. (1997) for France and the United Kingdom.

TABLE 5					
Business cycle characteristics of the log of the					
Country	Number of peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
FRANCE	5	14.75 (3.52)	3.25 (0.41)	0.11 (0.03)	0.01 (0.002)
UNITED KINGDOM	5	17.00 (5.62)	4.00 (0.94)	0.18 (0.05)	0.02 (0.003)
UNITED STATES	5	21.25 (3.44)	5.00 (1.28)	0.16 (0.04)	0.02 (0.01)

Standard errors in parenthesis.

TABLE 6

Best ARFIMA model specification for the log of the real GDP series

	ARFIMA	t-tests		AR coefficients			MA coefficients			Robinson's tests		
Country	(p, d, q)	d=1	d=2	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	d = 1	$d = d^*$	d = 2
FRANCE	(0, 1.47, 2)	9.40	-10.60	---	---	---	-0.86	0.20	---	1.99	1.23'	-2.31
UNITED KINGDOM	(1, 1.38, 2)	5.42	-8.85	-0.87	---	---	0.58	-0.38	---	1.73	0.07'	-1.69
UNITED STATES	(0, 1.36, 0)	3.60	-6.40	---	---	---	---	---	---	2.16	-0.79'	-2.34

The last column corresponds to the tests of Robinson (1994), testing H_o: $d = d_o$, where d_o is the maximum likelihood estimated of d obtained in previous tables. ' means non-rejection values at the 95% significance level.

TABLE 7

Best ARI(1)MA model specifications for the log of the real GDP

	ARIMA	AR coefficients			MA coefficients		
Country	(p, d, q)	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
FRANCE	(1, 1, 2)	0.99	---	---	-1.35	0.42	---
UNITED KINGDOM	(1, 1, 2)	0.95	---	---	-0.90	0.10	---
UNITED STATES	(1, 1, 1)	0.95	---	---	-0.63	---	---

Best ARI(2)MA model specifications for the log of the real GDP

	ARIMA	AR coefficients			MA coefficients		
Country	(p, d, q)	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
FRANCE	(0, 2, 2)	---	---	---	-1.35	0.42	---
UNITED KINGDOM	(0, 2, 1)	---	---	---	-0.99	---	---
UNITED STATES	(1, 2, 1)	0.30	---	---	-0.98	---	---

TABLE 8

Simulated business cycle characteristics of the log of the real GDP series with ARFIMA MODELS

Country	Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
FRANCE	10 (0.087)	11.7804 (2.063)	7.3447 (1.818)	0.0150 (0.0041)	0.013 (0.0035)
UNITED KINGDOM	5 (0.0446)	22.8476 (4.7520)	16.0179 (4.752)	0.0803 (0.0234)	0.0685 (0.023)
UNITED STATES	9 (0.0761)	13.6392 (2.7195)	8.8280 (2.4950)	0.0294 (0.0101)	0.0237 (0.0089)

Simulated business cycle characteristics of the log of the real GDP series with ARI(2)MA MODELS

Country	Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
FRANCE	6 (0.0664)	15.086 (2.827)	17.216 (2.909)	0.0311 (4.0073)	0.0283 (0.008)
UNITED KINGDOM	3 (0.033)	24.066 (3.800)	16.031 (3.760)	0.4389 (0.0971)	0.314 (0.090)
UNITED STATES	3 (0.035)	27.423 (4.215)	16.9742 (3.875)	.2502 (0.056)	0.1707 (0.051)