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Modelling the US, the UK and Japanese  
unemployment rates. Fractional integration and  
structural breaks

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Fractional integration and data frequency  
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#### ABSTRACT

A general procedure for fractional integration and structural breaks at unknown points in time is used, which allows for different orders of integration and deterministic components in each subsample. First, the procedure is extended to the non-linear case, and is showed by means of Monte Carlo experiments that it performs well in a non-linear environment. Second, it is applied to test for a single break in the unemployment rate in the US, the UK and Japan. The results shed some light on the empirical relevance of alternative unemployment theories for these countries. Specifically, a structuralist interpretation appears more appropriate for the US and Japan, whilst a hysteresis model accounts better for the UK experience (and also for the Japanese one in the second subsample). These findings are interpreted in terms of structural instability in labour markets with different features.

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## 1. Introduction

There is an extensive literature documenting asymmetric behaviour of the unemployment rate, namely the fact that it appears to rise faster in recessions than it falls during recoveries. Possible explanations include asymmetric adjustment costs of labour, especially in the case of Europe (see Bentolilla and Bertola, 1990), and asymmetry in job destruction (i.e., the fact that jobs disappear at a higher rate during recessions than expansions – see Caballero and Hammour, 1994), and/or in capital destruction (see Bean, 1989). Various non-linear models have been proposed to model this type of behaviour (see, e.g., Acemoglu and Scott, 1994; Skalin and Teräsvirta, 2002; Proietti, 2003; etc.). With a few exceptions (such as Van Dijk et al., 2002; Davidson and Teräsvirta, 2002; Caporale and Gil-Alana, 2007), these models generally consider the standard cases of  $I(0)$  and/or  $I(1)$  variables, but do not allow for fractional integration.

In this paper we investigate the possibility of structural breaks at unknown points in time in univariate fractionally integrated (possibly non-linear) time series. In particular, we focus on the study of the unemployment rate series in the US, UK and Japan, three of the main OECD economies. Our contribution is twofold. The first is of a methodological nature. As in Bai and Perron (1998), Zeileis et al. (2003) and others, our approach is based on the least square principle. However, it is more general, since it allows for different (fractional) orders of integration as well as different deterministic trends in each subsample. Further, it allows for non-linear structures. Incorporating non-linearities into a fractional integration framework with breaks at unknown points in time has not been attempted before, and represents a methodological innovation. We investigate the properties of our method in the non-linear case by means of Monte Carlo simulation

techniques. The second contribution is to provide useful empirical evidence to discriminate among different theories of unemployment. Note that under hysteresis (or persistence – see, e.g., Blanchard and Summers, 1986, 1987 and Cross, 1987) the order of integration (denoted by  $d$ ) should be equal to or at least close to 1, implying that the effects of the shocks are permanent (if  $d = 1$ ) or at least highly persistent (if  $d < 1$  though close to 1), whilst infrequent breaks would give support to the structuralist view (Phelps, 1994). On the other hand, a value of  $d$  close to 0 would favour NAIRU theories (see, e.g., Friedman, 1968). We obtain empirical results using our method to detect breaks, and interpret the findings in terms of differences in the structure of the labour markets of the countries we analyse.

Although structural instability is often been found in the unemployment rate, relatively few papers have focused on detecting break points in this series. One of the few exceptions is Konya (2000)), who tests the unit-root versus stationarity hypotheses in the case of Australia allowing for the possibility of two endogenous breaks in the deterministic trend function under the stationarity alternative hypothesis. He finds the logarithm of the unemployment rate to be trend stationary with possibly two structural breaks. In a more general setting, Stock and Watson (1996) found evidence of structural instability in macroeconomic relationships, while Douglas, Stock and Watson (1997) focus on the analysis of the interaction between unemployment and monetary policy. In another paper, Koop and Potter (2001) also conclude that non-linearities in many economic series are due to structural instability, while other authors (e.g. Holden and Nymoen, 2002) argue that unemployment instability might be due to misspecification of the model. Our study contributes to this relatively limited literature by implementing our general testing method (allowing for both fractional parameters and non-linearities) and

providing new evidence of unemployment instability for three of the main industrialised countries (the US, the UK and Japan).

The layout of the paper is as follows. Section 2 briefly reviews the recent empirical literature testing alternative theories of unemployment. Section 3 describes the econometric approach. Section 4 reports Monte Carlo evidence on the performance of our method in the non-linear case. Section 5 presents the empirical results. Section 6 summarises the main findings and offers some concluding remarks.

## **2. Testing unemployment theories: a brief review of the literature**

Alternative unemployment theories have different implications for the time series properties of unemployment. For instance, the natural rate theory (see Friedman, 1968, and Phelps, 1967, 1968) implies that the unemployment rate should fluctuate around a stationary equilibrium level (the natural rate, also known as NAIRU), which is determined by economic fundamentals. In “structuralist” models (see Phelps, 1994) the natural rate is “endogenised”: as in NAIRU models, unemployment is viewed as having an equilibrium level to which it generally reverts when hit by shocks, but it is also thought to be subject to infrequent structural breaks, resulting from changes in economic fundamentals, which affect the equilibrium itself. Hence the unemployment series should be stationary (or, more generally, mean-reverting) provided one allows for breaks. However, the adequacy of both these theories to account for the behaviour of unemployment has been questioned in recent decades, owing to the observed high persistence of unemployment in Europe. Therefore, hysteresis models have been developed (see Blanchard and Summers, 1986, 1987, and Barro, 1988), which characterise unemployment as a path-dependent variable, with temporary shocks having permanent or very highly persistent effects. The implication is that the

unemployment rate should be a stochastic process with long memory, exhibiting a (near) unit root.

Several empirical papers have used time series and panel techniques to discriminate between the different unemployment theories. Initially, standard unit root tests (such as Augmented Dickey-Fuller (ADF, 1979) or Phillips-Perron (PP, 1988)) were carried out (see, e.g., Blanchard and Summers, 1986, and Alogoskoufis and Manning, 1988), the results generally being consistent with the hysteresis hypothesis. Gordon (1989) defined full hysteresis as the case of a unit root and persistence as AR stationarity with roots being close to the unit circle, and did not find any evidence of full hysteresis in five countries (France, Germany, USA, Japan and the UK) for the time period 1873-1986. Graafland (1991) concluded that, in the 80s, the labour market in the Netherlands was characterised by a high and persistent level of unemployment. Lopez et al. (1996) reported that monthly unemployment in Spain (1976M6-1994M10) was consistent with hysteresis. Nott (1996) did not find evidence of hysteresis in Canada, while Wilkinson (1997) did. Subsequent studies allowed for structural breaks as well (see, e.g., Mitchell, 1993, Bianchi and Zoega, 1998, and Papell et al, 2000), using, for instance, the method developed by Zivot and Andrews (1992). The evidence presented in these papers mostly gave support to structuralist rather than hysteresis theories, as it suggested that unemployment can be adequately modelled as a stationary series with an infrequently changing equilibrium level.

In order to deal with the well-known problem of the low power of standard unit root tests (see Campbell and Perron, 1991 and DeJong, 1992), more recent studies have performed panel unit root tests. Again, some of these contributions do not address the issue of possible breaks – examples are the papers of Song and Wu (1998) and Leon-Ledesma (2002), where tests developed by Levin, Lin and Chu (2002, LLC hereafter) and

Im, Pesaran and Shin (2003) respectively are implemented. In most cases, such studies conclude that hysteresis theories are most appropriate for the European experience, whilst NAIRU models appear to work better for the US. By contrast, other papers take into account the possibility of breaks in a panel context. Prominent examples are Murray and Papell (2000) and Strazicich, Tieslau and Lee (2001), applying to OECD data, respectively, the LLC test and a panel LM t-statistic with up to two level breaks introduced by Im, Lee and Tieslau (2005). Allowing for breaks is generally found to lead to a rejection of the hysteresis hypothesis, and to be consistent instead with structuralist explanations of the behaviour of unemployment. Various theoretical models have been put forward to endogenise the natural rate of unemployment. They rely alternatively on productivity growth (Pissarides, 1990), real interest rates (Blanchard, 1999), stock prices (Phelps, 1999), institutional variables (Nickell, 1998 and Nickell and Van Ours, 2000), or the interaction between institutional and macroeconomic variables (Blanchard and Wolfers, 2000).

Another recent strand of the literature has exploited new developments in econometrics to study unemployment persistence using fractionally integrated (e.g., ARFIMA) models (see, for instance, Tschernig and Zimmermann, 1992; Crato and Rothman, 1996; Gil-Alana, 2001a, 2002; etc.). This approach, unlike earlier ones focusing exclusively on integer degrees of differentiation, i.e.,  $d = 0$  (stationarity) and  $d = 1$  (nonstationarity), has the advantage of allowing for fractional degrees of integration. Moreover, it is well known that standard unit root testing procedures (ADF, PP, etc.) have extremely low power if the alternatives are of a fractional form (Diebold and Rudebusch, 1991, Hassler and Wolters, 1994; Lee and Schmidt, 1996, etc.) and therefore fractional integration techniques could be very useful to model unemployment.

The present study falls into the same category, and adopts a framework which enables us to investigate the relevance of the three types of unemployment theories mentioned above; since it allows for fractional orders of integration, it is appropriate for both stationary processes (NAIRU models), and highly persistent/nonstationary ones (hysteresis hypothesis), and by incorporating structural breaks it can also be used to model processes exhibiting regime change (structuralist theories). For instance, suppose that it is found that, as a result of including a break, the degree of persistence appears to be different in the two subsamples, i.e. under one regime shocks have persistent though not permanent effects on the unemployment rate, (i.e.,  $d < 1$ ) whilst under the other those effects are permanent ( $d \geq 1$ ). This would indicate that the behaviour of unemployment is well captured by structural and hysteresis models respectively in the two subsamples, shedding light on their empirical relevance. However, the value of  $d$  by itself does not provide conclusive evidence in favour of any particular theory. For example, the NAIRU and the structuralist approach (with breaks) assume the existence of an equilibrium level to which the series converges, implicitly assuming that unemployment is mean-reverting ( $d < 1$ ). Persistence of unemployment under the hysteresis hypothesis, though, can also be consistent with values of  $d$  smaller than though close to 1, with the effect of the shocks disappearing in the very long run.

Non-linearities in the unemployment rate are also well documented. Specifically, unemployment has been found to rise faster in recessions than it falls during recoveries (see, e.g., Rothman, 1991, 1996, for the US case). Possible explanations are asymmetries in adjustment costs (Bentolilla and Bertoli, 1990), job destruction (Caballero and Hammour, 1994), and capital destruction (Bean, 1989). Such non-linearities have been modelled using Markov-switching models (see, e.g., Bianchi and Zoega, 1998), Smooth Transition AutoRegressive (STAR) models (see, e.g., Skalin and Teräsvirta, 2002),



unobserved component models (Proietti, 2003), or a non-linear fractional integration framework (see Caporale and Gil-Alana, 2007). Given their possible importance, in the next section we extend the procedure to detect breaks adopted by Gil-Alana (2008) in order to allow for fractional integration with non-linear structures as well.

### 3. The econometric approach

In this section we present a procedure that enables us to examine the stationarity/nonstationarity nature of the series of interest in a very general framework. Firstly, instead of restricting ourselves to the standard I(0) (stationarity) or I(1) (nonstationarity) cases, we consider the possibility of fractional orders of integration. Assuming that a sequence  $\{u_t, t = 0, \pm 1, \dots\}$  is I(0), defined as a covariance stationary process with spectral density function that is positive and finite, we define an I(d) process as:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where  $d$  can be any real number, and  $x_t = 0, t \leq 0$ . This latter condition is standard in applied work and is related to the Type II as opposed to the Type I definition of fractional integration (see Robinson and Marinucci, 2001, and more recently, Gil-Alana and Hualde, 2008). The differencing parameter  $d$  plays a crucial role from a statistical viewpoint. Thus, if  $d \in (0, 0.5)$ , the series is covariance stationary and mean-reverting, with shocks disappearing in the long run; if  $d \in [0.5, 1)$ , the series is no longer stationary but still mean-reverting, while  $d \geq 1$  indicates nonstationarity and non-mean-reversion. It is therefore crucial to examine if  $d$  is smaller than or equal to or higher than 1. These processes (with  $d > 0$ ) were initially introduced by Robinson (1978), Granger (1980,

1981) and Hosking (1981), and they have been widely employed in recent years to describe the dynamic behaviour of economic and financial data. (Diebold and Rudebusch, 1989; Baillie, 1996; Gil-Alana and Robinson, 1997, Haldrup and Nielsen, 2007, Morana, 2008, Ruiz and Veiga, 2008, etc.). Secondly, our framework also allows for the inclusion of deterministic terms, like intercepts, linear trends or even non-linear structures of the Threshold AutoRegressive (TAR) or Momentum Threshold AutoRegressive (M-TAR)-form (see, e.g. Enders and Granger, 1998; Enders and Siklos, 2001). Finally, the possibility of structural breaks at unknown points in time is also taken into account.

Gil-Alana (2008) proposes a simple procedure for estimating fractional orders of integration with deterministic linear trends and structural breaks at unknown dates. Following that approach, we assume that  $y_t$  is the observed time series, generated by the model

$$y_t = \alpha_1 + \beta_1 t + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1, \dots, T_b, \quad (2)$$

and

$$y_t = \alpha_2 + \beta_2 t + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1, \dots, T, \quad (3)$$

where the  $\alpha$ 's and the  $\beta$ 's are the coefficients corresponding respectively to the intercept and the linear trend;  $d_1$  and  $d_2$  may be real values,  $u_t$  is  $I(0)$ , and  $T_b$  is the time of the break that is assumed to be unknown. Note that given the difficulties in distinguishing between models with fractional orders of integration and those with broken deterministic trends, (Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.), it is important to consider estimation procedures that deal with fractional unit roots in the presence of broken deterministic trends. The model in equations (2) and (3) can also be written as:

$$(1 - L)^{d_1} y_t = \alpha_1 \tilde{I}_t(d_1) + \beta_1 \tilde{I}_t(d_1) + u_t, \quad t = 1, \dots, T_b, \quad (4)$$

and

$$(1 - L)^{d_2} y_t = \alpha_2 \tilde{1}_t(d_2) + \beta_2 \tilde{t}_t(d_2) + u_t, \quad t = T_b + 1, \dots, T, \quad (5)$$

where  $\tilde{1}_t(d_i) = (1 - L)^{d_i} 1$ , and  $\tilde{t}_t(d_i) = (1 - L)^{d_i} t$ ,  $i = 1, 2$ .

The approach taken in this article is based on the least square principle and is similar to the method proposed by Bai and Perron (1998), but extended to the fractional case. First, we choose a grid for the values of the fractionally differencing parameters  $d_1$  and  $d_2$ , for example,  $d_{10} = 0, 0.01, 0.02, \dots, 2$ ,  $i = 1, 2$ . Then, for a given partition  $\{T_b\}$  and given  $d_1, d_2$ -values,  $(d_{10}^{(i)}, d_{20}^{(i)})$ , we estimate the  $\alpha$ 's and the  $\beta$ 's by minimising the sum of squared residuals,

$$\min \sum_{t=1}^{T_b} \left[ (1-L)^{d_{10}^{(j)}} y_t - \alpha_1 \tilde{1}_t(d_{10}^{(j)}) - \beta_1 \tilde{t}_t(d_{10}^{(j)}) \right]^2 + \sum_{t=T_b+1}^T \left[ (1-L)^{d_{20}^{(j)}} y_t - \alpha_2 \tilde{1}_t(d_{20}^{(j)}) - \beta_2 \tilde{t}_t(d_{20}^{(j)}) \right]^2,$$

*w.r.t.  $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$*

for uncorrelated  $u_t$ , or, alternatively, using GLS for weakly autocorrelated disturbances. Note that when using this procedure we are assuming that the two subsamples are independent. Thus,  $\tilde{1}_t(d_2)$  and  $\tilde{t}_t(d_2)$  above are computed for subsamples starting at  $T_b+1$ , i.e., based on the Type II definition of fractional integration.

Let  $\hat{\alpha}(T_b; d_{10}^{(1)}, d_{20}^{(1)})$  and  $\hat{\beta}(T_b; d_{10}^{(1)}, d_{20}^{(1)})$  denote the resulting estimates for partition  $\{T_b\}$  and initial values  $d_{10}^{(1)}$  and  $d_{20}^{(1)}$ . Substituting these estimated values in the objective function, we obtain  $RSS(T_b; d_{10}^{(1)}, d_{20}^{(1)})$ , and minimising this expression for all values of  $d_{10}$  and  $d_{20}$  in the grid we obtain:  $RSS(T_b) = \arg \min_{\{i, j\}} RSS(T_b; d_{10}^{(i)}, d_{20}^{(j)})$ . Then, the estimated break date,  $\hat{T}_k$ , is such that  $\hat{T}_k = \arg \min_{i=1, \dots, m} RSS(T_i)$ , where

the minimisation is over all partitions  $T_1, T_2, \dots, T_m$ , such that  $T_i - T_{i-1} \geq |\epsilon T|$ . The regression parameter estimates are the associated least-squares estimates of the estimated  $k$ -partition, i.e.,  $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$ ,  $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$ , and their corresponding differencing parameters,  $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$ , for  $i = 1$  and  $2$ .

In Gil-Alana (2008) it is shown that the rates of convergence of the estimates are similar to those in Bai and Perron (1998), since the values are chosen in such a way as to minimise the residual sum of squares and, under the appropriate specification,  $u_t$  should follow an  $I(0)$  process. Moreover, several Monte Carlo experiments conducted in that study show that the procedure performs well even in relatively small samples.

This model can easily be extended to allow for multiple breaks. One then considers the following specification:

$$y_t = \alpha_j + \beta_j t + x_t; (1 - L)^{d_j} x_t = u_t, \quad t = T_{j-1} + 1, \dots, T_j,$$

for  $j = 1, \dots, m+1$ ,  $T_0 = 0$  and  $T_{m+1} = T$ , and  $m$  stands for the number of breaks. The break dates  $(T_1, \dots, T_m)$  are explicitly treated as unknown and for  $i = 1, \dots, m$ , we have  $\lambda_i = T_i/T$ , with  $\lambda_1 < \dots < \lambda_m < 1$ . Following the same lines as in the previous case, for each  $j$ -partition,  $\{T_1, \dots, T_j\}$ , denoted  $\{T_j\}$ , the associated least-squares estimates of  $\alpha_j$ ,  $\beta_j$  and the  $d_j$  are obtained by minimising the sum of squared residuals in the  $d_j$ -differenced models,

$$\text{i.e., } \sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (1 - L)^{d_j} (y_t - \alpha_j - \beta_j t)^2, \quad \text{where } \hat{\alpha}_j(T_j), \hat{\beta}_j(T_j) \text{ and } \hat{d}_j(T_j) \text{ denote the}$$

resulting estimates. Substituting them in the new objective function and denoting the sum of squared residuals as  $RSS_T(T_1, \dots, T_m)$ , the estimated break dates  $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m)$  are obtained by:  $\min_{(T_1, T_2, \dots, T_m)} RSS_T(T_1, \dots, T_m)$  where the minimisation is again obtained

over all partition  $(T_1, \dots, T_m)$ . See Olsen et al. (2008) for another recent paper for detecting short and long range change points in time series.

In the present paper we extend the above procedure to include non-linearities. That is, we consider for each subsample a model of the form

$$y_t = f(z_t; \theta^i) + x_t, \quad t = 1, 2, \dots, \quad i = 1, 2, \quad (6)$$

where  $f$  may be of a non-linear nature,  $z_t$  is a vector of non-stochastic regressors,  $\theta^i$ ,  $i = 1, 2$ , represents the unknown coefficients, and  $x_t$  is driven by  $(1 - L)^{d_i} x_t = u_t$ . The main problem with this equation lies in the interaction between the fractional polynomial  $(1 - L)^{d_i}$  and the possibly non-linear function  $f$ , and the estimation of the parameters involved in such a relationship. For the purpose of the present study, let us assume that  $f(z_t; \theta^i) = (\theta^i)' g(z_t)$ , where  $g$  is of a non-linear nature. In such a case, the model becomes:

$$(1 - L)^{d_i} y_t = (\theta^i)' w_{it} + u_t, \quad t = 1, 2, \dots, \quad i = 1, 2, \quad (7)$$

where  $w_{it} = (1 - L)^{d_i} g(z_t)$ , and hence, the "non-linearity" is not in terms of the parameters, but in terms of a non-linear function of the variables  $z_t$ . We can obtain the OLS estimate of  $\theta^i$  and residuals:

$$\hat{u}_t = (1 - L)^{d_i} y_t - (\hat{\theta}^i)' w_{it}, \quad \hat{\theta}^i = \left( \sum_t w_{it} w_{it}' \right)^{-1} \sum_t w_{it} (1 - L)^{d_i} y_t,$$

and the same type of analysis as in Gil-Alana (2008) can be conducted here. Although, as explained above for the linear case, this procedure could be extended to the case of multiple breaks (see again Gil-Alana, 2008), in the present study we focus instead on a single break to explain the stochastic nature of unemployment. The reason is the following. Structuralist theories imply infrequent breaks in the unemployment series.

Therefore, there could be more than a single break. However, for the validity of the type of long-memory (fractional integration) model we use for unemployment it is necessary that the data span a sufficiently long period of time to detect the dependence across time of the observations; given the sample size of the series employed here, the inclusion of two or more breaks would result in relatively short subsamples, thereby invalidating the analysis based on fractional integration. Moreover, other recent empirical studies on unemployment in the US and UK come to the conclusion that a single break is sufficient to describe the behaviour of these series. For instance, Papell et al. (2000) examine sixteen OECD countries, and find that for most of them the unit root hypothesis can be rejected when one or two breaks are incorporated. Coakley et al. (2001) also identify a single structural break in the UK (1980, in Germany as well), and in the US (1973). Similarly, Anderton (1998) focuses on a single break when modelling unemployment.

#### 4. Monte Carlo results

In this section we examine by means of Monte Carlo simulations the performance of the procedure described in Section 3 in the case of non-linear structures. We assume that the Data Generating Process (DGP) is the following:

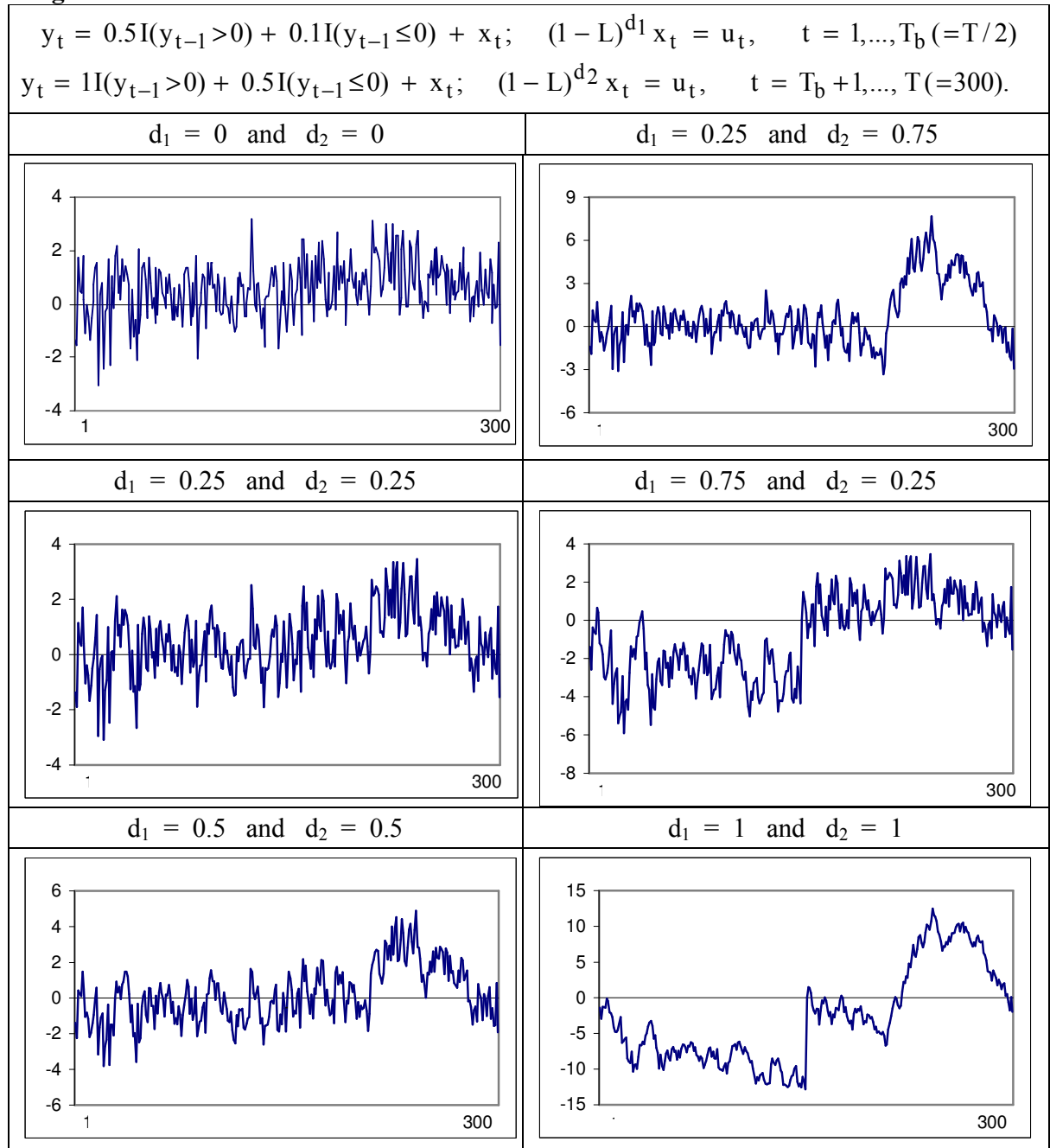
$$y_t = 0.5I(y_{t-1} > 0) + 0.1I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{d1} x_t = u_t, \quad t = 1, \dots, T_b, \quad (8)$$

and

$$y_t = 1I(y_{t-1} > 0) + 0.5I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{d2} x_t = u_t, \quad t = T_b + 1, \dots, T, \quad (9)$$

where  $I(x)$  stands for the indicator function, and  $u_t$  is a white noise process.

**Figure 1: Examples of simple realisations with non-linear terms, fractional integration and structural breaks**



We generate Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986). The experiment was also carried out using non-zero values for the threshold parameters in (8) and (9), and the results were fairly similar to those presented here. The same conclusions were reached when other values for the coefficients in equations (8) and (9) were considered, suggesting that the size of the non-linear effects in the procedure is not very important.

Figure 1 contains plots of simple realisations of the model given by (8) and (9) with  $T = 300$ ,  $T_b = 150$ , and  $(d_1, d_2) = (0, 0), (0.25, 0.25), (0.5, 0.5), (0.25, 0.75), (0.75, 0.25)$  and  $(1, 1)$ . It can be seen that, when the possibility of fractional integration is not considered (i.e.,  $d_1 = d_2 = 0$ ), visual inspection of the series does not clearly reveal the occurrence of a structural break. Hence, the non-linear structure does not seem to help to detect the presence of breaks in the data in  $I(0)$  contexts. By contrast, when  $d_1 = d_2 = 0.25$  or  $0.5$ , the break is clearly noticeable, and even more so for higher orders of integration (e.g.,  $d_1 = d_2 = 1$ ). It can also be clearly detected when the orders of integration are different for each subsample.

Tables 1 – 3 report the probabilities of correctly determining the timing of the break and the fractional differencing parameters in the model given by (8) and (9). In Table 1 it is assumed that in the true DGP,  $T_b = T/2$ ,  $d_1 = 0.2$  and  $d_2 = 0.4$ . Thus, the two subsamples are covariance stationary, though with a component of long-memory behaviour. In Table 2,  $T_b$  is still equal to  $T/2$ ,  $d_1 = 0.7$  and  $d_2 = 0.3$ . Finally, in Table 3, the break is assumed to take place at  $T/4$ , with  $d_1 = 0.6$  and  $d_2 = 0.8$ , and hence the two subsamples are now nonstationary. In all cases, we perform the procedure described in Section 3 for a grid of  $d_1, d_2$  values =  $0, 0.2, 0.4, \dots, 2$ . Of course, we could also have considered the case with  $d_1, d_2$  equal to  $0, 0.1, \dots, 1$ , or even used a grid of 0.01 increments, but in such cases the



probability of correctly determining the break would be substantially reduced by this refinement in the procedure, leading to higher probabilities for the parameter values close to the true one. We take values for the break  $T^* = (T_b - T/5), \dots, (1), \dots, (T_b + T/5)$ , where  $T_b$  is the correct time of the break. The number of replications is equal to 10,000 in each case.

It is apparent from these tables that the adopted procedure determines accurately the break date in virtually all cases. We find zero-probabilities for all values of  $d_1$  and  $d_2$  if  $T^*$  is smaller than  $T_b - 2$  or higher than  $T_b + 2$ . Thus, we report in the tables only the probabilities corresponding to  $T^* = T_b - 2, T_b - 1, T_b, T_b + 1, \text{ and } T_b + 2$ .

In Tables 1 and 2 the break is assumed to take place at  $T/2$ . First, we consider the case of  $d_1 = 0.2$  and  $d_2 = 0.4$  (Table 1). It can be seen that, if  $T = 100$ , the procedure yields the correct specification of the model in 37.45% of the cases. The other two cases with a large percentage correspond both to  $T^* = T_b$ , with  $d_1 = d_2 = 0.2$  (32.07%), and  $d_1 = 0.2$  and  $d_2 = 0.6$  (12.35%). For this sample size, the sum of the probabilities of correctly detecting the break-time is 93.19%. Increasing the sample size appears to increase the probability of correctly specifying the model: this is equal to 57.91% with  $T = 200$ ; 70.15% with  $T = 300$ , and 84.45% with  $T = 500$ . In this last case, the probability of accurately determining the break point is equal to 95.68%.

**Table 1: Probabilities of detecting the true model: break at  $T/2$ ;  $d_1 = 0.2$ ;  $d_2 = 0.4$**

$$y_t = 0.5I(y_{t-1} > 0) + 0.1I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.2}x_t = u_t, \quad t = 1, \dots, T_b = (T/2),$$

$$y_t = 1I(y_{t-1} > 0) + 0.5I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.4}x_t = u_t, \quad t = T_b + 1, \dots, T.$$

$T^*$	$d_1$	$d_2$	$T = 100$	$T = 200$	$T = 300$	$T = 500$
$T_b - 2$	0.2	0.4	0.0019	0.0019	0.0000	0.0037
	0.2	0.6	0.0019	0.0019	0.0000	0.0018
	0.2	0.2	0.0099	0.0000	0.0038	0.0000
$T_b - 1$	0.2	0.4	0.0119	0.0039	0.0174	0.0244
	0.2	0.6	0.0059	0.0115	0.0000	0.0000
	0.4	0.4	0.0019	0.0000	0.0000	0.0000
	0.2	0.2	0.3207	0.2316	0.1356	0.0676
$T_b$	0.2	0.4	0.3745	0.5791	0.7015	0.8445
	0.2	0.6	0.1235	0.0888	0.0542	0.0261
	0.2	0.8	0.0039	0.0000	0.0000	0.0000
	0.4	0.2	0.0518	0.0094	0.0058	0.0018
	0.4	0.4	0.0418	0.0405	0.0368	0.0150
	0.4	0.6	0.0099	0.0077	0.0019	0.0018
	0.6	0.4	0.0039	0.0000	0.0000	0.0000
	0.6	0.6	0.0019	0.0000	0.0000	0.0000
$T_b + 1$	0.2	0.2	0.0159	0.0057	0.0096	0.0018
	0.2	0.4	0.0119	0.0135	0.0251	0.0073
	0.2	0.6	0.0019	0.0039	0.0019	0.0037
	0.4	0.2	0.0019	0.0000	0.0000	0.0000
	0.4	0.4	0.0019	0.0000	0.0019	0.0000
$T_b + 2$	0.2	0.4	0.0000	0.0000	0.0039	0.0000

In bold, the probabilities corresponding to the true model.

In Table 2 we assume nonstationarity for the first subsample ( $d_1 = 0.7$ ) and stationarity for the second one ( $d_2 = 0.3$ ), still with a break at  $T/2$ . Here the highest probabilities are in all cases those corresponding to the true model, followed closely by those for the local departures ( $d_1 = 0.7$  with  $d_2 = 0.1$  and  $0.5$ ), with  $T^* = T/2$ . In Table 3 it is assumed that the break takes place at  $T/4$ , and that the two subsamples are nonstationary ( $d_1 = 0.6$  and  $d_2 = 0.8$ ). Here, the probabilities corresponding to the true model are slightly smaller than in the previous cases, though still sufficiently high to detect the true DGP, especially if the sample size is large enough.

On the whole, the evidence presented in this section seems to suggest that our procedure for fractional integration with a structural break and non-linear structures

performs well in finite samples. Integer orders of differentiation ( $d_i = 0$ , or 1, or both, for  $i = 1, 2$ ) were also considered in the simulations, and the results were even more precise than those presented here for the fractional case.

**Table 2: Probabilities of detecting the true model: break at  $T/2$ ;  $d_1 = 0.7$ ;  $d_2 = 0.3$**

$$y_t = 0.5I(y_{t-1} > 0) + 0.1I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.7} x_t = u_t, \quad t = 1, \dots, T_b = (T/2),$$

$$y_t = 1I(y_{t-1} > 0) + 0.5I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.3} x_t = u_t, \quad t = T_b + 1, \dots, T.$$

$T^*$	$d_1$	$d_2$	$T = 100$	$T = 200$	$T = 300$	$T = 500$
$T_b - 2$	0.3	0.1	0.0036	0.0000	0.0000	0.0000
	0.3	0.3	0.0019	0.0000	0.0000	0.0000
	0.5	0.1	0.0036	0.0000	0.0000	0.0000
	0.5	0.3	0.0036	0.0093	0.0015	0.0013
	0.7	0.1	0.0055	0.0062	0.0015	0.0013
	0.7	0.3	0.0055	0.0139	0.0185	0.0215
	0.7	0.5	0.0018	0.0015	0.0015	0.0000
	0.9	0.1	0.0018	0.0000	0.0000	0.0000
	0.9	0.3	0.0036	0.0000	0.0000	0.0000
$T_b - 1$	0.3	0.3	0.0019	0.0015	0.0000	0.0000
	0.5	0.1	0.0092	0.0015	0.0000	0.0000
	0.5	0.3	0.0055	0.0062	0.0061	0.0013
	0.7	0.1	0.0092	0.0030	0.0030	0.0026
	0.7	0.3	0.0147	0.0170	0.0231	0.0269
	0.7	0.5	0.0018	0.0000	0.0015	0.0013
	0.9	0.3	0.0036	0.0015	0.0000	0.0000
	0.1	0.3	0.0036	0.0000	0.0000	0.0000
	0.3	0.1	0.0220	0.0019	0.0000	0.0000
$T_b$	0.3	0.3	0.073	0.0000	0.0000	0.0000
	0.3	0.5	0.036	0.0000	0.0000	0.0000
	0.5	0.1	0.0662	0.0481	0.0169	0.0081
	0.5	0.3	0.1104	0.0729	0.0601	0.0377
	0.5	0.5	0.0313	0.0124	0.0030	0.0000
	0.7	0.1	0.1694	0.1770	0.1203	0.0661
	0.7	0.3	0.1915	0.3742	0.5540	0.7125
	0.7	0.5	0.0497	0.0465	0.0354	0.0148
	0.9	0.1	0.0773	0.0326	0.01008	0.0026
$T_b + 1$	0.9	0.3	0.0681	0.0652	0.0478	0.0229
	0.9	0.5	0.0202	0.0077	0.0092	0.0013
	0.5	0.1	0.0019	0.0046	0.0015	0.0000
	0.5	0.3	0.0147	0.0015	0.0060	0.0039
	0.5	0.5	0.0019	0.0015	0.0000	0.0000
	0.7	0.1	0.0239	0.0046	0.0169	0.0107
	0.7	0.3	0.0184	0.0496	0.0447	0.0553
	0.7	0.5	0.0092	0.0093	0.0092	0.0013
	0.9	0.1	0.0165	0.0030	0.0015	0.0000
$T_b + 2$	0.9	0.3	0.0073	0.0061	0.0045	0.0054
	0.5	0.5	0.0019	0.0031	0.0000	0.0000
	0.7	0.1	0.0055	0.0155	0.0000	0.0000

In bold, the probabilities corresponding to the true model.

**Table 3: Probabilities of detecting the true model: break at T/4; d<sub>1</sub> = 0.6; d<sub>2</sub> = 0.8**

$$y_t = 0.5I(y_{t-1} > 0) + 0.1I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.6}x_t = u_t, \quad t = 1, \dots, T_b = (T/4),$$

$$y_t = 1I(y_{t-1} > 0) + 0.5I(y_{t-1} \leq 0) + x_t; \quad (1-L)^{0.8}x_t = u_t, \quad t = T_b + 1, \dots, T.$$

T*	d <sub>1</sub>	d <sub>2</sub>	T = 100	T = 200	T = 300	T = 500
T <sub>b</sub> - 2	0.4	0.6	0.0044	0.0000	0.0000	0.0000
	0.4	0.8	0.0088	0.0079	0.0075	0.0051
	0.4	1.0	0.0044	0.0000	0.0000	0.0000
	0.6	0.8	0.0021	0.0177	0.0132	0.0323
	0.8	0.6	0.0021	0.0000	0.0000	0.0000
	0.8	0.8	0.0044	0.0059	0.0075	0.0034
	0.4	0.6	0.0088	0.0000	0.0000	0.0000
	0.4	0.8	0.0154	0.0177	0.0150	0.0034
	0.6	0.6	0.0088	0.0019	0.0000	0.0000
	0.8	0.8	0.0022	0.0197	0.0189	0.0357
T <sub>b</sub> - 1	0.6	0.8	0.0022	0.0197	0.0189	0.0357
	0.8	0.6	0.0088	0.0019	0.0018	0.0017
	0.8	0.8	0.0022	0.000	0.0113	0.0017
	0.4	0.6	0.0927	0.0256	0.0189	0.0034
	0.4	0.8	0.2980	0.2984	0.1988	0.1499
	0.4	1.0	0.0794	0.0256	0.0056	0.0017
	0.6	0.6	0.0463	0.0494	0.0284	0.0051
	0.6	0.8	0.1346	0.3379	0.4659	0.5928
	0.6	1.0	0.0463	0.0276	0.0113	0.0034
	0.8	0.4	0.0066	0.0000	0.0000	0.0000
T <sub>b</sub>	0.8	0.6	0.0198	0.0079	0.0037	0.0000
	0.8	0.8	0.0684	0.0869	0.0928	0.0562
	0.8	1.0	0.0198	0.0118	0.0075	0.0000
	1.0	0.6	0.0110	0.0000	0.0000	0.0000
	1.0	0.8	0.0286	0.0019	0.0000	0.0000
	1.0	1.0	0.0088	0.0000	0.0000	0.0000
	0.4	0.6	0.0066	0.0000	0.000	0.0000
	0.4	0.8	0.0088	0.0059	0.0056	0.0034
	0.6	0.6	0.0088	0.0039	0.0000	0.0000
	0.8	0.8	0.0022	0.0197	0.0340	0.0579
T <sub>b</sub> + 1	0.6	0.6	0.0022	0.0197	0.0340	0.0579
	0.8	0.6	0.0176	0.0000	0.0000	0.0000
	0.8	0.8	0.0066	0.0079	0.0208	0.0102
	1.0	0.6	0.0044	0.0000	0.0018	0.0000
T <sub>b</sub> + 2	0.6	0.6	0.0044	0.0039	0.0000	0.0000
	0.6	0.8	0.0021	0.0059	0.0189	0.0255
	0.6	1.0	0.0000	0.0039	0.0000	0.0017
	0.8	0.8	0.0044	0.0019	0.0094	0.0052

In bold, the probabilities corresponding to the true model.

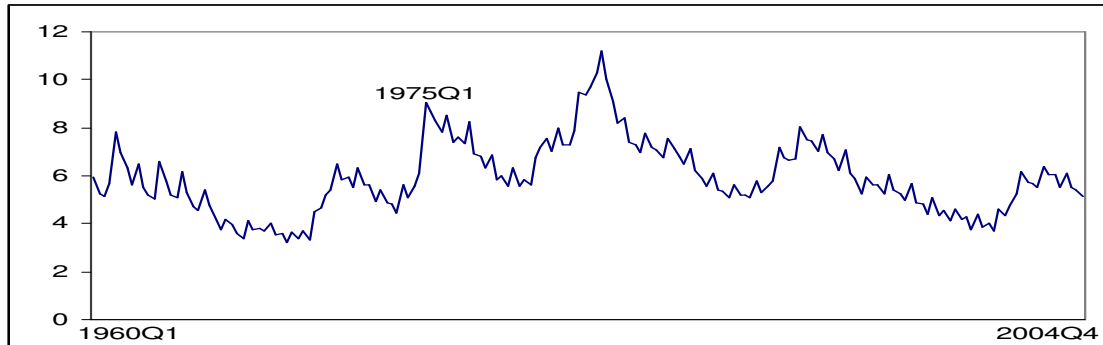
## 5. Empirical results

All three unemployment rates are seasonally adjusted. The US series is quarterly, and covers the period 1960Q1-2004Q4; the source is the IMF's International Financial Statistics. The UK series is monthly, for the time period 1971M1 – 2005M9, and is

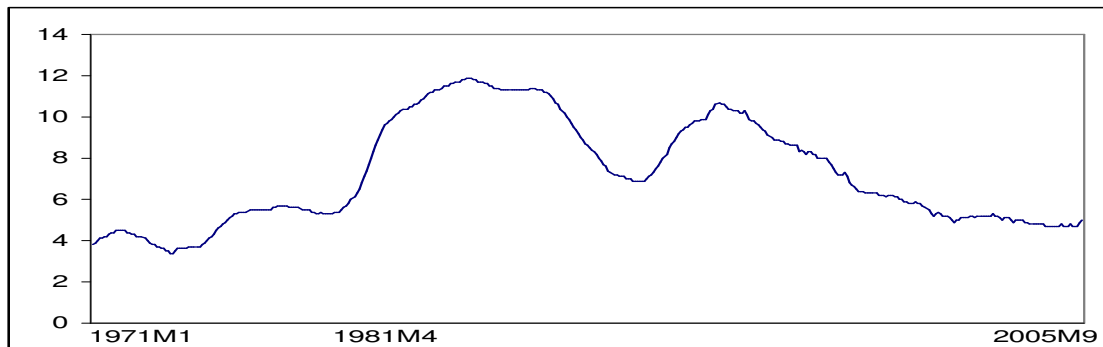
obtained from the Labour Force Survey (<http://www.statistics.gov.uk>). In the case of Japan, the quarterly series covers the period 1973Q1-2004Q2, and the data source is the OECD statistics.

**Figure 2: Unemployment rates**

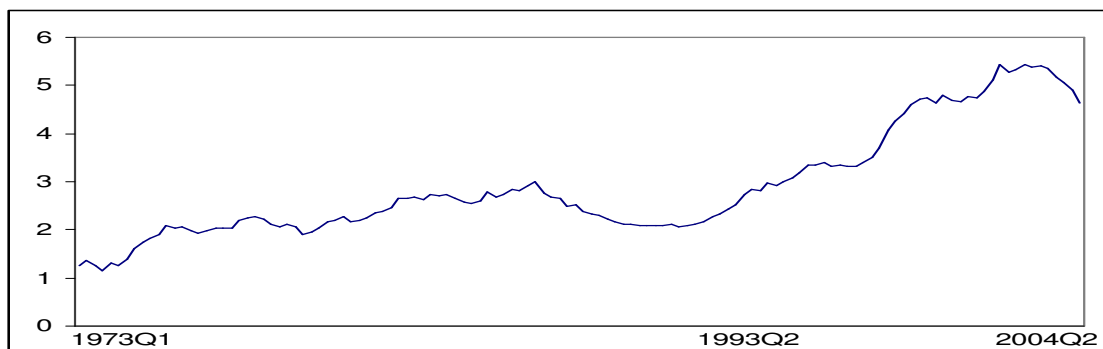
i) United States



i) United Kingdom



i) Japan



**Table 4: Results for the United States**

	Model 1		Model 2		Model 3		Model 4		Model 5	
	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)
$T_b$	73Q4	75Q1	75Q1	74Q4	75Q1	75Q1	72Q2	74Q3	74Q4	74Q3
$D_1$	0.88	0.92	0.68	0.71	0.67	0.72	0.72	1.01	0.74	0.66
CI $d_1$	(0.67, 0.98)	(0.75, 1.16)	(0.58, 0.86)	(0.54, 0.86)	(0.59, 0.98)	(0.59, 0.97)	(0.48, 0.99)	(0.88, 1.22)	(0.53, 0.99)	(0.41, 0.93)
$d_2$	0.96	0.92	0.83	1.03	0.80	1.11	0.96	0.86	0.86	1.03
CI $d_2$	(0.82, 1.00)	(0.87, 1.07)	(0.70, 0.91)	(0.88, 1.12)	(0.71, 0.91)	(0.92, 1.33)	(0.88, 1.09)	(0.77, 1.00)	(0.76, 0.98)	(0.93, 1.11)
$\alpha_1$	---	---	5.758 (10.48)	4.994 (4.36)	5.799 (10.16)	4.887 (3.36)	---	---	---	---
$\alpha_2$	---	---	8.837 (15.42)	16.34 (8.31)	10.70 (6.86)	10.94 (2.82)	---	---	---	---
$\beta_1$	---	---	---	---	-0.007 (-0.27)	0.0052 (0.16)	---	---	---	---
$\beta_2$	---	---	---	---	-0.003 (-1.32)	-0.038 (-0.68)	---	---	---	---
$\gamma_1^1$	---	---	---	---	---	---	5.342 (8.93)	0.503 (1.03)	---	---
$\gamma_1^2$	---	---	---	---	---	---	0.350 (1.18)	0.363 (1.47)	---	---
$\gamma_2^1$	---	---	---	---	---	---	5.419 (9.30)	0.606 (1.27)	---	---
$\gamma_2^2$	---	---	---	---	---	---	0.598 (2.09)	0.601 (2.51)	---	---
$\delta_1^1$	---	---	---	---	---	---	---	---	-0.609 (-1.07)	-0.536 (-0.43)
$\delta_1^2$	---	---	---	---	---	---	---	---	2.973 (5.17)	11.48 (5.26)
$\delta_2^1$	---	---	---	---	---	---	---	---	-0.327 (-0.56)	-0.234 (-0.19)
$\delta_2^2$	---	---	---	---	---	---	---	---	3.123 (5.29)	11.50 (5.20)
$\tau_1$	---	- 0.162	---	-0.053	---	-0.044	---	-0.095	---	0.100
$\tau_2$	---	- 0.111	---	-0.316	---	-0.418	---	-0.002	---	-0.291

t-values in parentheses. In bold, significant coefficients at the 95% significance level.

For the U.S. series, the main results are the following (see Table 4). The break date is almost the same in all models, ranging from 72Q2 to 75Q1 (it is 75Q1 in 4 out of the 10 models presented). It clearly corresponds to the first oil price shock. The values of  $d_1$  and  $d_2$  (the order of integration of the first and second subsample respectively) are in most cases between 0 and 1, providing evidence of fractional integration and persistence. The estimates of the parameter  $d_1$  range between 0.66 and 1.01, and those of  $d_2$  between 0.80 and 1.11. In general, we observe higher orders of integration in the second subsample, implying that the degree of persistence is higher after the break. The white noise and AR(1) specifications yield rather similar results. When a linear time trend is included in the model (model 3), the slope coefficients are statistically insignificant in both subsamples. Therefore, model 3 can be discarded in favour of model 2 (with an intercept). As for the non-linear structures, the coefficients of model 4 are significant for the first subsample ( $\gamma_1^1$  and  $\gamma_2^1$ ) in case of white noise errors and for  $\gamma_2^2$  in the two cases of white noise and AR(1)  $u_t$ , while in model 5 they are significant for the second subsample only ( $\delta_1^2$  and  $\delta_2^2$ ). The similarities between the coefficients of the non-linear models in the two subsamples seem to indicate that the adjustment process is symmetric in the case of US unemployment. Note, for example, that in model 4 the significant coefficients are 5.342 and 5.419 for the first subsample. In model 5, the values are 2.973 and 3.123 for the case of white noise  $u_t$ , and 11.485 and 11.501 with autocorrelated disturbances. In all these cases, standard F-tests were performed, providing evidence that the null hypotheses of  $\gamma_1^i = \gamma_2^i$  and  $\delta_1^i = \delta_2^i$  could not be rejected at conventional statistical levels.

A drawback of the procedure presented here is that it does not provide confidence intervals for the fractional differencing parameters. However, they can be easily obtained by using Robinson's (1994) parametric approach for each subsample. The results in this table indicate that the unit root null cannot be rejected in only two cases for the first subsample, while the same hypothesis cannot be rejected in seven out of ten cases for the second subsample.



**Table 5: Results for the United Kingdom**

	Model 1		Model 2		Model 3		Model 4		Model 5	
	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)
$T_b$	73m11	73m11	83m1	81m4	83m1	81m1	73m9	80m3	82m12	81m3
$d_1$	1.03	1.03	1.03	1.09	1.04	1.09	0.99	1.10	1.02	1.03
CI $d_1$	(0.89, 1.24)	(0.80, 1.39)	(0.97, 1.12)	(1.01, 1.21)	(0.97, 1.12)	(0.99, 1.21)	(0.48, 1.02)	(0.93, 1.21)	(0.95, 1.09)	(0.94, 1.09)
$d_2$	1.18	1.17	0.97	1.06	0.97	1.06	1.19	1.07	0.98	0.98
CI $d_2$	(1.06, 1.23)	(1.09, 1.25)	(0.91, 1.06)	(0.97, 1.18)	(0.91, 1.05)	(0.95, 1.19)	(0.48, 1.02)	(0.89, 1.15)	(0.85, 1.04)	(0.85, 1.04)
$\alpha_1$	---	---	3.065 (70.33)	3.013 (37.72)	3.019 (65.11)	3.027 (25.45)	---	---	---	---
$\alpha_2$	---	---	11.09 (152.2)	11.02 (45.55)	11.11 (2.88)	11.05 (2.23)	---	---	---	---
$\beta_1$	---	---	---	---	0.051 (1.33)	-0.020 (0.54)	---	---	---	---
$\beta_2$	---	---	---	---	-0.023 (-0.32)	-0.038 (-1.17)	---	---	---	---
$\gamma_1^1$	---	---	---	---	---	---	0.479 (2.04)	-0.007 (-0.56)	---	---
$\gamma_1^2$	---	---	---	---	---	---	0.035 (0.20)	0.014 (1.12)	---	---
$\gamma_2^1$	---	---	---	---	---	---	-0.005 (-0.02)	-0.084 (-0.03)	---	---
$\gamma_2^2$	---	---	---	---	---	---	0.041 (2.38)	0.057 (0.54)	---	---
$\delta_1^1$	---	---	---	---	---	---	---	---	-3.382 (-44.22)	-3.52 (-33.40)
$\delta_1^2$	---	---	---	---	---	---	---	---	4.030 (3.20)	1.972 (9.90)
$\delta_2^1$	---	---	---	---	---	---	---	---	-3.401 (-63.76)	-3.534 (-3.71)
$\delta_2^2$	---	---	---	---	---	---	---	---	4.024 (3.10)	1.964 (9.71)
$\tau_1$	---	0.035	---	-0.115	---	-0.116	---	0.016	---	-0.518
$\tau_2$	---	-0.107	---	-0.146	---	-0.146	---	-0.031	---	-0.266

t-values in parentheses. In bold, significant coefficients at the 95% significance level.

**Table 6: Results for Japan**

	Model 1		Model 2		Model 3		Model 4		Model 5	
	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)	W. N.	AR(1)
$T_b$	93Q1	92Q4	93Q2	93Q1	93Q2	93Q1	92Q3	92Q2	93Q1	92Q4
$d_1$	1.07	1.06	0.93	0.04	0.95	0.03	1.02	1.16	0.93	0.14
CI $d_1$	(0.96, 1.18)	(0.95, 1.20)	(0.78, 1.16)	(-0.03, 0.16)	(0.79, 1.14)	(-0.05, 0.10)	(0.88, 1.18)	(0.93, 1.28)	(0.79, 1.11)	(0.01, 0.48)
$d_2$	1.11	1.16	1.34	1.00	1.33	1.00	1.16	1.17	1.31	1.00
CI $d_2$	(1.00, 1.22)	(1.03, 1.34)	(1.15, 1.59)	(0.84, 1.16)	(1.14, 1.58)	(0.89, 1.20)	(1.01, 1.42)	(1.02, 1.45)	(1.09, 1.64)	(0.85, 1.33)
$\alpha_1$	---	---	1.2711 (4.75)	2.701 (6.47)	1.283 (4.78)	3.428 (4.03)	---	---	---	---
$\alpha_2$	---	---	2.3941 (22.01)	-151.6 (10.8)	2.370 (21.3)	-47.710 (-18.10)	---	---	---	---
$\beta_1$	---	---	---	---	- 0.020	-0.013 (-1.08)	---	---	---	---
$\beta_2$	---	---	---	---	0.045 (0.74)	0.046 (2.30)	---	---	---	---
$\gamma_1^1$	---	---	---	---	---	---	0.276 (3.94)	0.032 (0.65)	---	---
$\gamma_1^2$	---	---	---	---	---	---	-0.146 (-0.74)	-0.361 (-1.65)	---	---
$\gamma_2^1$	---	---	---	---	---	---	0.246 (3.56)	0.008 (0.17)	---	---
$\gamma_2^2$	---	---	---	---	---	---	-0.016 (-0.08)	-0.413 (-1.71)	---	---
$\delta_1^1$	---	---	---	---	---	---	---	---	-1.417 (-4.3)	-0.211 (-0.81)
$\delta_1^2$	---	---	---	---	---	---	---	---	-0.627 (-4.2)	-153.25 (-18.60)
$\delta_2^1$	---	---	---	---	---	---	---	---	-1.435 (-5.6)	-0.275 (-1.09)
$\delta_2^2$	---	---	---	---	---	---	---	---	-0.475 (-4.5)	-153.19 (-18.59)
$\tau_1$	---	0.038	---	0.969	---	0.956	---	-0.378	---	0.925
$\tau_2$	---	-0.793	---	0.016	---	0.054	---	-0.704	---	0.016

t-values in parentheses. In bold, significant coefficients at the 95% significance level.

The results for U.K. are reported in Table 5. As can be seen, in most cases the break occurs in the early 80s, namely a decade later than in the US (only in three of the estimated models there is an earlier break, more precisely in 1973). Further, unlike in the US case, virtually all the fractional parameters are estimated to be equal to or higher than 1, implying permanent deviations from equilibrium and a much higher degree of persistence in the UK unemployment rate. Anderton (1998) also finds persistence in the UK unemployment, though in his study the break occurs slightly earlier, namely in 1979.

Specifically, the values of  $d_1$  are found to range between 0.99 and 1.10, and those of  $d_2$  between 0.97 and 1.19. This is also in line with previous papers on UK unemployment, finding orders of integration above 1 (Gil-Alana, 2001b, c). As in the US case, there is no evidence of asymmetries when estimating non-linear TAR and M-TAR models. For example, in model 5, the significant coefficients in the two subsamples are -3.382, 4.030 and -3.401, 4.024, respectively in the case of white noise disturbances, and -3.522, 1.972 and -3.534, 1.964 with AR(1) disturbances.

Table 6 reports the results for Japan. It can be seen that in this case all the estimated break points are between 1992Q2 and 1993Q2. In fact, in four out of the ten models considered, the break takes places in the first quarter in 1993, namely one decade later than in the UK, and almost two decades later than in the US. It can be argued that this might be a consequence of the sample period considered, which starts in 1973Q1. Therefore, as a robustness check, we also applied our procedure to annual data, which are available from 1960, and the break was again found to occur in 1993. Anderton (1998) reports that a break took place in 1974, with an increase in unemployment persistence. Our results are more mixed.

The orders of integration are clearly different in the two subsamples, before and after the break. Specifically, before 1993, most of the estimated values are strictly smaller than 1, while after that date they are equal to or higher than 1. If  $u_t$  follows an AR(1) process, the estimated values of  $d_1$  are very close to 0 in three out of the five models, being equal to 0.04, 0.03 and 0.14 for model 2, 3, and 5 respectively. In these cases, however, it is clear that the low order of integration found in the first subsample is associated with large AR coefficients describing the time dependence across the observations, these coefficients being equal to 0.969, 0.959 and 0.925 for models 2, 3 and 5 respectively. Finally, note that while the unit root null hypothesis is almost never rejected during the first subsamples, this hypothesis is strongly rejected in favor of higher orders of integration after the breaks.

Next, we select for each country the best model on statistical grounds. In the case of the US, a time trend appears not to be required, since the slope coefficients are not significantly different from zero. Moreover, the estimated coefficients for the two non-linear models are rather similar in the two subsamples, and these models can be ruled out on the basis of standard specification tests. In the model with an intercept (model 2), the estimated order of integration is slightly above 1 in the second subsample for the case with AR(1) disturbances, and the AR coefficients are close to zero in the two subsamples. Therefore, the following specification is chosen for the US:

$$y_t = 5.758 + x_t; \quad (1 - L)^{0.68} x_t = \varepsilon_t, \quad t = 1, 2, \dots, T_b = 75Q1,$$

and

$$y_t = 8.837 + x_t; \quad (1 - L)^{0.83} x_t = \varepsilon_t, \quad t = T_b + 1, \dots, T,$$

implying nonstationary mean-reverting behaviour in the two subsamples and a higher degree of dependence after the break. According to this model, unemployment in the US rose in 1975Q1 and has not come down since then. However, this does not rule out the possibility of decreasing unemployment in the future since, consistently with the structuralist view, other breaks may occur in the future.

Note that in this model the unit root null hypothesis is rejected in the two subsamples in favour of lower orders of integration.

Moving on to the UK, again it appears that the non-linear models can be discarded because of the insignificant coefficients (in model 4) and the similarities between the coefficients in the two subsamples (in model 5). Model 3 can also be discarded on the grounds of the insignificance of the slope coefficients. Thus, we focus on model 2 (with an intercept): the coefficients for the orders of integration are slightly above 1 in all cases except for the second subsample with white noise  $u_t$ , and they are also higher in the first subsamples. We choose the specification with autocorrelated disturbances because of the significant coefficients and other likelihood criteria. Therefore, the selected model is the following:

$$y_t = 3.013 + x_t; (1 - L)^{1.09} x_t = u_t; u_t = -0.115u_{t-1} + \varepsilon_t, t = 1, 2, \dots, T_b=81M4,$$

and

$$y_t = 11.023 + x_t; (1 - L)^{1.06} x_t = u_t; u_t = -0.146u_{t-1} + \varepsilon_t, t = T_b+1, \dots, T.$$

Note that in this case the unit root null is rejected in the first subsample, though not in the second.

Finally, in the case of Japan, using the same type of arguments as before, we focus on the model with an intercept (model 2). When  $u_t$  is specified as a white noise the orders

of integration are 0.93 and 1.34 respectively for the two subsamples, whilst, if  $u_t$  is modelled as an AR(1) process, the corresponding values are 0.04 and 1.00. (As previously mentioned, the low order of integration in case of the first subsample is due to the competition with the AR coefficient in describing the dependence between the observations). For this country we choose the following model:

$$y_t = 1.271 + x_t; \quad (1 - L)^{0.93} x_t = \varepsilon_t, \quad t = 1, 2, \dots, T_b = 93Q2,$$

and

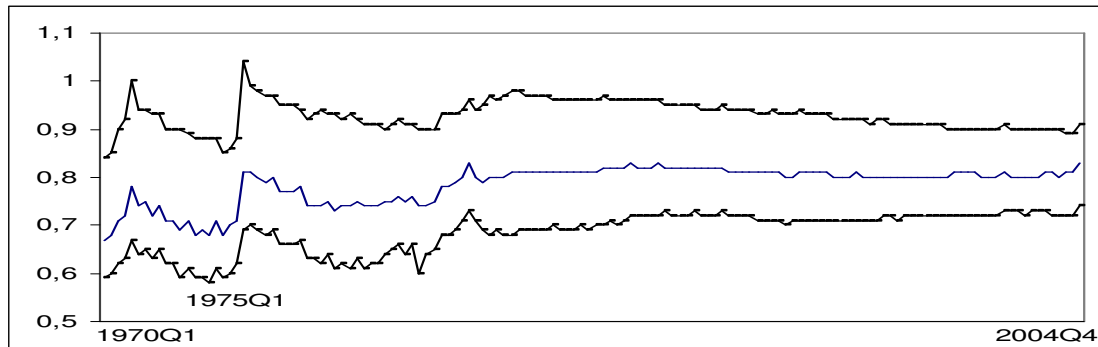
$$y_t = 2.394 + x_t; \quad (1 - L)^{1.34} x_t = \varepsilon_t, \quad t = T_b + 1, \dots, T.$$

Several diagnostic tests were carried out for each selected model and subsample. In particular, we found no evidence of serial correlation using the tests of Durbin (1970) and Godfrey (1978a,b) in any of the cases. Also, standard F-tests were conducted in the three series to check for the existence of a break and the results support this view in the three cases.

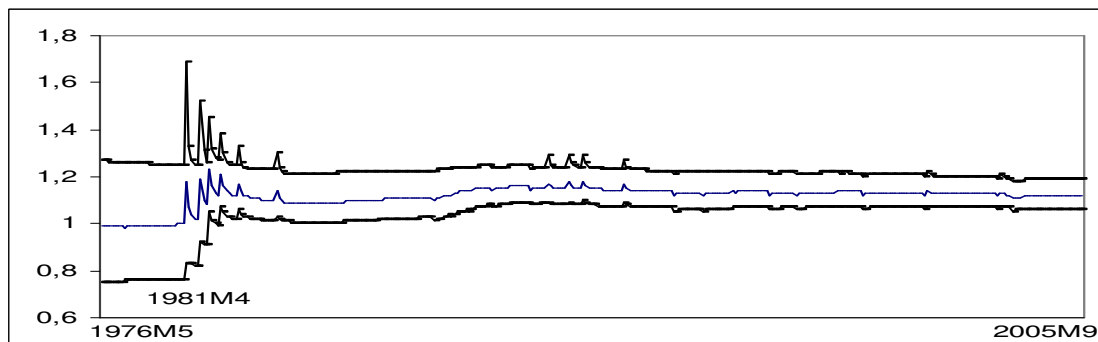
The results presented so far clearly indicate that the three series are characterised by a certain degree of instability, with different orders of integration once a structural break is taken into account. In what follows, we examine this issue more in depth by computing recursive estimates of the fractional differencing parameter to check whether this has a changing pattern. In particular, we adopt for each country the model specification selected above, and estimate  $d$  starting with a short sample (of about 60 observations) adding recursively one observation at a time and re-estimating the model for these different subsamples. The Whittle estimates along with the 95% confidence intervals are reported in Figure 3.

**Figure 3: Recursive Estimates of  $d$  and 95% confidence bands**

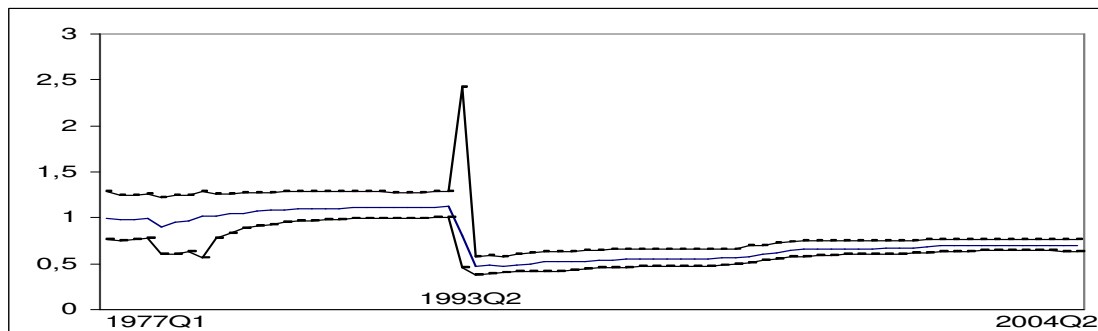
i) United States



ii) United Kingdom



iii) Japan



The results point to exactly the same break points as before. Specifically, in the case of the US there seems to be an increase in  $d$  after the occurrence of a break in 1975Q1 and the values are below unity in all cases. For the UK, the most unstable period is around 1981M4 and the estimates of  $d$  are above 1 in practically all cases. Finally, for Japan there

is a sharp drop in  $d$  around 1993 and the estimates fall from about 1 (with the sample ending in 1993) to strictly below 1 when the sample increases.

**Figure 4: Impulse response functions corresponding to the selected models for each country and each subsample**

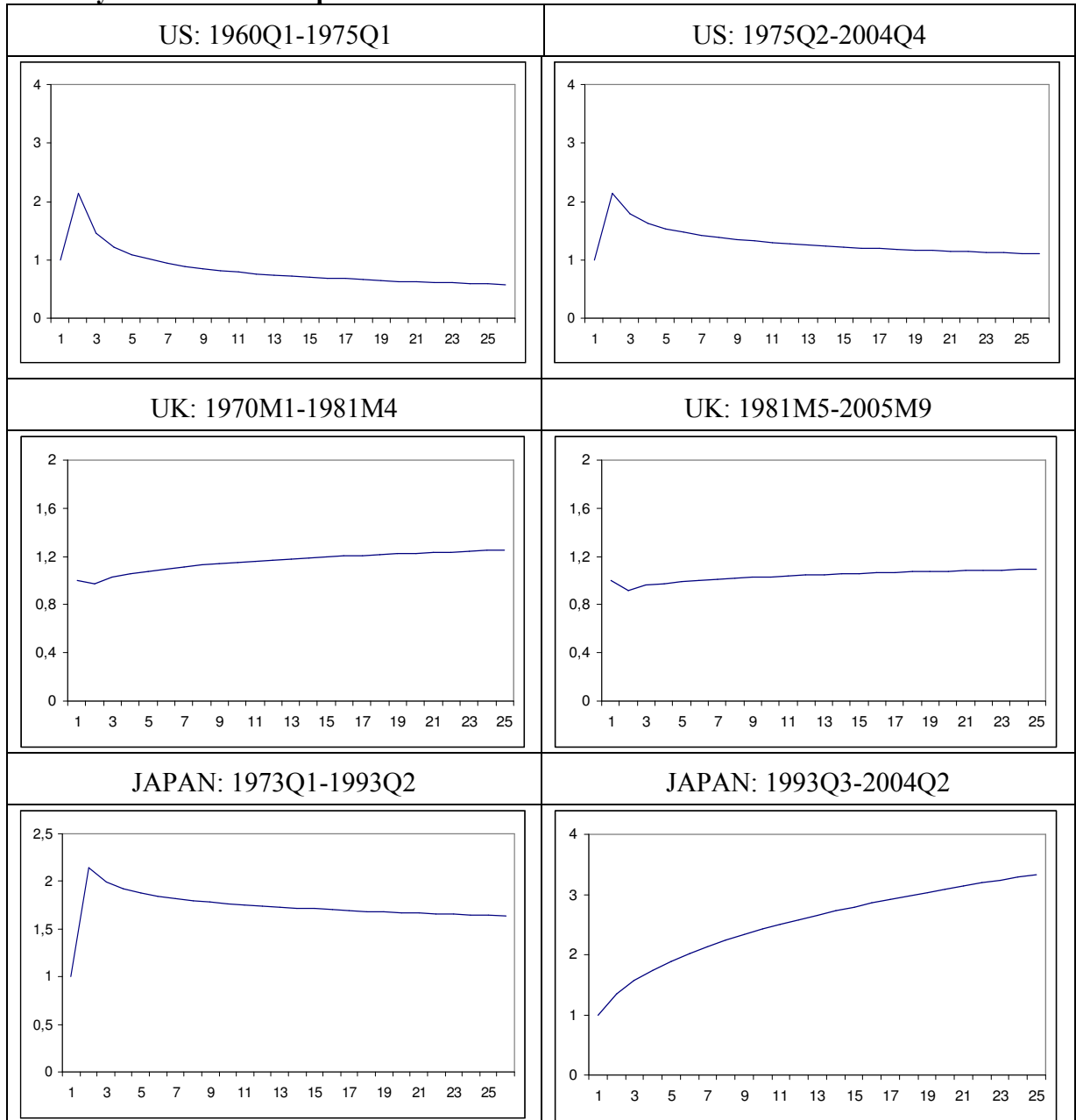




Figure 4 shows the impulse responses to a unit shock computed for twenty-five periods for each country and each subsample, based on the selected models. It can be seen that in the US (and also in Japan in the first subsample) the size of the response increases at first and then starts decreasing very slowly. Also, in the US the convergence process is slower during the second subsample. By contrast, in the remaining cases (the two subsamples in UK and especially the second one in Japan) the process is explosive and not mean-reverting even in the long run, consistently with the earlier findings on persistence.

To sum up, the empirical results indicate that, if we assume the existence of a single structural break in the data, the break point is different for each country, specifically it is in the early 70s in the UK, a decade later in the US, and two decades later in Japan. Also, interestingly, it appears that if the structural break is taken into account non-linearities vanish. On the whole, our results could be seen as *prima facie* evidence that a structuralist model might be appropriate to describe unemployment behaviour in these countries. However, a closer look at the order of integration of the series suggests that this might not be the case for all three economies. Specifically, the fact that the estimated fractional parameters are consistently higher than 1 in the UK means that in this country the economic environment is such that shocks to unemployment (and hence macroeconomic policy) have permanent effects, thereby giving support to a hysteresis model. An order of integration higher than 1 is also found in the case of Japan in the second subsample, indicating a change in the labour market that has resulted in hysteresis. By contrast, in the first subsample, and in both periods in the case of the US, the estimated degree of persistence implies that, although the speed of adjustment towards equilibrium is slow, unemployment exhibits mean reversion, consistently with the natural rate hypothesis,

appearing to be a near-unit root process, with shocks having long-lasting but not permanent effects. In Anderton (1998), the unemployment persistence parameter is estimated to be highest in the UK, and bigger in Japan than in the US, as in the present paper. This finding, combined with the evidence of a break, gives support to a structuralist view of unemployment behaviour. Hence it appears that different models (hysteresis and structuralist) are appropriate to account for the unemployment experience of the different countries and periods under investigation. The impulse response analysis also confirms the earlier findings.

Overall, our results, and the persistence ranking, are in line with earlier studies (e.g., Alogoskoufis and Manning, 1988), also reporting that Japan and the US typically display lower degrees of unemployment persistence than European countries. The mixed evidence on whether persistence has decreased or increased in the UK since the early 1980s might at first seem surprising in view of the labour market reforms (aimed at eliminating rigidities) implemented by the Conservative government led at the time by Mrs. Thatcher. However, other authors, such as Blanchflower and Freeman (1994), have reported a slower transition from unemployment to employment in the Thatcher years.

These results can be explained in terms of labour market and institutional differences. It is usually argued that the poorer unemployment performance of the European economies compared to the US is due to imperfections in the labour market (see, e.g., Layard et al, 1991). Features such as decentralised wage determination (see Calmfors and Driffill, 1988), low social security and trade union density, and minimum employment protection are often thought to account for the better labour market outcomes in the US. Note, however, that the neoclassical paradigm (with the associated deregulation policies) has been criticised as exhibiting some theoretical weaknesses, such

as second best problems, externalities etc. (see, e.g., Gregg and Manning, 1997). Also, it has been pointed out that, in addition to the degree of centralisation, other features of the bargaining process, such as the degree of unionisation and coordination, as well as the coverage of bargaining, are important (see OECD, 1997). But in the period 1970-1990 Japan outperformed even the US in terms of unemployment. In the Layard et al. (1991) study, wage flexibility in the small business sector and the fact that female workers exit the labour market during recessions were highlighted as important factors accounting for the absence of hysteresis in Japan. An OECD study (1994) attributed instead the successful Japanese experience to some other key features of the Japanese labour market, in particular long-term employment relationships, high investment in training and worker loyalty. As for the higher degree of persistence since the early 90s, this might be due to intensive on-the-job training and the resulting firm-specific skills leading to high costs of hiring and firing: in response to possibly temporary negative shocks, firms might be reluctant to fire employees with highly specialised skills, who would have to be replaced in the upturn by new workers requiring costly additional training. This reduces both job creation and destruction, and hence increases unemployment persistence. (For a more extensive discussion of the Japanese case, see Brunello, 1990).

## **6. Conclusions**

In this paper we have made a twofold contribution. First, we have extended to the non-linear case a general procedure to detect structural breaks at unknown points in time which allows for different orders of integration and deterministic components in each subsample (see Gil-Alana, 2008). The suggested procedure has been shown by means of Monte Carlo experiments to be able to determine accurately the timing of the break in a

non-linear, fractionally integrated framework. Second, we have applied it to test for a structural break in the US, UK and Japanese unemployment rates, and assessed the empirical relevance of alternative unemployment theories in each case.

We are able to identify a break in the three countries under study, and also find that non-linearities do not play a very important role. Moreover, unemployment appears to exhibit a higher degree of persistence in the UK compared to the US. In the case of the US persistence has risen since the beginning of the 80s, and in Japan hysteresis is found in the period starting in 1993. Overall, it seems that a structuralist interpretation (see Phelps, 1994) is more appropriate for the US and Japan, whilst a hysteresis model (see Blanchard and Summers, 1986, 1987, and Barro, 1998) accounts better for the UK experience (and also for the Japanese one in the second subsample). The persistence ranking and the results in general can be interpreted in terms of the different characteristics of the labour market in the countries being analysed. In particular, imperfections and rigidities preventing or slowing down labour market adjustment and clearing (despite the Thatcher reforms) might be responsible for the inferior unemployment performance of the UK compared to the US and Japan. The better labour market outcomes achieved in the two latter countries could be attributed to higher flexibility and deregulation in the case of the US, whilst long-term employment relationships and other related factors might play a role in the case of Japan (see Layard et al., 1991).

Our analysis could be extended by estimating a multivariate fractional model, including regressors such as real oil prices, output gap and real interest rates, which might account for the observed behaviour of the unemployment rate, and also allowing for possible cross-country linkages. This could be particularly informative when analysing

the impulse response of unemployment to various types of shocks (e.g. price shocks), including shocks affecting unemployment in other countries in the first instance. However, this is beyond the scope of the present study, and is left for future research.

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