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Are you a good employee or simply a good guy? Influence Costs and Contract Design

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ABSTRACT

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Abstract

We develop a principal-agent model in which the principal has access to hard and soft information about the agent's level of effort. We model the soft signal as being informative about the agent's level of effort but manipulable by the agent at a cost. We show that the presence of influence activities increases the cost of implementing the efficient level of effort for the principal. We also show that the manipulability of the soft signal leads to wage compression. However, when influence costs affect negatively the agent's productivity we establish that the design of influence-free contracts by the principal may lead to high-powered incentives. This result implies that high-productivity workers may face incentives schemes that are more responsive and give more weight to hard evidence compared to low-productivity workers.

1 Introduction

1.1 Hard and soft information

Recent financial scandals raise the question of the manipulability of quantitative information. A comprehensive analysis of this issue requires a precise understanding of the relation between the concepts of hard and quantitative information. In the finance literature, hard information is defined as being quantitative (Berger, Miller, Petersen, Rajan and Stein, 2001; Stein, 2002; Petersen, 2004; Liberti and Mian, 2009). In addition, hard information is easy to store, transmitted in impersonal ways and independent of the collection process; all these features making it a priori difficult for hard information to be manipulated. In contrast, the definition of hard and soft information operated by microeconomists does not rely on the distinction between quantitative and qualitative information but on the distinction between verifiable and unverifiable information. In particular, research on supervision and delegation in principal-agent models refer to hard information as being verifiable but potentially hideable (Tirole, 1986) whereas soft information is unverifiable (Baliga, 1999; Faure-Grimaud, Laffont and Martimort, 2003). These models allow for quantitative information to be either distorted when modeled as soft information or simply hided when modeled as hard information. Supervision models are characterized by three-tier structures based on Tirole's model (1986) in which the principal has the possibility to hire a supervisor to monitor the agent's privately observed level of effort. These models analyze whether it is optimal for the principal to hire a supervisor in a context in which agents and supervisors can collude. In particular, Faure-Grimaud, Laffont and Martimort (2003) consider the case in which the information held by the supervisor is unverifiable, that is the supervisor possesses soft information about the agent. In that context, the authors stress the equivalence in terms of efficiency between decentralized structures in which the principal contracts only with the supervisor and centralized structures in which the principal can directly contract with both agents and supervisors.

Our approach differs from supervision models as we do not consider a three-tier structure but a standard bilateral relationship between a principal and an agent. We then leave aside issues of collusion while extending the principal-agent model by allowing the agent, at the cost of undertaking influence activities, to manipulate certain pieces of information. In contrast, the literature in supervision considers the possibility to hide and distort one's own information whereas the current paper allows agents to manipulate others' information sets. In the rest of the paper we will refer to manipulable pieces of information as soft information in line with previous research in both finance and microeconomics.[1]

Our approach is also related to the literature in subjective evaluations as it uses a principal-agent model with unverifiable information (Baker, Gibbons and Murphy, 1994; MacLeod, 2003). In our model, the principal can propose contingent contracts that depend on a hard signal (determined by the level of production that is assumed to be verifiable) as well as on a soft (i.e. unverifiable) signal, which provides additional information about the agent's level of effort. In line with Baker, Gibbons and Murphy (1994), our paper analyzes incentives setting in a principal-agent model with both hard and soft information. In contrast with the previous authors the link between soft and hard signals follows in our paper from the existence of influence activities that improve the principal's perception of the agent-related soft information while undermining

[1] We do not need to interpret soft information as being uniquely qualitative.

the agent's level of performance at the same time (see Section 5). More importantly, our model departs from this literature as it allows for unverifiability to be endogenous. In our model, part of the information available to the principal can be made unverifiable by the agent's influence activities. The agent can manipulate information in order to affect positively the principal's evaluation of his work. As a result, influence activities do not simply distort information but allow agents to improve the principal's view of their performance level. In our model, influence activities tend to reduce aggregate welfare as they increase information asymmetry between the agent and the principal. In that respect our approach differs from the model developed by Maggi and Rodríguez-Clare (1995) in which agents can distort the principal's private pieces of information so as to reduce information asymmetry. As a result, information distortion may actually allow for falsification of information in equilibrium and may increase aggregate welfare as a result. Relatedly, Lacker and Weinberg (1989) consider a sharecropping model focusing on optimal risk-sharing when agents have the possibility to misreport the volume of the crop. The authors find that under some conditions, the principal should induce some misreporting in equilibrium so as to improve risk-sharing.

1.2 Influence and manipulability of information

Influence activities have been identified as actions completed by organizational members in order to bias in their favor the decisions of managers responsible for pay and promotion (Milgrom, 1988; Milgrom and Roberts, 1988; Milgrom and Roberts, 1992).[2] The authors develop a theory in which influence activities consist in gathering credentials for a promotion to a new job. In contrast with our model, the authors consider the case in which influence activities generate valuable information. As a general principal, this approach suggests that influence costs can be reduced by limiting the discretion of decision makers for those decisions that have a significant impact on the distribution of rents inside the organizations but that have minor impact on the firm's efficiency. Milgrom (1988) also mentions the possibility to use compensation schemes as one of the possible instruments to reduce influence. In their analysis, the compression of wage differentials between current jobs and promotion jobs is an effective strategy to reduce incentives to influence the manager's promotion decision. In our paper, we focus on optimal contracts rather than on organizational design as a potentially economical instru-

[2] Also, notice that influence costs have been considered as a key element of the theory of the firm (Gibbons, 2005).

ment to reduce influence costs. In our model the influence activity is effective in manipulating the principal's belief because it is unverifiable. This allows the agent to take advantage of the principal's cognitive biases. First, unverifiability implies that incentives contracts cannot get rid of influence by simply punishing observable attempts to manipulate the principal. The second element that allows influence behaviors to effectively raise the principal's assessment of the agent's work is the assumption that, under influence, the principal suffers from cognitive biases in their perception of the agent's soft signal. In particular, the principal may wrongly perceive negative signals about an agent's performance as positive signals when the agent undertakes apparently positive actions. Although modeled in a reduced form, our approach is related to psychological models of persuasion under which the principal may be manipulated.[3] In a recent paper, Mullainathan, Schwartzstein, Shleiffer (2008) propose a psychological account for persuasion using the concept of associative thinking under which individuals group situations into categories, and transfer the informational content of a given message from situations in a category where it is useful to those where it is not. Applying this concept to our model, we can see the principal as being unable to distinguish the following positive pieces of information "*The agent is a hard-working employee*" and *"The agent is a good person"* that belong to two different categories, work abilities and personality, respectively. The difficulty for the principal is to disentangle signals that concern the contribution of their employee to the firm and signals that relate to personal characteristics. We model persuasion as a reduced form of coarse thinking by considering the case in which the principal suffers from biased information processing in the spirit of Bénabou and Tirole (2002). The principal misperceives with a certain probability a negative soft signal about the agent's level of effort as being positive.

1.3 Incentives schemes under influence

In this paper we focus on the design of incentives contracts that implement the efficient level of effort.

We first show that the cost of implementing the efficient equilibrium increases as the soft signal is more manipulable and influence activities are more pervasive. This is the case because the principal relies on less informative signals to incentivate the agent and will have to increase the variance of wages to maintain incentives intact. This implies that a larger rent will have to be paid to the risk-adverse agent to ensure that the participation constraint holds. This result follows from Kim (1995)

[3]Persuasion has also been modeled using an informational approach (Milgrom and Roberts, 1986; Dewatripont and Tirole, 1999).

when taking into account that the efficiency of the information structure decreases in the manipulability of the soft signal.

When considering influence costs as privately incurred by the agent we show that wages become more compressed and less volatile as the soft signal becomes more manipulable. Also, more weight tends to be given to the hard signal in the payment scheme as the soft signal is more manipulable. These results are closely related to the sufficient statistic theorem (Holmström, 1979) under which incentives contracts should include all pieces of information that are informative about the agent's level of effort. As a consequence, incentives schemes will be less responsive to the soft signal as this signal becomes more manipulable and therefore less informative. Given that wages are less responsive to the soft signal, the range of possible wages as well as the variance of wages decreases. This finding is related to the result established in MacLeod (2003) in which wage compression occurs when the agent's measures of performance are subjective. However, the mechanism behind wage compression in MacLeod (2003) is distinct from our analysis. In their model, wage compression results from the additional constraint that requires the agents to truthfully reveal their signals to the principal. The idea of wage compression is also present in Milgrom's (1988) model of influence activities in promotion decisions in which reducing differential wages between available jobs is derived as an optimal response to counter influence activities by employees.

We finally extend our analysis to the case in which influence entails costs in terms of the firm's productive activities as is suggested by the initial definition in Milgrom (1988).

> "That time of course is valuable; if it were not wasted in influence activities, it could be used for directly productive or simply consumed as leisure."

In this context the principal will have to choose between accepting some influence activities in equilibrium or designing influence-free contracts that eliminate destructive manipulation attempts. We have to emphasize that, in our framework, all optimal contracts cannot be replicated by influence-free contracts so that an influence-proofness principle does not apply. Intuitively, this is the case because taking both private and productivity-based influence costs to be arbitrarily close to zero, we know that any influence-free contract would have to pay the agent a fixed wage. This is a necessary condition to eliminate the agent's incentives to boost his actual contribution. However, fixed wages contract do not satisfy the incentive-compatibility constraint implying that the agent will exert no effort in equilibrium. As a result, influence-free contracts may not implement the efficient equilibrium in that case. In general, principals designing influence-free contracts can rely on two possible strategies to dissuade influence activities. The first one consists in designing incentives contracts that are less responsive to the soft signal so as to reduce the expected benefits associated to influence activities. This first strategy would induce even greater wage compression than in the case of private influence costs. The cost of providing incentives to the agent would increase for the principal under this strategy. The second strategy that is actually followed by the principal in equilibrium consists in increasing the expected costs associated to influence activities by increasing the incentives associated to the hard signal. In that case, influence activities become less attractive as it reduces the probability that the agent will get the high payment associated to a high level of performance on the hard signal. As a result, we show that principals may be interested in designing high-powered incentives to avoid influence activities. More specifically, we show that high-powered incentives and influence-free contracts are more likely to be assigned to agents for which influence is especially costly in terms of firm productivity. In short, we expect high-productivity workers to be paid according to influence-free contracts whereas low-productivity agents are likely to be rewarded with contracts allowing for some level of influence activities. As a result, we show that high-productivity agents incentives contracts tend to be more responsive to the hard signal compared to low-productivity agents. In this version of our model, the substitutability between hard and soft information follows from the fact that improving the soft signal through influence activities is detrimental to the value of the hard signal. In Baker, Gibbons and Murphy (1994) as in our model with private influence costs the substitutability between different types of signals follows from the fact that highly precise hard signals are sufficient to ensure the implementation of the efficient equilibrium independently of the reception of soft signals.

The rest of this paper is organized as follows. We present our model in the case of rational supervisors in Section 2 and solve the corresponding model in the third section. The analysis of the model with influence is developed in Section 4. We extend our model for the case in which the influence activity is costly for the organization in Section 5. We conclude in Section 6. All proofs are available in the appendix.

2 The significant model

2.1 Players, payoffs and actions

We consider a principal-agent model characterized by the following four stages of the game:

- In Stage 1, the principal [she] sets a contract $\mathbf{w}$ that will be used to pay the agent [he] in the last stage of the game. The revenues for the risk-neutral principal $R(y)$ are positively related to the level of production in the organization y in $Y \equiv \{0,1\}$. This production depends on the level of effort (e) exerted by the agent on the productive task where e is in $\{e_L, e_H\}$, and $e_H > e_L$. As usual, the agent's level effort is unobservable for the principal. The principal, however, is able to observe the output (y) where $P[y = 0 \mid e = e_L] = P[y = 1 \mid e = e_H] = \rho_y$, and the precision of the signal ρ_y is in $\left[\frac{1}{2}, 1\right]$ where $R(1) > R(0) \geq 0$. Then, the level of output can be interpreted as a hard and non-manipulable signal.

  At this stage, the principal also decides whether to engage in supervising the agent $(s = 1)$ or not $(s = 0)$ in order to obtain an additional signal v on his actual level of effort. This signal is obtained at a cost $\phi_s > 0$. If the principal engages in supervision she will observe the signal v in $V \equiv \{G, B\}$ defined as follows: $P[v = B \mid e = e_L] = P[v = G \mid e = e_H] = \rho_v$, where the precision of the signal ρ_v is in $\left[\frac{1}{2}, 1\right]$. This piece of information obtained by the supervisor can be interpreted as a subjective evaluation of the supervisor about the employee's performance where B means *"The agent is a lazy (bad) employee"* and G means *"The agent is a hard-working (good) employee"*. As a result, the supervision message can be interpreted as soft information (Berger, Miller, Petersen, Rajan and Stein 2001, Stein 2002, Petersen 2004). If the agent is not being supervised then $v = \{\varnothing\}$.

- In Stage 2, the agent decides whether to exert a high level of effort $(e = e_H)$ or a low level of effort $(e = e_L)$ on the productive task. The cost of effort on the productive task is denoted by $C(e) \geq 0$. We denote $C \equiv C(e_H)$ and without loss of generality $C(e_L) = 0$. The agent is assumed to be risk adverse.

- In Stage 3, the agent decides whether to undertake an influence activity $(a = 1)$ or not $(a = 0)$. The private cost of effort associated to the influence activity is denoted by $\phi(a) \geq 0$, where $\phi_a \equiv \phi(1) > 0$ and $\phi(0) = 0$.

As we explain in the next subsection on information and influence, the influence activity affects the principal's qualitative assessment of the agent's actual level of effort but it is not contractible.

- In Stage 4, the principal pays the agent relying upon the contract chosen in Stage 1. The contract bases on the signals received in Stages 2 and 3 about the agent's levels of effort.

Figure 1: Timeline for the supervision and influence model

The payoffs for the principal are determined as follows.

$U_P \equiv R(y) - w - s\phi_s$, where $s \in \{0,1\}$ denotes whether supervision takes place ($s = 1$) or not ($s = 0$).

The payoffs for the agent are determined as follows.

$U_A \equiv V(w,e) = u(w) - C(e) - \phi(a) > 0$ where $u' > \varepsilon > 0$, $u'' < 0$.[4]

We denote $\bar{u} > 0$ the agent's outside option.

2.2 Information and influence

The principal does not observe the agent's level of effort on the productive task, $e \in \{e_L, e_H\}$ but she infers this effort by observing the output (y). The principal may also decide to get an additional signal (v) about the agent's performance. This signal $v \in \{G, B\}$ can be obtained after supervising the agent. As explained in the introduction, we leave aside issues of collusion and consider that the principal and the supervisor embody the same person.

We assume that the supervisor's perception of the agent's level of effort can be distorted by the agent's influence activity (a). We model agents' influence on supervisors' assessments as a case of biased attribution (Bénabou and Tirole 2002) in which the principal, with some probability, will mistakenly perceive a negative signal about his employee as being positive as a result of the latter influence action (a). This biased attribution process may occur as a result of an interpersonal relationship between the agent and the principal. We can also think of trust as an important feature that biases the supervisor's perception.[5]

We state this hypothesis as follows. We denote v_S the principal's perception of the supervision signal v.

[4]We assume that agent's utility is separable in effort and in the influence cost. These are standard assumptions (MacLeod 2003).

[5]The psychological behavioral process is also known as *transference* (Mullainathan, Schwartzstein and Schleiffer, 2008). See Shavell (1979) to find situations in which the principal and the agent have an interpersonal relationship and Hosmer (1995) to read about trust in organizations.

Assumption I (Influence)

If agent decides to undertake an influence activity ($a = 1$), the principal will perceive with probability π in $(0, 1)$ any signal v as if it were good.

With probability $(1 - \pi)$ the principal uses standard Bayesian updating.

In the case of rational supervision, $\pi = 0$ so that $v_S \equiv v$.

The principal's bias creates incentives for the agent to manipulate the soft signal through the influence activity. Our model relies then upon the principal's difficulty to disentangle positive actions and the positive supervision message $v = G$. This approach can be extended to other situations such as document falsification, which involves the agent performing activities to affect the output signal.

In this paper we consider the case in which the principal and the agent are fully aware of the principal's bias. We state this assumption as follows.

Assumption A (Awareness of Biases)

Under Assumption A, the principal updates his belief about the soft signal as follows.

$$\begin{cases} P\left[v = G \mid v_S = G, e = e_H\right] = \frac{\rho_v}{\pi(1-\rho_v)+\rho_v} \\ P\left[v = G \mid v_S = G, e = e_L\right] = \frac{1-\rho_v}{(1-\rho_v)+\pi\rho_v} \\ P\left[v = G | v_S = B, e\right] = 0 \end{cases}$$

This assumption is used in Bénabou and Tirole (2002) and is referred to as metacognition. Under this assumption, the principal knows that perceiving her employee positively ($v_S = G$) may not systematically imply that the soft signal was positive given that, with probability π, the principal being under the agent's influence ($a = 1$) perceives his contribution positively.

Assumption O (Observability of actions and signals)

i) The influence action ($a \in \{0, 1\}$) is observable by the supervisor but it is non-verifiable.

ii) The agent knows whether he is being supervised but the supervisor's perception of the agent's work (v) is not observable by the agent.

When signals are observable and contractible, the standard principal-agent model arises (Holmström 1979). Our Assumption O, however, implies that the influence activity is unverifiable by a third party. As a result, the contract cannot rely upon the observed action $a \in \{0, 1\}$ (e.g., if you invite your boss for a coffee she will observe this action and actually know that can be influenced in her assessment of your work but she cannot set a contract using this piece of information). We further assume that at the time you have to decide whether to take a coffee with your boss, you do not know her impression on your work. If this was not the case, the agent would influence only after observing $v = B$ and this would automatically reveal the soft signal to the principal.

We also consider the following assumption about the efficient level of effort. Exerting an effort is efficient in the following sense.

Assumption E (Efficiency)

$e_H = \arg \max_{e \in \{e_L, e_H\}} \{U_P + U_A\}$

The efficient level of effort is then $e = e_H$ so that an efficient equilibrium is achieved whenever the wage scheme implements e_H.

We focus our paper on the implementation of the efficient level of effort, that is we study the contract that elicits high effort (e_H) at lowest cost. Hereafter, we consider the following contingent contracts.

Definition 1 *A contingent contract is a vector* $\mathbf{w}$ *such that the agent is paid as a function of the hard and the soft signals* $(y, v) \in S$ *where* $S \in \{(0, 1) \times (B, G, \varnothing)\}$*, so that* $w_{yv} \equiv w(y, v)$*.*

We denote $\mathbf{P}_1[\mathbf{P}_0]$ the probability of receiving each of the payments in $\mathbf{w} = [w_{1G}, w_{1B}, w_{0G}, w_{0B}]^\top$ when the agent is supervised and exerts a high [low] level of effort on the productive task. When no supervision takes place, the contingent contract is reduced to $\mathbf{w}_N = [w_{1,\varnothing}, w_{0,\varnothing}]^\top$ and the probability vector associated to respective wages is denoted by $\mathbf{P}_1^N[\mathbf{P}_0^N]$.

The principal can always choose $v = \varnothing$ (no supervision). In that case (*status quo*), the principal gets rid of the soft signal which contains pieces of manipulable information. The soft signal, however, includes additional information that may be useful for the principal. This feature raises the question of whether the principal should actually supervise or not. Our next definition sheds light on this issue.

Definition 2 *We say that the supervision signal is valuable to the principal in order to implement the efficient level of effort as long as there exists* $\mathbf{w}^\top$ *in* $\mathbb{R}^4$ *such that* $\mathbf{w}^\top \mathbf{P}_1 - \mathbf{w}_N^\top \mathbf{P}_1^N < 0$ *for any* $\mathbf{w}_N^\top$ *in* $\mathbb{R}^2$*.*

Hence, if the supervision signal is costless, the principal should use this information whenever it is informative about the agent's level of effort. In that case, the optimal contract should not be based on $\mathbf{w}_N = [w_{1\varnothing}, w_{0\varnothing}]^\top$ but on $\mathbf{w} = [w_{1G}, w_{1B}, w_{0G}, w_{0B}]^\top$ (Laffont and Martimort, 2002).

3 Rational supervision

In this section we solve the principal-agent model in the absence of influence. First notice that when $\pi = 0$ (no influence), the agent will never engage in an influence activity ($a^* = 0$) in Stage 3 given that he cannot distort the soft signal.

We denote by $\mathbf{w}^{**}$ the optimal contract determined by the principal in the model without influence that implements the efficient level of effort e_H. We show in the next proposition and in the next corollary how wages are set by the principal.

Proposition 1 (Optimal wages under supervision) *If the principal supervises the agent in the model without influence, the optimal contract $\mathbf{w}^{**}$ is such that:*
- For $\rho_v \leq \rho_y$, $w_{0B}^{**} < w_{0G}^{**} \leq w_{1B}^{**} < w_{1G}^{**}$
- For $\rho_v > \rho_y$, $w_{0B}^{**} < w_{1B}^{**} < w_{0G}^{**} < w_{1G}^{**}$

Our first proposition follows from the fact that wages are non-decreasing in either the hard or the soft signal.[6] That is, $w_{1k}^{**} \geq w_{0k}^{**}$ for any k in $\{B, G\}$ and $w_{lG}^{**} \geq w_{lB}^{**}$ for any l in $\{0, 1\}$. Also, the relative weight given to each signal will depend on the relative precision of the respective signal. If the soft signal is less precise than the hard signal $\rho_v \leq \rho_y$ then more weight is going to be assigned to the hard signal in the optimal contingent contract designed by the principal. This is because in case of conflicting signals (i.e. w_{1B}^{**} and w_{0G}^{**}) optimal wages are set according to the hard evidence so that $w_{0G}^{**} \leq w_{1B}^{**}$. That is, in case of conflicting information wages are increasing in the hard signal (level of production) whereas wages are decreasing in the soft signal. A general definition is stated below.

Definition 3 (Respective weights of hard and soft signals)
i) We say that more weight is assigned to the hard (soft) signal in the optimal contingent contract if $w_{0G} \leq w_{1B}$ ($w_{0G} > w_{1B}$).
ii) We say that an increase in a parameter τ rises the weight that is assigned to the hard (soft) signal in the optimal contingent contract if $\frac{\partial}{\partial \tau}(w_{1B} - w_{0G}) > 0$ ($\frac{\partial}{\partial \tau}(w_{0G} - w_{1B}) > 0$).

[6] This is the case because our signals satisfy the monotone likelihood ratio property.

In the next corollary we analyze the sensibility of optimal wages to the precision of the signals. We state that as the precision of a signal increases wages increase (decrease) whenever this signal brings good (bad) news about the agent's level of effort.

Corollary 1 (Wages and precision of the signals) *>From Proposition* 1 *we get the following relationship between the precision of the hard and soft signals and supervision wages.*

$$\begin{cases} \frac{\partial w_{0B}^{**}}{\partial \rho_v} < 0, \frac{\partial w_{0,G}^{**}}{\partial \rho_v} > 0, \frac{\partial w_{1B}^{**}}{\partial \rho_v} < 0, \frac{\partial w_{1G}^{**}}{\partial \rho_v} > 0 \\ \frac{\partial w_{0B}^{**}}{\partial \rho_y} < 0, \frac{\partial w_{0G}^{**}}{\partial \rho_y} < 0, \frac{\partial w_{1B}^{**}}{\partial \rho_y} > 0, \frac{\partial w_{1G}^{**}}{\partial \rho_y} > 0 \end{cases}$$

From Proposition 1 and Corollary 1, we deduce the following relationship between wages under supervision and wages in the absence of supervision, where the optimal wages in the absence of supervision are denoted by $\mathbf{w}_N^* = \left[w_{1\varnothing}^*, w_{0\varnothing}^*\right]^\top$.[7]

Corollary 2 (Wages comparison)

- *If* $\frac{1}{2} < \rho_v \leq \rho_y \Longrightarrow w_{0B}^{**} < w_{0\varnothing}^* < w_{0G}^{**} < w_{1B}^{**} < w_{1\varnothing}^* < w_{1G}^{**}$
- *If* $\rho_v > \rho_y \Longrightarrow w_{0B}^{**} < w_{0\varnothing}^* < w_{1B}^{**} < w_{0G}^{**} < w_{1\varnothing}^* < w_{1G}^{**}$
- *If* $\rho_v = \frac{1}{2} \Longrightarrow w_{0B}^{**} = w_{0\varnothing}^* = w_{0G}^{**}$ *and* $w_{1B}^{**} = w_{1\varnothing}^* = w_{1G}^{**}$

We conclude in the following corollary that supervision will be completed by the principal as long as the cost of supervision ϕ_s is lower than the benefits obtained from supervision, where the latter depend on the efficiency costs that the principal saves when obtaining more information about the agent's level of effort.

Corollary 3 (Supervision decision) *The principal will decide to supervise whenever the following condition holds.*

$(\mathbf{w}_N^*)^\top \mathbf{P}_1^N - (\mathbf{w}^{**})^\top \mathbf{P}_1 \geq \phi_s$ *where* $(\mathbf{w}_N^*)^\top \mathbf{P}_1^N - (\mathbf{w}^{**})^\top \mathbf{P}_1 > 0$ *for any* $\rho_v \in (\frac{1}{2}, 1]$.

We then observe that the supervision signal is informative about the agent's level of effort when $\rho_v > \frac{1}{2}$. Hereafter, we assume for simplicity that $\rho_v > \frac{1}{2}$. We show next that as the precision of a signal increases the principal saves costs to implement the efficient level of effort. We also analyze how the decision to supervise the agent is affected by the precision of the hard and the soft signals.

[7] Notice that we can interpret the case in which no supervision occurs as a special case of the model with two signals where the soft signal is uninformative, that is $\rho_v = \frac{1}{2}$.

Corollary 4 (Efficiency cost and precision of the signals)

i) As the precision of the hard or the soft signal increases the cost of achieving the efficient level of effort decreases.

ii) $\frac{\partial \mathsf{w}_N^\top \mathsf{P}_1^N - \mathsf{w}^\top \mathsf{P}_1}{\partial \rho_v} > 0.$

iii) $\frac{\partial \mathsf{w}_N^\top \mathsf{P}_1^N - \mathsf{w}^\top \mathsf{P}_1}{\partial \rho_y} < 0$

This corollary implies that supervision is going to be less pervasive when the hard signal is more precise while the reverse is true when the soft signal is more precise. As a consequence, the precision of the signals is key to determine the optimal schemes and the supervision decision in the context of no influence.

4 Supervision and Influence

In this section we consider the case in which the supervisor can be influenced by the agent, that is $\pi > 0$.

We first determine the condition under which the agent undertakes the costly influence activity $(a = 1)$. This condition is captured in the following lemma.[8]

Lemma 1 (Influence activity) *If the agent is being supervised in an efficient equilibrium* $(e = e_H)$ *then he will perform the influence activity* $(a = 1)$ *whenever:*

(IA) $\rho_y \left[u(w_{1G}) - u(w_{1B})\right] + (1 - \rho_y)\left[u(w_{0G}) - u(w_{0B})\right] > \frac{\phi_a}{\pi(1-\rho_v)}$

We can therefore see that the agent does not only rely upon the magnitude of the principal's bias $\pi > 0$ to decide whether to perform the influence activity or not but he accounts for the precision of the signals as well. In particular, as the quality of the hard signal (ρ_y) rises the incentives for the agent to undertake the influence activity decrease. This occur because as ρ_y increases, the principal focuses relatively more on the hard signal (y) than on the soft signal (v_s), as pointed out by our previous results. The opposite is true when the precision of the soft signal increases (ρ_v). Finally, the agent's choice is being affected by the influence cost (ϕ_a) so that an increase in ϕ_a reduces the incentives for the agent to undertake the influence activity.

One of the questions to be addressed is how does the principal set the optimal contract and what is the effect of the principal's bias $\pi > 0$ on this contingent contract.

[8] We assume that the agent does not undertake the influence activity in the case of indifference. The fact that wages are non-decreasing in the soft signal is necessary for the existence of this condition.

Definition 4 (Wage compression and the power of incentives) *i) We say that the power of incentives increases (decreases) in the hard signal y with respect to parameter κ whenever $\frac{\partial w_{1i}}{\partial k} \geq 0$ (≤ 0) and $\frac{\partial w_{0i}}{\partial k} < 0$ (> 0) for any $i \in \{B, G\}$.*

ii) We say that the power of incentives increases (decreases) in the soft signal v with respect to the parameter κ whenever $\frac{\partial w_{iG}}{\partial k} \geq 0$ (≤ 0) and $\frac{\partial w_{iB}}{\partial k} < 0$ (> 0) for any $i \in \{0, 1\}$.

This definition can be related to strong wage compression and strong responsiveness. We use these concepts to improve the clarity of the exposition of our next results.

To keep our discussion on influence interesting, we assume in this section that the optimal level of wages in the absence of influence ($\mathbf{w}^{**}$) is not a solution to the optimization problem with influence. In this setup, the principal has two different options. On the one hand, the principal may propose influence contracts ($\mathbf{w}^I$) for which she anticipates that agents will be willing to manipulate the soft signal. On the other hand, the principal may deter influence by proposing influence-free contracts ($\mathbf{w}^F$). In this case, the principal's optimization problem includes an additional constraint to avoid the influence activity.

Influence contracts

The principal can allow for influences activities by choosing the optimal contract $\hat{\mathbf{w}}^I = [\hat{w}^I_{1G}, \hat{w}^I_{1B}, \hat{w}^I_{0G}, \hat{w}^I_{0B}]^\top$ that maximizes her expected payoffs, that is $\hat{\mathbf{w}}^I$ is the least expensive contract that satisfies the condition (IA), meaning that: $\hat{\mathbf{w}}^I = \arg\min_{\mathbf{w}^I \in Z} (\mathbf{w}^I)^\top \mathbf{P}^I_1$, where $\mathbf{P}^I_1$ is the probability vector associated to the case in which the agent exerts a high level of effort on the productive task and the principal accepts influence from the agent, and Z is the subset of contingent wages such that:

$$\rho_y[u\left(w^I_{1G}\right) - u\left(w^I_{1B}\right)] + (1 - \rho_y)[u\left(w^I_{0G}\right) - u\left(w^I_{0B}\right)] > \frac{\phi_a}{\pi(1-\rho_v)}$$

We show in the following proposition that influence contracts may be optimal for the principal as the supervision message may be informative about the agent's level of effort. In the proposition, we also present the relationship between the optimal wages under influence.

Proposition 2 (Optimal wages under influence contracts) *If the Principal supervises the agent in an efficient equilibrium under influence then $\mathbf{w}_N^\top \mathbf{P}^N_1 - (\hat{\mathbf{w}}^I)^\top \mathbf{P}^I_1 > 0$. Moreover, there exists $\pi_0(\rho_y, \rho_v)$ such that the optimal wage scheme $\hat{\mathbf{w}}^I$ satisfies the following conditions:*

(a) If $\frac{1}{2} < \rho_v \leq \rho_y$ then $\hat{w}^I_{0B} < \hat{w}^I_{0G} \leq \hat{w}^I_{1B} < \hat{w}^I_{1G}$

(b) If $\rho_v \in (\rho_y, \bar{\rho}_y)$ then:

(i) $\hat{w}^I_{0B} < \hat{w}^I_{1B} \leq \hat{w}^I_{0G} < \hat{w}^I_{1G}$ for $\pi \leq \pi_0$

(ii) $\hat{w}^I_{0B} < \hat{w}^I_{0G} < \hat{w}^I_{1B} < \hat{w}^I_{1G}$ *for* $\pi > \pi_0$
(c) If $\rho_v \geq \bar{\rho}_y$ *then* $\hat{w}^I_{0B} < \hat{w}^I_{1B} < \hat{w}^I_{0G} < \hat{w}^I_{1G}$

Again, wages are non-decreasing in either the hard or the soft signal so that $\hat{w}^I_{1k} \geq \hat{w}^I_{0k}$ for any k in $\{B, G\}$ and $\hat{w}^I_{lG} \geq \hat{w}^I_{lB}$ for any l in $\{0, 1\}$. In addition, more weight is assigned to the hard signal when its precision (ρ_y) is higher than the precision of the soft signal (ρ_v). When the opposite is true, there exists a trade-off for the Principal since $\rho_v < \rho_y$ pushes $\hat{w}^I_{1B}$ above $\hat{w}^I_{0G}$ as stated in Proposition 1 but it also increases the incentives for the agent to undertake the influence activity. As a result, we observe that more weight is assigned to the soft signal if either (i) $\rho_y < \rho_v < \bar{\rho}_y \equiv \frac{\rho_y^2}{1+2\rho_y(\rho_y-1)}$ and $\pi \leq \pi_0$ or (ii) the precision of the soft signal is sufficiently high (i.e., $\rho_v > \bar{\rho}_y$). On the contrary, more weight tends to be given to the hard signal compared to the soft signal if the manipulability of the soft signal is superior to π_0, even though $\rho_v \in (\rho_y, \bar{\rho}_y)$. We can interpret in that case that the soft signal is more precise than the output signal, but its precision is not high enough to compensate the principal's bias. We then conclude that influence contracts compared to rational supervision contracts are characterized by a greater weight assigned to the hard signal.

The following corollary relates the contingent contract under influence $\hat{\mathbf{w}}^I$ and the magnitude of the Principal's bias, $\pi > 0$. We also derive the relationship between $\hat{\mathbf{w}}^I$ and the precision of the signals.

Corollary 5 (Relative weights and wage compression)

i) The optimal influence contract is such that an increase in the principal's bias (π) *rises the weight that is assigned to the hard signal.*

ii) The optimal influence contract is such that the power of incentives decreases (wage compression) in the soft signal (v) *with respect to the principal's bias* (π)*. As a result, the variance of wages decreases in the principal's bias.*

This corollary follows directly from the definition of wage compression and the following relationship between the principal's bias and optimal wages under influence: $\frac{\partial \hat{w}^I_{0B}}{\partial \pi} = 0$, $\frac{\partial \hat{w}^I_{0G}}{\partial \pi} < 0$, $\frac{\partial \hat{w}^I_{1B}}{\partial \pi} = 0$ and $\frac{\partial \hat{w}^I_{1G}}{\partial \pi} < 0$. An increase in the principal's bias reduces the likelihood ratio associated to the soft signal implying that contingent on observing $v_s = G$ it is less likely that the agent has exerted a high level of effort in the first place.[9] As a result, we have that both $\frac{\partial \hat{w}^I_{0G}}{\partial \pi} < 0$ and $\frac{\partial \hat{w}^I_{1G}}{\partial \pi} < 0$ meaning that optimal wages are less responsive to the positive soft signal as π

[9] Indeed, this likelihood ratio is equal to $1 - \frac{1-\rho_v+\pi\rho_v}{\rho_v}$.

increases. In contrast, notice that the likelihood ratio associated to a negative soft signal $v_s = B$ is not affected by the principal's bias since $P\left[e = e_H \mid v = B\right] = P\left[e = e_H \mid v_s = B\right]$. Then, by applying definitions 3 and 4 we are able to establish that more weight is assigned to the hard signal and that the power of incentives decreases in the soft signal (v) as the principal's bias (π) increases. These results imply that the variance of wages decreases in the principal's bias. Our corollary is related to the wage compression established by MacLeod (2003) in the case of subjective assessments in a principal-agent model. The principal is willing to use the hard signal more intensively relative to the soft signal as π increases since the informativeness of the soft signal decreases in the principal's bias.[10]

We show in the next proposition that the manipulability of the soft signal tends to increase the efficiency costs suffered by the principal. That is, the more manipulable is the soft signal the less effective is supervision as a disciplining device for the agent since the prevision of the soft signal tends to decrease in π.

Proposition 3 (Efficiency cost and principal bias) *As the precision of the principal bias (π) rises the cost of achieving the efficient level of effort for the principal increases.*

We can interpret from previous Proposition that as the principal's bias increases (π) supervision will be less likely. But will any supervision actually occur if the principal is under influence? The supervision decision is characterized as follows.

Corollary 6 (Supervision decision under influence) *If it is optimal for the agent to influence the principal, the latter will supervise the agent if and only if:*

i) $(\mathbf{w}_N^*)^\top \mathbf{P}_1^N - (\hat{\mathbf{w}}^{\mathrm{I}})^\top \mathbf{P}_1^I \geq \phi_s$

ii) In addition, $\left(\hat{\mathbf{w}}^I\right)^\top \mathbf{P}_1^I - (\mathbf{w}^{**})^\top \mathbf{P}_1 > 0.$

As it was shown in Proposition 2, the supervision signal is informative about the agent's performance. Then, condition (i) states that this signal should be included in the optimal contract whenever the cost of supervision ϕ_s is smaller than the benefits obtained after deviating from the *status quo* $(v = \varnothing)$. At comparing $\mathbf{w}^I \mathbf{P}_1^I$ and $\hat{\mathbf{w}}^\top \mathbf{P}_1$ we find that

[10] This behavior can be related to the empirical evidence provided by Liberti and Mian (2009) in the context of credit decisions. Liberti and Mian (2009) observe that higher hierarchical distance between the decision-maker and the agent who collects the information yields less reliance on soft information.

more influence is always worse for the principal. Thereby, we can conclude that the principal prefers a situation in which influence is deterred. This solution works in line with closing the communication channels to avoid influence activities (Milgrom, 1988; Milgrom and Roberts, 1988) and may explain the organization structure in some cases. For instance, we can explain the existence of firms in which the principal and the agents work in different buildings as well as the empirical evidence provided by Berger, Miller, Petersen, Rajan and Stein (2001) showing that larger banks are more likely to communicate with borrower impersonally. However, closing the communication channels may not be feasible (or it may be too expensive). In that case, the principal will be willing to supervise the agent for ϕ_s small enough, even though the supervision message contains pieces of manipulable information.

In the next corollary we show that the efficiency cost incurred by the principal tends to increase in the presence of influence compared to the rational supervision case whenever the soft signal becomes more manipulable and whenever the soft or the hard signal becomes less precise.

Corollary 7 (Efficiency cost and precision of the signals)

i) $\frac{\partial\left(\hat{\mathsf{w}}^I\right)^\top \mathsf{P}_1^I - (\mathsf{w}^{**})^\top \mathsf{P}_1}{\partial \pi} > 0$

ii) $\frac{\partial\left(\hat{\mathsf{w}}^I\right)^\top \mathsf{P}_1^I - (\mathsf{w}^{**})^\top \mathsf{P}_1}{\partial \rho_v} < 0$

iii) $\frac{\partial\left(\hat{\mathsf{w}}^I\right)^\top \mathsf{P}_1^I - (\mathsf{w}^{**})^\top \mathsf{P}_1}{\partial \rho_y} < 0$

Influence-free contracts

The principal needs not to accept influence as she may offer influence-free contracts ($\mathbf{w}^F$) that discourage the agent from choosing $a = 1$. We denote $\hat{\mathbf{w}}^F = (\hat{w}_{1B}^F, \hat{w}_{1G}^F, \hat{w}_{0G}^F, \hat{w}_{0B}^F)$ the optimal contract such that the agent is not willing to distort the supervisor's assessment on his work. Recall that we denote $\mathbf{P}_1^I$ the probability vector associated to the case in which the principal accepts influence from the agent. We denote $\mathbf{P}_1^F$ the probability vector when the principal designs influence-free contract to implement the efficient equilibrium.[11] It follows that the principal designs influence-free contracts as long as the following condition is satisfied:

$$\left(\hat{\mathbf{w}}^I\right)^\top \mathbf{P}_1^I \geq \left(\hat{\mathbf{w}}^F\right)^\top \mathbf{P}_1 \tag{1}$$

Therefore when (1) does not hold it is optimal for the principal to accept influence. Otherwise, the principal sets influence-free contracts

[11] By definition we know that $\mathsf{P}_1^F \equiv \mathsf{P}_1$.

$\hat{\mathbf{w}}^F$ to achieve efficiency. In that case, the principal discourages the influence activity by imposing an additional constraint on her optimization problem. This condition is summarized in the following definition.

Definition 5 (Influence-free contracts) *A contract is influence-free as long as the following restriction (IF) is imposed to the principal's problem:*

(IF) $\mathbf{u}\left(\mathbf{w}^F\right)^\top \mathbf{P}_1^F \geq \mathbf{u}\left(\mathbf{w}^F\right)^\top \mathbf{P}_1^I - \phi_a$

We can then interpret that (IF) requires the agent accepting the influence-free contract.[12] We characterize in the following proposition the optimal influence-free contracts that implement the efficient equilibrium.

Corollary 8 (Influence-free contracts and wage compression)

i) The optimal influence-free contract is such that an increase in the principal's bias (π) *increases the weight that is assigned to the hard signal.*

ii) The optimal influence-free contract is such that the power of incentives decreases (wage compression) in the soft signal (v) *with respect to the principal's bias* (π)*. As a result, the variance of wages decreases in the principal's bias.*

This corollary follows directly from definition 3 and the following relationship between the principal's bias and optimal influence-free wages: $\frac{\partial \hat{w}_{0B}^F}{\partial \pi} > 0$, $\frac{\partial \hat{w}_{0G}^F}{\partial \pi} < 0$, $\frac{\partial \hat{w}_{1B}^F}{\partial \pi} > 0$ and $\frac{\partial \hat{w}_{1G}^F}{\partial \pi} < 0$. The result is similar to the case of influence contracts but wage compression is actually stronger in this case given that $\frac{\partial \hat{w}_{0B}^F}{\partial \pi} > 0$ and $\frac{\partial \hat{w}_{1B}^F}{\partial \pi} > 0$ whereas $\frac{\partial \hat{w}_{0B}^I}{\partial \pi} = \frac{\partial \hat{w}_{1B}^I}{\partial \pi} = 0$. That is, we observe that if the principal's bias (π) rises the optimal wages upon receiving the bad supervision signal only react for influence-free contracts. This strategy, which increases $\hat{w}_{0B}^F$ and $\hat{w}_{1B}^F$, compensates the agent for choosing $a = 0$ after the increasing in π. The rationale for this result is that when π rises the agent's marginal utility in $\hat{w}_{0G}^F$ and $\hat{w}_{1G}^F$ increases, whereas $\frac{\partial u'(\hat{w}_{0B}^F)}{\partial \pi}$ and $\frac{\partial u'(\hat{w}_{1B}^F)}{\partial \pi}$ decrease.

5 Influence costs and the value of the firm

Influence activities are costly for the organization as they detract workers from their productive task (Milgrom and Roberts 1992). We provide in this section an analysis of influence in which this activity is time consuming and actually undermining the quality of the work of the agent.

[12] In other words, the agent does not take the influence activity so that (IA) does not hold when (IF) does.

This translates into the following assumption in which the probability of obtaining the high level of output is reduced by the influence activity.

> **Assumption C (Influence costs and the value of the firm)**
>
> If the agent decides to undertake an influence activity ($a = 1$), then $P[y = 1 \mid e = e_H] = (1-\alpha)\rho_y$ and $P[y = 1 \mid e = e_L] = (1-\alpha)\left(1-\rho_y\right)$ where $\alpha \in [0,1]$ measures the influence cost.

We aim at comparing in this section the costs and benefits for the principal associated to two possible strategies: designing influence-free contracts by imposing the influence-free restriction in her maximization problem or accepting influence from the agent and relaxing the optimization problem.

Influence contracts

We first characterize the influence contract $\mathbf{w}^{\iota} = (w_{1G}^{\iota}, w_{1B}^{\iota}, w_{0G}^{\iota}, w_{0B}^{\iota})$ when influence activities are costly. We consider that the influence-free constraint is binding, that is the efficient contract $\mathbf{w}^{**}$ is not a solution to the optimization problem with influence.[13]

Corollary 9 (Influence contracts and wage compression)

i) The optimal influence contract is such that either an increase in the principal's bias (π) *or an increase in influence costs* (α) *rises the weight that is assigned to the hard signal in the optimal contingent contract.*

ii) The optimal influence contract is such that the power of incentives decreases (wage compression) in the soft signal (v) *with respect to the principal's bias* (π) *and the power of incentives decreases in the hard signal with respect to influence costs* (α)*.*

The first part of the corollary follows from the fact that under influence the likelihood ratio of the signals $v_s = G$ and $y = 0$ decrease in π and α respectively, so that an increase in the principal's bias π (influence cost α) rises the weight that is assigned to the hard (soft) signal. The result also follows from the definition of wage compression and the following relationship between the principal's bias, influence costs and optimal wages under influence: $\frac{\partial \hat{w}_{0B}^{\iota}}{\partial \pi} = 0$, $\frac{\partial \hat{w}_{0G}^{\iota}}{\partial \pi} < 0$, $\frac{\partial \hat{w}_{1B}^{\iota}}{\partial \pi} = 0$, $\frac{\partial \hat{w}_{1G}^{\iota}}{\partial \pi} < 0$ and $\frac{\partial \hat{w}_{0B}^{\iota}}{\partial \alpha} > 0$, $\frac{\partial \hat{w}_{0G}^{\iota}}{\partial \alpha} > 0$, $\frac{\partial \hat{w}_{1B}^{\iota}}{\partial \alpha} = 0$, $\frac{\partial \hat{w}_{1G}^{\iota}}{\partial \alpha} = 0$. This implies that the variance of wages decreases in the principal's bias and in the influence

[13] Formally, this means that $\mathsf{u}(\mathsf{w}^{**})^{\top} \mathsf{P}_1 < \mathsf{u}(\mathsf{w}^{**})^{\top} \mathsf{P}_1^{\iota}$, where P_1^{ι} denotes the probability vector when the principal accepts influence but the influence activity is costly for the organization.

costs. The rationale for this result follows from the relationship between the agent's marginal utility and the principal's bias π (influence cost α) as explained above.

Influence-free contracts

We provide now the condition under which the principal is willing to design influence-free contracts to implement the efficient equilibrium. We denote by $\mathbf{w}^f = (w^f_{1G}, w^f_{1B}, w^f_{0G}, w^f_{0B})$ $\left[\mathbf{P}^f_1\right]$ the influence-free wage contract [probability vector] and by $\mathbf{w}^\iota = (w^\iota_{1G}, w^\iota_{1B}, w^\iota_{0G}, w^\iota_{0B})$ $[\mathbf{P}^\iota_1]$ the wage contract [probability vector] when the principal accepts influence from the agent.[14] It should be clear that it is optimal for the principal to design influence-free contracts as long as:

$$\alpha R(y) + (\mathbf{w}^\iota)^\top \mathbf{P}^\iota_1 \geq \left(\mathbf{w}^f\right)^\top \mathbf{P}_1 \tag{2}$$

Then, the principal designs influence-free contracts when (2) is satisfied. Otherwise, the principal's best option is to accept influence from the agent. We determine in the next proposition a series of conditions under which influence-free contracts will be chosen by the principal.

Proposition 4 (Influence-free strategy)

i) For $\alpha \geq \alpha_f$ the principal will choose to use influence-free contracts.

ii) For $R(y) \geq R_f$ the principal will choose to use influence-free contracts.

The proposition states that as the cost associated to implement influence contracts increases above a certain threshold (that is as α or $R(y)$ rises above α_f and R_f respectively) the principal will find it optimal to set influence-free contracts. As a result, high-productivity workers j such that $R_j(y) \geq R_f$ are going to be paid according to influence-free contracts whereas low-productivity agents l such that $R_l(y) < R_f$ are likely to be rewarded with contracts allowing for some level of influence activities.

We provide a deeper characterization of the optimal influence-free contract in the following proposition. We denote $\alpha^+ = max\{\alpha_0; \alpha_1; \alpha_f\}$ where $\alpha_0 = \frac{\pi(1-\rho_y)}{(1-\pi)\rho_y}$ and $\alpha_1 = \frac{\pi(1-\rho_v)}{(1-\pi)\rho_v+\pi}$. The expressions for α_0 and α_1 represent thresholds above which $\frac{\partial w^F_{1G}}{\partial \alpha} > 0$ and $\frac{\partial w^F_{0B}}{\partial \alpha} < 0$, respectively. Then, by applying definition 4 we conclude that above these threshold values the optimal influence-free contract is such that the power of incentives increases in the hard signal with respect to influence costs (α).

[14]By definition we know that $\mathsf{P}^f_1 \equiv \mathsf{P}_1$.

Notice that for π low (i.e., for α_0 and α_1 low) the wage compression becomes less relevant since the expected benefits associated to influence are reduced as influence is less likely.

Corollary 10 (Influence-free contracts and wage responsiveness) *(i) The optimal influence-free contract in the case of influence costs is such that an increase in the costs* (α) *rises the weight that is assigned to the hard signal.*

(ii) The optimal influence-free contract in the case of influence costs is such that for any $\alpha \geq \alpha^+$*, the power of incentives increases in the hard signal with respect to influence costs* (α)*. As a result, the variance of wages decreases in influence costs* (α)*.*

The rationale is that for any $\alpha \geq \alpha^+$ we have that $\frac{\partial w_{1G}^F}{\partial \alpha} > 0$, $\frac{\partial w_{1B}^F}{\partial \alpha} > 0$, $\frac{\partial w_{0G}^F}{\partial \alpha} < 0$ and $\frac{\partial w_{0B}^F}{\partial \alpha} < 0$. The result follows from the relationship between the agent's marginal utility in the optimal contract and the influence costs (α). A detailed analysis is provided in the appendix.

We show in the next corollary that agents with different levels of productivity may face different incentives schemes.

Corollary 11 (Influence-free contract and agent's productivity) *For any* $\alpha \geq \alpha^+$*, there exists a level of productivity* $\bar{R}$ *above which wages offered to low-productivity agents* $\left(R(y) < \bar{R}\right)$ *are less responsive to the hard signal compared to high-productivity agents* $\left(R(y) \geq \bar{R}\right)$*.*

This corollary follows from the last two results where $\bar{R}$ is the level of productivity such that for a given value of α the principal is indifferent between supervising the agent or not. We show below that a consequence of our analysis is that the principal may prefer to focus on the hard signal and avoid supervision even when it is actually costless ($\phi_s = 0$). The rationale is that for high levels of the principal's bias ($\pi \geq \bar{\pi}$) the agent will attempt to influence the principal and the latter will not be getting a large amount of additional information as a result of supervision. The benefits of supervision are then arbitrarily low whereas the cost of supervision ($\alpha R(y)$) is strictly positive.

Corollary 12 (Supervision and influence) *For any* $\pi \geq \bar{\pi}$ *the principal will no be willing to supervise even though the monitoring cost is equal to zero.*

This result can be interpreted as the supervision signal not being informative about the agent's level of effort when $\pi \geq \bar{\pi}$. In that case, the principal focuses on hard information and gets rid of the supervision message v, so the *status quo* ($v = \varnothing$) is implemented.

6 Conclusion

In this paper we analyzed the design of incentives contracts in a principal-agent model in which the agent had the possibility to manipulate soft evidence about his actual performance. We considered successively the cases in which influence activities entailed a private cost to the agent and the case in which such activities diverted the agent from producing for the principal. In both contexts, we showed that an increase in the manipulability (i.e. "softness") of the signal increases information asymmetry between the agent and the principal and increases the cost of implementing the efficient level of effort as a result. We also established that when influence costs are particularly high, the principal prefers to offer influence-free contracts so as to eliminate influence activities in equilibrium. To that purpose, the principal follows opposite strategies depending on the origin of the influence costs. In the case in which influence costs as privately incurred by the agent the optimal contract specifies wages that become more compressed and less volatile as the soft signal becomes more manipulable. Wages being less responsive to the soft signal, the range of possible wages as well as the variance of wages decreases. This result is in line with MacLeod (2003) in which wage compression occurs when the agent's measures of performance are subjective. In contrast, when influence activities reduce the performance of the agent on the productive task, the principal's optimal strategy consists in increasing the expected costs associated to influence activities by raising the incentives associated to the hard signal. As a result, we show that principals may be interested in designing high-powered incentives to avoid influence activities. More specifically, we show that high-powered incentives and influence-free contracts are more likely to be assigned to agents for which influence is especially costly in terms of firm productivity. We then predict that high-productivity workers are likely to be paid according to influence-free contracts whereas low-productivity agents are likely to be rewarded with contracts allowing for some level of influence activities.

Although our model provides a generalization of the principal-agent model to the case in which some signals are manipulable, we deliberately abstract away from the interesting case of multi-agents frameworks. However, in their definition of influence activities, Milgrom and Roberts (1992) envision not only personal attempts to manipulate the principal's view of one-self but also the time devoted by organizational members to countervail the manipulation attempts of their coworkers. In order to apprehend influence activities at the organizational level, extending our analysis to the case of multi-agent models with team production and hierarchies may be a fruitful area for future research.

7 Appendix

Proof of Proposition 1.

1- If the Principal supervises: $v \in \{B, G\}$

We denote $\mathbf{w} = [w_{1G}, w_{1B}, w_{0G}, w_{0B}]^\top$ the contingent contract offered by the principal and we denote $\mathbf{P}_1$ $[\mathbf{P}_0]$ the probability of receiving each of these payments when exerting a high [low] level of effort, that is:

$$\mathbf{P}_1 \equiv (p_{i1})_{i\in\{1,\ldots,4\}} = \begin{bmatrix} \rho_y\rho_v \\ \rho_y(1-\rho_v) \\ (1-\rho_y)\rho_v \\ (1-\rho_y)(1-\rho_v) \end{bmatrix}$$

And $\mathbf{P}_0 \equiv (p_{i0})_{i\in\{1,\ldots,4\}} = \begin{bmatrix} (1-\rho_y)(1-\rho_v) \\ (1-\rho_y)\rho_v \\ \rho_y(1-\rho_v) \\ \rho_y\rho_v \end{bmatrix}.$

The optimal contract solves the following problem:

$$\begin{cases} (1)\ \ \mathbf{w}^{**} = \min\limits_{\mathbf{w}\in\mathbb{R}^4} \mathbf{w}^\top \mathbf{P}_1 \\ (2)\ \ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \bar{u} \qquad \textbf{IR} \\ (3)\ \ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \mathbf{u}(\mathbf{w})^\top \mathbf{P}_0 \qquad \textbf{IC} \end{cases}$$

In order to ensure that the optimization program is concave we will write the optimization program as a function of $h = u^{-1}$ the inverse function of $u(\cdot)$, which is increasing and convex, that is $h' > 0$ and $h'' > 0$.[15] We then define $u_{1G} = u(w_{1G}), u_{1B} = u(w_{1B}), u_{0G} = u(w_{0G})$ and $u_{0B} = u(w_{0B})$ so that $w_{1G} = h(u_{1G})$, $w_{1,B} = h(u_{1B}), w_{0G} = h(u_{0G})$ and $w_{0B} = h(u_{0B})$. Thereby, the Principal solves:

$$\begin{cases} (1)\ \ \mathbf{w}^{**} = \min\limits_{\{(u_0,u_1)\}} p_{11}h(u_{1G}) + p_{21}h(u_{1B}) + p_{31}h(u_{0G}) + p_{41}h(u_{0B}) \\ (2)\ \ p_{11}u_{1G} + p_{21}u_{1B} + p_{31}u_{0G} + p_{41}u_{0B} - C \geq \bar{u}_i \qquad \textbf{IR} \\ (3)\ \ (p_{11}-p_{10})u_{1G} + (p_{21}-p_{20})u_{1B} + (p_{31}-p_{30})u_{0G} \qquad \textbf{IC} \\ \qquad +(p_{14}-p_{04})u_{0B} - C \geq 0 \end{cases}$$

We denote $\lambda \geq 0$ and $\mu \geq 0$ the Lagrange multipliers associated with the incentive constraint and the individual rationality constraint. We then get the following first order conditions.

$$\begin{cases} (1_{0B})\ \ h'(u_{1G}) = \frac{\lambda p_{11} + \mu(p_{11}-p_{10})}{p_{11}} \\ (1_{0G})\ \ h'(u_{1B}) = \frac{\lambda p_{21} + \mu(p_{21}-p_{20})}{p_{21}} \\ (1_{1B})\ \ h'(u_{0G}) = \frac{\lambda p_{31} + \mu(p_{31}-p_{30})}{p_{31}} \\ (1_{1G})\ \ h'(u_{0B}) = \frac{\lambda p_{41} + \mu(p_{41}-p_{40})}{p_{41}} \end{cases}$$

Since $h^{'}(x) = 1/u'(x)$ we can write:

[15] We use this change of variable $h = u^{-1}$ following Laffont and Mortimort (2002) so as to ensure the concavity of the optimization problem solved by the principal.

$$\begin{cases} (1_{1G}) \ u'(w_{1G}^{**}) = \frac{\rho_y\rho_v}{\lambda\rho_y\rho_v+\mu(\rho_y+\rho_v-1)} \\ (1_{1B}) \ u'(w_{1B}^{**}) = \frac{(1-\rho_v)\rho_y}{\lambda(1-\rho_v)\rho_y+\mu(\rho_y-\rho_v)} \\ (1_{0G}) \ u'(w_{0G}^{**}) = \frac{(1-\rho_y)\rho_v}{\lambda(1-\rho_y)\rho_v+\mu(\rho_v-\rho_y)} \\ (1_{0B}) \ u'(w_{0B}^{**}) = \frac{(1-\rho_y)(1-\rho_v)}{\lambda(1-\rho_y)(1-\rho_v)+\mu(1-\rho_y-\rho_v)} \end{cases}$$

In addition, we get the feasibility and Slackness conditions:

(2_{IR}) $\mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C - \bar{u} \geq 0$

(3_{IC}) $\mathbf{u}(\mathbf{w})^\top (\mathbf{P}_1 - \mathbf{P}_0) - C \geq 0$

(4_λ) $\lambda[\mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C - \bar{u}] = 0$

(5_μ) $\mu[\mathbf{u}(\mathbf{w})^\top (\mathbf{P}_1 - \mathbf{P}_0) - C)] = 0$

CASE 1. It should be clear that $\lambda = \mu = 0$ is not a solution for the problem above because it would imply $u'(w) = \infty$.

CASE 2. If $\mu > 0$ and $\lambda = 0$ then,

$$\left\{ (1_{0B}) \quad u'(w_{0B}^{**}) = \frac{(1-\rho_y)(1-\rho_v)}{\mu(1-\rho_y-\rho_v)} > 0 \text{ iff } \rho_y + \rho_v < 1 \right.$$

But $\rho_y + \rho_v < 1$ contradicts $\rho_y, \rho_v \in (\frac{1}{2}, 1]$

CASE 3. If $\mu = 0$ and $\lambda > 0$ then,

$$\left\{ u'(w_{0B}^{**}) = u'(w_{0G}^{**}) = u'(w_{1B}^{**}) = u'(w_{1G}^{**}) = \frac{1}{\lambda} > 0 \right.$$

In this case, the Principal's optimal choice is to propose a fixed wage contract but the agent will not perform high effort because (3_{IC}) does not hold.

CASE 4. Therefore for the solution to exist, $\mu > 0$ and $\lambda > 0$ so (**IC**) and (**IR**) are binding constraints.[16]

Thus,

$$\begin{cases} (1_{0B}) \quad u'(w_{0B}^{**}) = \frac{(1-\rho_y)(1-\rho_v)}{\lambda(1-\rho_y)(1-\rho_v)+\mu(1-\rho_y-\rho_v)} \\ (1_{0G}) \quad u'(w_{0G}^{**}) = \frac{(1-\rho_y)\rho_v}{\lambda(1-\rho_y)\rho_v+\mu(\rho_v-\rho_y)} \\ (1_{1B}) \quad u'(w_{1B}^{**}) = \frac{(1-\rho_v)\rho_y}{\lambda(1-\rho_v)\rho_y+\mu(\rho_y-\rho_v)} \\ (1_{1G}) \quad u'(w_{1G}^{**}) = \frac{\rho_y\rho_v}{\lambda\rho_y\rho_v+\mu(\rho_y+\rho_v-1)} > 0 \\ (2) \ \mathbf{u}(\mathbf{w}^{**})^\top \mathbf{P}_1 - C - \bar{u}_i = 0 \\ (3) \ \mathbf{u}(\mathbf{w}^{**})^\top (\mathbf{P}_1 - \mathbf{P}_0) - C = 0 \end{cases}$$

In order to ensure that $u'(\cdot) > 0$ we would need the denominator being positive. For instance, when $\rho_y \geq \rho_v$ we would need $\frac{\mu}{\lambda} < \frac{(1-\rho_y)(1-\rho_v)}{(\rho_y+\rho_v-1)}$

Besides,

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[16] MacLeod (2003) and Holmström (1979) find exactly the same result. Hereafter, we focus on the case of $\mu > 0$ and $\lambda > 0$.

$$\begin{cases} u'\left(w_{0G}^{**}\right) \geq u'\left(w_{1B}^{**}\right) \text{ for } \rho_y \geq \rho_v \\ \Leftrightarrow w_{0G}^{**} \leq w_{1B}^{**} \text{ for } \rho_y \geq \rho_v \\ \text{Then, for } \rho_y \geq \rho_v\text{:} \\ u'\left(w_{1G}^{**}\right) < u'\left(w_{1B}^{**}\right) \leq u'\left(w_{0G}^{**}\right) < u'\left(w_{0B}^{**}\right) \\ \Leftrightarrow w_{0B}^{**} < w_{0G}^{**} \leq w_{1B}^{**} < w_{1G}^{**} \end{cases}$$

2- If the Principal does not supervise ($v = \oslash$)

This can be interpreted as a special case of the derivations above where $\rho_v = \frac{1}{2}$. In that case, the contingent contract offered by the principal ($\mathbf{w}^*$) to the agent is defined by two contingent payments that are respectively denoted: $w_{1\varnothing}^*$ and $w_{0\varnothing}^*$. Another way to consider the case $v = \{\varnothing\}$ is to repeat the analysis in 1- with $\rho_v = \frac{1}{2}$.

In that case, we obtain the following optimal contract.

$$(1_L) \qquad u'\left(w_{0G}^{**}\right) = u'\left(w_{0B}^{**}\right) = \frac{\frac{1}{2}\left(1-\rho_y\right)}{\frac{1}{2}\lambda\left(1-\rho_y\right)+\mu(\frac{1}{2}-\rho_y)} = u'\left(w_{0\varnothing}^*\right)$$

$$(1_H) \qquad u'\left(w_{1G}^{**}\right) = u'\left(w_{1B}^{**}\right) = \frac{\frac{1}{2}\rho_y}{\frac{1}{2}\lambda\rho_y+\mu(\rho_y-\frac{1}{2})} = u'\left(w_{1\varnothing}^*\right) \qquad \blacksquare$$

$$\Leftrightarrow w_{0\varnothing}^* < w_{1\varnothing}^*$$

Proof of Corollary 1. Comparative statics

If we use the Implicit Function Theorem in equations (1_{0B}), (1_{0G}), (1_{1B}) and (1_{1G}) above, we get that:

$$\begin{cases} \frac{\partial w_{0B}^{**}}{\partial \rho_v} = -\frac{\mu\rho_y(\rho_y-1)}{u''\left(w_{0,B}^{**}\right)(\lambda\left(\rho_y-1\right)(\rho_v-1)+\mu(1-\rho_y-\rho_v))^2} < 0 \\ \frac{\partial w_{0G}^{**}}{\partial \rho_v} = \frac{\mu\rho_y\left(\rho_y-1\right)}{u''(w_{0G})(\lambda\left(\rho_y-1\right)\rho_v+\mu\left(\rho_y-\rho_v\right))^2} > 0 \\ \frac{\partial w_{1B}^{**}}{\partial \rho_v} = -\frac{\mu\rho_y\left(\rho_y-1\right)}{u''(w_{1B})(\lambda(\rho_v-1)\rho_y+\mu\left(\rho_v-\rho_y\right))^2} < 0 \\ \frac{\partial w_{1G}^{**}}{\partial \rho_v} = \frac{\mu\rho_y(\rho_y-1)}{u''(w_{1G})(\lambda\rho_y\rho_v+\mu(\rho_y+\rho_v-1))^2} > 0 \end{cases}$$

Similarly,

$$\begin{cases} \frac{\partial w_{0B}^{**}}{\partial \rho_y} = -\frac{\mu\rho_v(\rho_v-1)}{u''\left(w_{0B}^{**}\right)(\lambda\left(\rho_y-1\right)(\rho_v-1)+\mu(1-\rho_y-\rho_v))^2} < 0 \\ \frac{\partial w_{0G}^{**}}{\partial \rho_y} = \frac{\mu\rho_v(\rho_v-1)}{u''\left(w_{0G}^{**}\right)(\lambda\left(\rho_y-1\right)\rho_v+\mu\left(\rho_y-\rho_v\right))^2} < 0 \\ \frac{\partial w_{1B}^{**}}{\partial \rho_y} = \frac{\mu\rho_v(\rho_v-1)}{u''\left(w_{1B}^{**}\right)(\lambda(\rho_v-1)\rho_y+\mu\left(\rho_v-\rho_y\right))^2} > 0 \\ \frac{\partial w_{1G}^{**}}{\partial \rho_y} = \frac{\mu\rho_v(\rho_v-1)}{u''\left(w_{1G}^{**}\right)(\lambda\rho_y\rho_v+\mu(\rho_y+\rho_v-1))^2} > 0 \end{cases}$$

Therefore,

$$\begin{cases} \frac{\partial w_{0B}^{**}}{\partial \rho_v} < 0, \frac{\partial w_{0G}^{**}}{\partial \rho_v} > 0, \frac{\partial w_{1B}^{**}}{\partial \rho_v} < 0, \frac{\partial w_{1G}^{**}}{\partial \rho_v} > 0 \end{cases}$$

$$\begin{cases} \frac{\partial w_{0B}^{**}}{\partial \rho_y} < 0, \frac{\partial w_{1B}^{**}}{\partial \rho_y} < 0, \frac{\partial w_{1G}^{**}}{\partial \rho_y} > 0, \frac{\partial w_{0G}^{**}}{\partial \rho_y} > 0 \end{cases} \quad \blacksquare$$

Proof of Corollary 2. This follows directly from Corollary 1 taking into account that the benchmark model corresponds to the case in which $\rho_v = \frac{1}{2}$. $\blacksquare$

Proof of Corollary 3. If the principal will decide to supervise the agent whenever the following condition holds:

$\phi_s + \mathbf{w}^{**\top}\mathbf{P}_1 \leq \mathbf{w}_N^{*\top}\mathbf{P}_1^N$

We can derive from the proofs of Proposition 1 and Corollary 2 that $(\mathbf{w}^{*}-\mathbf{w}^{**})^{\top}\mathbf{P}_1 > 0$ for any $\rho_v \in \left(\frac{1}{2},1\right]$ taking into account that the benchmark model corresponds to the case in which $\rho_v = \frac{1}{2}$. Indeed only for $\rho_v = \frac{1}{2}$ we have that $w_{0B}^{**} = w_{0\varnothing}^{*} = w_{0G}^{**}$ and $w_{1B}^{**} = w_{1\varnothing}^{*} = w_{1G}^{**}$. This implies that cost of implementing the efficient level of effort for any $\rho_v \neq \frac{1}{2}$ is strictly lower than in the benchmark model. ■

Proof of Corollary 4. *i)* As ρ_v or ρ_y increases the cost of implementing the efficient level of effort decreases. This can be shown using the Blackwell efficiency theorem. We consider the case of ρ_y (the case of ρ_v is symmetric).

We take the following information structure $(\mathbf{P}_1, \mathbf{P}_0)$ that corresponds to the supervision case with $\mathbf{P}_1$ $[\mathbf{P}_0]$ the probability of receiving each of these payments when exerting a high [low] level of effort, that is:

$$\mathbf{P}_1 = \begin{bmatrix} \rho_y\rho_v \\ \rho_y(1-\rho_v) \\ (1-\rho_y)\rho_v \\ (1-\rho_y)(1-\rho_v) \end{bmatrix} \text{ and } \mathbf{P}_0 = \begin{bmatrix} (1-\rho_y)(1-\rho_v) \\ (1-\rho_y)\rho_v \\ \rho_y(1-\rho_v) \\ \rho_y\rho_v \end{bmatrix}$$

Also, we consider the following information structure where the precision of the soft signal is decreased to $\rho_y - \varepsilon$, where $\varepsilon > 0$.

$$\mathbf{P}_{1\varepsilon} = \begin{bmatrix} (\rho_y-\varepsilon)\rho_v \\ (\rho_y-\varepsilon)(1-\rho_v) \\ (1-\rho_y+\varepsilon)\rho_v \\ (1-\rho_y+\varepsilon)(1-\rho_v) \end{bmatrix} \text{ and } \mathbf{P}_{0\varepsilon} = \begin{bmatrix} (1-\rho_y+\varepsilon)(1-\rho_v) \\ (1-\rho_y+\varepsilon)\rho_v \\ (\rho_y-\varepsilon)(1-\rho_v) \\ (\rho_y-\varepsilon)\rho_v \end{bmatrix}$$

If we are able to show that the information structure $(\mathbf{P}_1, \mathbf{P}_0)$ is sufficient, in the sense of Blackwell, for the information structure $(\mathbf{P}_{1\varepsilon}, \mathbf{P}_{0\varepsilon})$ for $\varepsilon > 0$ then we can conclude using the Blackwell sufficiency theorem that the cost of implementing the efficient level of effort decreases in ρ_y.

To show that $(\mathbf{P}_1, \mathbf{P}_0)$ is sufficient, in the sense of Blackwell, for $(\mathbf{P}_{1\varepsilon}, \mathbf{P}_{0\varepsilon})$ we have to show that there exists a transition matrix $Q = (q_{ij}), (i,j) \in \{1,...,4\}^2$ that is independent of the level of effort such that $p_{j1\varepsilon} = \sum_{j=1}^{4} q_{ij}p_{j1}$ and $p_{j0\varepsilon} = \sum_{j=1}^{4} q_{ij}p_{j0}$.

This can be shown taking the following transition matrix:

$$Q = \begin{bmatrix} 1-\frac{\varepsilon}{2\rho_y-1} & 0 & \frac{\varepsilon}{2\rho_y-1} & 0 \\ 0 & 1-\frac{\varepsilon}{2\rho_y-1} & 0 & \frac{\varepsilon}{2\rho_y-1} \\ \frac{\varepsilon}{2\rho_y-1} & 0 & 1-\frac{\varepsilon}{2\rho_y-1} & 0 \\ 0 & \frac{\varepsilon}{2\rho_y-1} & 0 & 1-\frac{\varepsilon}{2\rho_y-1} \end{bmatrix}$$

ii) For the second part of the corollary we have to show that: $\frac{\partial\left(\hat{\mathsf{w}}^{I}-\mathsf{w}^{**}\right)^{\top}\mathsf{P}_1}{\partial\rho_y} < 0$. We use the result established by Kim (1995), showing that an information structure $\mathbf{P}$ is more efficient than an information structure $\mathbf{\Pi}$ if its likelihood ratio is a mean preserving spread of that of Π.

We compute the following function:
$\Phi\left(\rho_v^p, \rho_v^\pi, \rho_y^p, \rho_y^\pi, \pi\right) \equiv \sum_{i \in S}\left(\frac{\pi_{i0}}{\pi_i 1} - \frac{p_{i0}}{p_i 1}\right)$

Where ρ_i^j stands for the precision of signal $i \in \{v, y\}$ of information structure $j \in \{\mathbf{P}, \mathbf{\Pi}\}$.

$\Phi\left(\rho_v, \rho_v, \rho_y, \rho_y, \pi\right) = \left(\frac{\left(1-\rho_y\right)(1-\rho_v+\pi\rho_v)}{\rho_y[\rho_v+\pi(1-\rho_v)]} + \frac{\rho_y(1-\rho_v+\pi\rho_v)}{\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]}\right)$
$- \left(\frac{\left(1-\rho_y\right)(1-\rho_v)}{\rho_y\rho_v} + \frac{\rho_y(1-\rho_v)}{\left(1-\rho_y\right)\rho_v}\right) > 0$

Since $\frac{\partial\Phi\left(\rho_v,\rho_v,\rho_y,\rho_y,\pi\right)}{\partial\rho_y} > 0$ for any $\pi > 0$ and $\frac{\partial^2\Phi\left(\rho_v,\rho_v,\rho_y,\rho_y,\pi\right)}{\partial\rho_y\partial\pi} > 0$.

As a result, for any ρ_y we need for the information structure $\mathbf{\Pi}\left(\pi, \rho_y\right)$ to be as efficient as $\mathbf{P}\left(\rho_v, \rho_y^p\right)$ that $\rho_y^p = \rho_y^-$ where $\rho_y^- < \rho_y$ so that $\Phi\left(\rho_v, \rho_v, \rho_y^=, \rho_y, \pi\right) = 0$. Also, for an increase in ρ_y to ρ_y^+ we know that Φ rises so that $\mathbf{\Pi}\left(\pi, \rho_y^+\right)$ will be as efficient as $\mathbf{P}\left(\rho_v, \rho_y^p\right)$ for $\rho_y^p = \rho_{y=}$ where $\rho_y^+ - \rho_{y=} > \rho_y^- - \rho_y$. We conclude that for an increase in ρ_y information systems $\mathbf{\Pi}$ and $\mathbf{P}$ are not affected similarly. In particular, an increase in ρ_y tends to favor information system $\mathbf{P}$ compared to $\mathbf{\Pi}$ since $\frac{\partial^2\Phi\left(\rho_v,\rho_v,\rho_y,\rho_y,\pi\right)}{\partial\rho_y\partial\pi} > 0$ that is the likelihood ratio of information system $\mathbf{P}$ is increased by a larger amount than the likelihood ratio of information system Π when ρ_y rises.

iii) The last part of the corollary is shown after getting that $\frac{\partial\Phi\left(\rho_v,\rho_v,\rho_y,\rho_y,\pi\right)}{\partial\rho_v} < 0$. ■

Proof of Lemma 1. Contingent wages will depend on both signals, i.e., $w \equiv w\left(y, v_s\right)$

Consider the case of an efficient equilibrium $(e = e_H)$. We denote $\mathbf{w}^I = \left[w_{1,G}^I, w_{1,B}^I, w_{0,G}^I, w_{0,B}^I\right]^\top$ the vector of contingent wages and $\mathbf{P}_1^I$ the probability of receiving each of these payments when the agent undertakes the influence activity. Then,

$$\mathbf{P}_1^I \equiv (p_{i1}^1)_{i\in\{1,\dots,4\}} = \begin{bmatrix} \rho_y[\rho_v + \pi(1-\rho_v)] \\ \rho_y(1-\pi)(1-\rho_v) \\ \left(1-\rho_y\right)[\rho_v + \pi(1-\rho_v)] \\ \left(1-\rho_y\right)(1-\pi)(1-\rho_v) \end{bmatrix}$$

Contrariwise, if the agent does not undertake the influence activity:

$$\mathbf{P}_1 = (p_{i1})_{i\in\{1,\dots,4\}} = \begin{bmatrix} \rho_y\rho_v \\ \rho_y(1-\rho_v) \\ \left(1-\rho_y\right)\rho_v \\ \left(1-\rho_y\right)(1-\rho_v) \end{bmatrix}$$

Therefore, the agent undertakes the influence activity if and only if

$u\left(\mathbf{w}^I\right)^\top (\mathbf{P}_1^I - \mathbf{P}_1) > \phi_a$.

That is,

$\rho_y\left[u\left(w_{1G}^I\right) - u\left(w_{1B}^I\right)\right] + (1-\rho_y)[u\left(w_{0G}^I\right) - u\left(w_{0B}^I\right)] > \frac{\phi_a}{\pi(1-\rho_v)}$ ■

Proof of Proposition 2. If the Principal supervises under influence: $v_s \in \{B, G\} = v$ with probability $(1-\pi)$ and $v_S = G$ otherwise. Recall that the optimal contract cannot depend on the influence activity $a \in \{0,1\}$ because it is non-verifiable. We denote the contingent contract under influence $\mathbf{w}^I = \left[w_{1,G}^I, w_{1,B}^I, w_{0,G}^I, w_{0,B}^I\right]^\top$ and denote $\mathbf{P}_1^I$ $\left[\mathbf{P}_0^I\right]$ the probability of receiving each of these payments when exerting a high [low] level of effort. Thus,

$$\mathbf{P}_1^I \equiv \left(p_{i1}^I\right)_{i\in\{1,\dots,4\}} = \begin{bmatrix} \rho_y[\rho_v + \pi(1-\rho_v)] \\ \rho_y(1-\pi)(1-\rho_v) \\ \left(1-\rho_y\right)[\rho_v + \pi(1-\rho_v)] \\ \left(1-\rho_y\right)(1-\pi)(1-\rho_v) \end{bmatrix}$$

and $$\mathbf{P}_0^I \equiv \left(p_{i0}^I\right)_{i\in\{1,\dots,4\}} = \begin{bmatrix} \left(1-\rho_y\right)(1-\rho_v+\pi\rho_v) \\ \left(1-\rho_y\right)\rho_v(1-\pi) \\ \rho_y\left(1-\rho_v+\pi\rho_v\right) \\ \rho_y\rho_v(1-\pi) \end{bmatrix}.$$

The first part of the proposition can be proven by showing that the signal v_s is informative about the agent's level of effort. Recall that $P\left[v_s = G \mid e = e_L\right] = 1-\rho_v+\pi\rho_v$ and $P\left[v_s = G \mid e = e_H\right] = \rho_v + \pi(1-\rho_v)$. Since, $P\left[v_s = G \mid e = e_L\right] < P\left[v_s = G \mid e = e_H\right]$ for any $\rho_v > \frac{1}{2}$, $\pi < 1$ the result follows (see Laffont and Martimort 2002, Section 4.6.1, p168).

We can then derive the optimal contract under influence ($\mathbf{w}^I$) which solves:

$$\begin{cases} (1)\ \hat{\mathbf{w}}^I = \min\limits_{\mathrm{w}\in\mathbb{R}^4} \mathbf{w}^\top \mathbf{\Pi}_1 \\ \quad (2)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1^I - C \geq \bar{u} \qquad \mathbf{IR} \\ \quad (3)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1^I - C \geq \mathbf{u}(\mathbf{w})^\top \mathbf{P}_0^I \quad \mathbf{IC} \end{cases}$$

We can define $u_{1G} = u(w_{1G}^I)$, $u_{1B} = u(w_{1B}^I)$, $u_{0G} = u(w_{0,G}^I)$ and $u_{0B} = u(w_{0,B}^I)$ so that $w_{1G}^I = h(u_{1G})$, $w_{1B}^I = h(u_{1B})$, $w_{0G}^I = h(u_{0G})$ and $w_{0B}^I = h(u_{0B})$.

Then, the first-order Kuhn-Tucker conditions are necessary and sufficient to determine the optimal contract

$$\begin{cases} (1)\ W^* = \min\limits_{\{(u_0,u_1)\}} p_{11}^I h(u_{1G}) + p_{21}^I h(u_{1B}) + p_{31}^I h(u_{0G}) + p_{41}^I h(u_{0B}) \\ \quad (2)\ p_{11}^I u_{1G} + p_{21}^I u_{1G} + p_{31}^I u_{1G} + p_{41}^I u_{1G} - C \geq \bar{u}\ \mathbf{IR} \\ \quad (3)\ p_{11}^I u_{1G} + p_{21}^I u_{1G} + p_{31}^I u_{1G} + p_{41}^I u_{1G} - C \geq \\ \qquad p_{10}^I u_{1G} + p_{20}^I u_{1G} + p_{30}^I u_{1G} + p_{40}^I u_{1G} \qquad \mathbf{IC} \end{cases}$$

We denote λ and μ the non-negative Lagrange multipliers associated respectively with the incentive compatibility (**IC**) constraint and the individual rationality (**IR** constraint.. If we use the arguments in

Proposition 1, we conclude that:

$$\begin{cases} (1_{1G})\ \ u'\left(\hat{w}_{1G}^{I}\right) = \frac{\rho_y(\rho_v+\pi(1-\rho_v))}{\lambda\rho_y(\rho_v+\pi(1-\rho_v))+\mu(\rho_v+\rho_y-1+\pi(\rho_y-\rho_v))} \\ (1_{1B})\ \ u'\left(\hat{w}_{1B}^{I}\right) = \frac{\rho_y(1-\pi)(1-\rho_v)}{\lambda\rho_y(1-\pi)(1-\rho_v)+\mu(\pi-1)(\rho_v-\rho_y)} \\ (1_{0G})\ \ \ \ u'\left(\hat{w}_{0G}^{I}\right) = \frac{\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]}{\lambda\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]+\mu(\rho_v-\rho_y-\pi(\rho_v+\rho_y-1))} \\ (1_{0B})\ \ \ \ u'\left(\hat{w}_{0B}^{I}\right) = \frac{\left(1-\rho_y\right)(1-\pi)(1-\rho_v)}{\lambda\left(1-\rho_y\right)(1-\pi)(1-\rho_v)+\mu(\pi-1)(\rho_v+\rho_y-1)} \end{cases}$$

And notice that $\lim_{\pi\to 0}\left(\hat{\mathbf{w}}^{I}-\mathbf{w}^{**}\right)=\vec{0}$. Indeed, for $\pi=0$ the optimal contingent contract $\hat{\mathbf{w}}^{I}=\left[\hat{w}_{1G}^{I},\hat{w}_{1B}^{I},\hat{w}_{0G}^{I},\hat{w}_{0B}^{I}\right]^{\top}$ coincides with the optimal scheme under rational supervision $\mathbf{w}^{**}=\left[w_{1G}^{**},w_{1B}^{**},w_{0G}^{**},w_{0B}^{**}\right]^{\top}$. For completeness, we can also observe that $\lim_{\pi\to 1}\hat{w}_{1,G}^{I}-\hat{w}_{1,B}^{I}=0$.

If we compare (1_{1G}), (1_{1B}), (1_{0G}) and (1_{0B}) above, we get that:

(1) $\hat{w}_{1G}^{I}>\hat{w}_{1B}^{I}>\hat{w}_{0B}^{I}\Leftrightarrow u'\left(\hat{w}_{1G}^{I}\right)<u'\left(\hat{w}_{1B}^{I}\right)<u'\left(\hat{w}_{0B}^{I}\right)$. The relationship holds for any $\rho_v>\frac{1}{2},\rho_y\in(\frac{1}{2},1)$ and $\pi<1$.

We also find that:

(2) $\hat{w}_{1G}^{I}>\hat{w}_{0G}^{I}>\hat{w}_{0B}^{I}\Leftrightarrow u'\left(\hat{w}_{1G}^{I}\right)<u'\left(\hat{w}_{0G}^{I}\right)<u'\left(\hat{w}_{0B}^{I}\right)\Leftrightarrow\rho_v>\frac{1}{2},\rho_y\in(\frac{1}{2},1)$ and $\pi<1$.

So we need to study whether more weight is assigned to the hard or the soft signal in the optimal contingent contract under influence. This relationship between $\hat{w}_{0G}^{I}$ and $\hat{w}_{1B}^{I}$ varies according to the Principal's bias (π) and the precision of the signals (ρ_v and ρ_y).

Let us define $\Lambda(\pi,\rho_v,\rho_y,\lambda_0,\mu_0)=u'\left(\hat{w}_{1B}^{I}\right)-u'\left(\hat{w}_{0G}^{I}\right)$

$\Lambda(\pi,\rho_v,\rho_y,\lambda_0,\mu_0)\equiv 0\Leftrightarrow\pi\in\{\bar{\pi}_0,1\}$

where $\bar{\pi}_0=\frac{(\rho_v-\rho_y)(\rho_v(2\rho_y-1)-\rho_y)}{\rho_v(\rho_v-1)(2\rho_y-1)}>0$ if and only if $\rho_v>\rho_y$

and $\bar{\pi}_0<1$ for $\rho_v<\frac{\rho_y^2}{1+2\rho_y(\rho_y-1)}=\bar{\rho}_y$

So it follows that

(i) If $\rho_v\in(\rho_y,\bar{\rho}_y)$ and $\pi<\bar{\pi}_0\Rightarrow\Lambda(\cdot)>0\Rightarrow\hat{w}_{1B}^{I}<\hat{w}_{0G}^{I}$

(ii) If $\rho_v\in(\rho_y,\bar{\rho}_y)$ and $\pi>\bar{\pi}_0\Rightarrow\Lambda(\cdot)<0\Rightarrow\hat{w}_{1B}^{I}>\hat{w}_{0G}^{I}$

(iii) If $\rho_v<\rho_y\Rightarrow\Lambda(\cdot)<0$ for all $\pi<1\Rightarrow\hat{w}_{1B}^{I}>\hat{w}_{0G}^{I}$

(iv) If $\rho_v>\bar{\rho}_y\Rightarrow\Lambda(\cdot)>0$ for all $\pi<1\Rightarrow\hat{w}_{1B}^{I}<\hat{w}_{0G}^{I}$ ■

Proof of Corollary 5. If we use the Implicit function theorem in equations (1_{1G}), (1_{1B}), (1_{0G}) and (1_{0B}) above it is easy to see that

$$\begin{cases} \frac{\partial\hat{w}_{1G}^{I}}{\partial\pi} = \frac{-(2\rho_v-1)(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}_{1G}^{I}\right)((\pi(\rho_v-1)-\rho_v)\rho_y\lambda+\mu(\rho_v+\rho_y+\pi(\rho_y-\rho_v))^2} < 0 \\ \frac{\partial\hat{w}_{0G}^{I}}{\partial\pi} = \frac{-(2\rho_v-1)(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}_{0B}^{I}\right)((\pi(\rho_v-1)-\rho_v)(1-\rho_y)\lambda-\mu(\rho_y-\rho_v+\pi(\rho_y+\rho_v-1))^2} < 0 \end{cases}$$

whereas

$$\left\{ \frac{\partial\hat{w}_{1B}^{I}}{\partial\pi} = \frac{\partial\hat{w}_{0B}^{I}}{\partial\pi} = 0 \right.$$

Using these equations, we can also derive the relationship between the wages and the signals' precision.

$$\begin{cases} \frac{\partial \hat{w}^I_{0B}}{\partial \rho_v} = -\frac{(1-\pi^2)(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}^I_{0B}\right)((\pi(\rho_v-1)-\rho_v)(1-\rho_y)\lambda-\mu(\rho_y-\rho_v+\pi(\rho_y+\rho_v-1))^2} > 0 \\ \frac{\partial \hat{w}^I_{0G}}{\partial \rho_v} = -\frac{(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}^I_{0G}\right)(\lambda\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]+\mu(\rho_v-\rho_y-\pi(\rho_v+\rho_y-1)))^2} < 0 \\ \frac{\partial \hat{w}^I_{1B}}{\partial \rho_v} = -\frac{(1-\pi^2)(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}^I_{1B}\right)(\lambda\rho_y(1-\pi)(1-\rho_v)+\mu(\pi-1)(\rho_v-\rho_y))^2} > 0 \\ \frac{\partial \hat{w}^I_{1G}}{\partial \rho_v} = -\frac{(\rho_y-1)\rho_y\mu}{u''\left(\hat{w}^I_{1G}\right)((\pi(\rho_v-1)-\rho_v)\rho_y\lambda+\mu(\rho_v+\rho_y+\pi(\rho_y-\rho_v))^2} < 0 \end{cases}$$

And

$$\begin{cases} \frac{\partial \hat{w}^I_{0B}}{\partial \rho_y} = \frac{(\pi(\rho_v-1)-\rho_v)(1+(\pi-1)\rho_v)\mu}{u''\left(\hat{w}^I_{0B}\right)((\pi(\rho_v-1)-\rho_v)(1-\rho_y)\lambda-\mu(\rho_y-\rho_v+\pi(\rho_y+\rho_v-1))^2} > 0 \\ \frac{\partial \hat{w}^I_{0G}}{\partial \rho_y} = \frac{(\rho_v-1)\rho_v\mu}{u''\left(\hat{w}^I_{0G}\right)(\lambda\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]+\mu(\rho_v-\rho_y-\pi(\rho_v+\rho_y-1)))^2} > 0 \\ \frac{\partial \hat{w}^I_{1B}}{\partial \rho_y} = -\frac{(\pi(\rho_v-1)-\rho_v)(1+(\pi-1)\rho_v)\mu}{u''\left(\hat{w}^I_{1B}\right)(\lambda\rho_y(1-\pi)(1-\rho_v)+\mu(\pi-1)(\rho_v-\rho_y))^2} < 0 \\ \frac{\partial \hat{w}^I_{1G}}{\partial \rho_y} = -\frac{(\rho_v-1)\rho_v\mu}{u''\left(\hat{w}^I_{1G}\right)((\pi(\rho_v-1)-\rho_v)\rho_y\lambda+\mu(\rho_v+\rho_y+\pi(\rho_y-\rho_v))^2} < 0 \end{cases}$$ ■

Proof of Proposition 3. We use the result established by Kim (1995), showing that an information structure $\mathbf{P}$ is more efficient than an information structure $\mathbf{\Pi}$ if its likelihood ratio is a mean preserving spread of that of $\mathbf{\Pi}$.

We compute the following function:

$$\Phi\left(\rho^p_v, \rho^\pi_v, \rho^p_y, \rho^\pi_y, \pi\right) \equiv \sum_{i\in S}\left(\frac{p^I_{i0}}{p^I_i 1} - \frac{p_{i0}}{p_i 1}\right)$$

Where ρ^j_i stands for the precision of signal $i \in \{v, y\}$ of information structure $j \in \{\mathbf{P}, \mathbf{\Pi}\}$ and $\Pi \equiv \mathbf{P}^I$ denotes the probability vector under influence.

$$\Phi\left(\rho_v, \rho_v, \rho_y, \rho_y, \pi\right) = \left(\frac{\left(1-\rho_y\right)(1-\rho_v+\pi\rho_v)}{\rho_y[\rho_v+\pi(1-\rho_v)]} + \frac{\rho_y(1-\rho_v+\pi\rho_v)}{\left(1-\rho_y\right)[\rho_v+\pi(1-\rho_v)]}\right) - \left(\frac{\left(1-\rho_y\right)(1-\rho_v)}{\rho_y\rho_v} + \frac{\rho_y(1-\rho_v)}{\left(1-\rho_y\right)\rho_v}\right) > 0$$

Since $\frac{\partial\left(\frac{(1-\rho_y)(1-\rho_v+\pi\rho_v)}{\rho_y[\rho_v+\pi(1-\rho_v)]}+\frac{\rho_y(1-\rho_v+\pi\rho_v)}{(1-\rho_y)[\rho_v+\pi(1-\rho_v)]}\right)}{\partial\pi} > 0$. At the same time, we have that $\frac{\partial\left(\frac{(1-\rho_y)(1-\rho_v)}{\rho_y\rho_v}+\frac{\rho_y(1-\rho_v)}{(1-\rho_y)\rho_v}\right)}{\partial\rho_v} < 0$. As a result, for any increase in the influence parameter from π^- to π^+ the information structure $\mathbf{P}\left(\rho_v\right)$ is not as efficient as $\mathbf{\Pi}\left(\pi^+, \rho_v\right)$ since then $\Phi > 0$. In order to make $\mathbf{\Pi}\left(\pi^+, \rho_v\right)$ as efficient as $\mathbf{P}$ we can consider the information structure $\mathbf{P}\left(\rho^-_v\right)$ where $\rho^-_v < \rho_v$ so that $\Phi\left(\rho^-_v, \rho_v, \rho_y, \rho_y, \pi^+\right) = 0$. As a result any increase in π reduces the efficiency of the information structure $\mathbf{\Pi}$. ■

Proof of Corollary 6. The first part of the Corollary is trivial. The second part can be derived from Proposition 3 taking into account that the model of rational supervision corresponds to case of supervision under influence when $\pi = 0$. ■

Proof of Corollary 7. It follows from the two previous results and Corollary 2. ■

Proof of Corollary 8. The optimal contract to detract workers from the influence activity ($\hat{\mathbf{w}}^F$) solves:

$$\begin{cases} (1)\ \hat{\mathbf{w}}^F = \min\limits_{\mathbf{w}\in\mathbb{R}^4} \mathbf{w}^\top \mathbf{P}_1 \\ (2)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \bar{u} & \textbf{IR} \\ (3)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \mathbf{u}(\mathbf{w})^\top \mathbf{P}_0 & \textbf{IC} \\ (4)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 \geq \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1^I - \phi_a & \textbf{IF} \end{cases}$$

The non-negative Lagrange multipliers are denoted $\lambda > 0$, $\mu > 0$ and $\delta > 0$. We know that all of them are positive because $\mathbf{w}^{**}$ is not a solution to the optimization problem. We consider the change of variable $u_{1G} = u(w_{1G}^F), u_{1B} = u(w_{1B}^F), u_{0G} = u(w_{0G}^F)$ and $u_{0B} = u(w_{0B}^F)$ to ensure concavity. We then solve for $\hat{\mathbf{w}}^F$ and get:

$$\begin{cases} (1_{1G})\, u'\left(\hat{w}_{1G}^F\right) = \frac{\rho_y\rho_v}{\lambda\rho_y\rho_v+\mu(\rho_y+\rho_v-1)+\delta\left(\pi(\rho_v-1)\rho_y\right)} \\ (1_{1B})\, u'\left(\hat{w}_{1B}^F\right) = \frac{(1-\rho_v)\rho_y}{\lambda(1-\rho_v)\rho_y+\mu\left(\rho_y-\rho_v\right)+\delta\pi\rho_y(1-\rho_v)} \\ (1_{0G})\ u'\left(\hat{w}_{0G}^F\right) = \frac{\left(1-\rho_y\right)\rho_v}{\lambda\left(1-\rho_y\right)\rho_v+\mu\left(\rho_v-\rho_y\right)+\delta\pi\left(\rho_y+\rho_v-1-\rho_y\rho_v\right)} \\ (1_{0B})\, u'\left(\hat{w}_{0B}^F\right) = \frac{\left(1-\rho_y\right)(1-\rho_v)}{\lambda\left(1-\rho_y\right)(1-\rho_v)+\mu(1-\rho_y-\rho_v)+\delta\pi(\rho_v-1)(\rho_y-1)} \end{cases}$$

Therefore, we can show that:

$$(1_{1G})\ \frac{\partial \hat{w}_{1G}^F}{\partial \pi} = -\frac{(\rho_v-1)\rho_v\rho_y^2\delta}{u''\left(\hat{w}_{1G}^F\right)(\pi(\rho_v-1)\rho_y\delta+\rho_v\rho_y\lambda+(\rho_y+\rho_v-1)\mu)^2} < 0$$

$$(1_{1B})\ \frac{\partial \hat{w}_{1B}^F}{\partial \pi} = -\frac{(\rho_v-1)^2\rho_y^2\delta}{u''\left(\hat{w}_{1B}^F\right)((\lambda+\pi\delta)(\rho_v-1)\rho_y+\mu(\rho_v-\rho_y))^2} > 0$$

$$(1_{0G})\ \frac{\partial \hat{w}_{0G}^F}{\partial \pi} = -\frac{(\rho_v-1)\rho_v(1-\rho_y)^2\delta}{u''\left(\hat{w}_{0G}^F\right)((\rho_y-1)(\pi(\rho_v-1)\delta+\rho_v\lambda)+(\rho_y-\rho_v)\mu)^2} < 0$$

$$(1_{0B})\ \frac{\partial \hat{w}_{0B}^F}{\partial \pi} = -\frac{(\rho_v-1)^2(\rho_y-1)^2\delta}{u''\left(\hat{w}_{0B}^F\right)((\lambda+\pi\delta)(\rho_v-1)(\rho_y-1)-\mu(\rho_v+\rho_y-1))^2} > 0$$

Similarly, we can derive the results for $\frac{\partial \hat{\mathbf{w}}^F}{\partial \rho_y}$ and $\frac{\partial \hat{\mathbf{w}}^F}{\partial \rho_v}$ by using the implicit function theorem.

Proof of Corollary 9. We denote $\mathbf{P}_1^\iota \equiv (p_{i1}^\iota)_{i\in\{1,\ldots,4\}}$ the probability vector when the agent undertakes the influence activity in the context of influence costs.

That is, $$\mathbf{P}_1^\iota \equiv (p_{i1}^\iota)_{i\in\{1,\ldots,4\}} = \begin{bmatrix} (1-\alpha)\,\rho_y[\rho_v + \pi(1-\rho_v)] \\ (1-\alpha)\,\rho_y(1-\pi)\,(1-\rho_v) \\ \left[1-(1-\alpha)\,\rho_y\right][\rho_v+\pi(1-\rho_v)] \\ \left[1-(1-\alpha)\,\rho_y\right](1-\pi)\,(1-\rho_v) \end{bmatrix} \blacksquare$$

and $$\mathbf{P}_0^\iota \equiv (p_{i0}^\iota)_{i\in\{1,\ldots,4\}} = \begin{bmatrix} (1-\alpha)\left(1-\rho_y\right)(1-\rho_v+\pi\rho_v) \\ (1-\alpha)\left(1-\rho_y\right)\rho_v(1-\pi) \\ \left[\alpha+(1-\alpha)\,\rho_y\right](1-\rho_v+\pi\rho_v) \\ \left[\alpha+(1-\alpha)\,\rho_y\right]\rho_v(1-\pi) \end{bmatrix}$$

We then have that:

$$\begin{cases} (1_{1B}) \ u'(\hat{w}^{\iota}_{1G}) = \frac{1}{\lambda+\mu(1-\frac{p^{\iota}_{10}}{p^{\iota}_{11}})} \\ (1_{1B}) \ u'(\hat{w}^{\iota}_{1B}) = \frac{1}{\lambda+\mu(1-\frac{p^{\iota}_{20}}{p^{\iota}_{21}})} \\ (1_{0G}) \ u'(\hat{w}^{\iota}_{0G}) = \frac{1}{\lambda+\mu(1-\frac{p^{\iota}_{30}}{p^{\iota}_{31}})} \\ (1_{0B}) \ u'(\hat{w}^{\iota}_{0B}) = \frac{1}{\lambda+\mu(1-\frac{p^{\iota}_{30}}{p^{\iota}_{31}})} \end{cases}$$

By taking derivatives and using simple algebra we get the results summarized in the corollary. ■

Proof of Proposition 4. It is optimal for the principal to design influence-free contracts as long as: $\alpha y + (\mathbf{w}^{\iota})^{\top} \mathbf{P}^{\iota}_1 \geq (\mathbf{w}^{F})^{\top} \mathbf{P}_1$. Also, we know by using a very similar proof to the one presented for Proposition 3 that $(\mathbf{w}^{\iota})^{\top} \mathbf{P}^{\iota}_1$ is increasing in both α and π and decreasing in the precision of both signals ρ_v and ρ_y. We then conclude that as α increases not only influence contracts tend to be more expansive but revenues will also decrease (αy rises).

The cost of implementing the efficient level of effort in the case of influence-free contracts depends on the solution to the following optimization program:

$$\begin{cases} (1) \ \hat{\mathbf{w}}^{f} = \min_{\mathbf{w}\in\mathbb{R}^4} \mathbf{w}^{\top}\mathbf{P}_1 & \\ (2) \ \mathbf{u}(\mathbf{w})^{\top} \mathbf{P}_1 - C \geq \bar{u}_i & \textbf{IR} \\ (3) \ \mathbf{u}(\mathbf{w})^{\top} \mathbf{P}_1 - C \geq \mathbf{u}(\mathbf{w})^{\top} \mathbf{P}_0 & \textbf{IC} \\ (4) \ \mathbf{u}(\mathbf{w}^{f})^{\top} \mathbf{P}_1 \geq \mathbf{u}(\mathbf{w}^{f})^{\top} \mathbf{P}^{\iota}_1 & \textbf{IF} \end{cases}$$

We consider that the influence-free constraint (**IF**) is binding, that is the efficient contract $\mathbf{w}^{**}$ is not a solution to the optimization problem with influence. We denote $IF = \mathbf{u}(\mathbf{w}^{f})^{\top} (\mathbf{P}_1 - \mathbf{P}^{\iota}_1)$. Also, by simple algebra we get the following comparative statics:

i) $\frac{\partial IF}{\partial \alpha} > 0$, *ii)* $\frac{\partial IF}{\partial \pi} < 0$, *iii)* $\frac{\partial IF}{\partial \rho_v} > 0$, *iv)* $\frac{\partial IF}{\partial \rho_y} > 0$ for low values of π whereas $\frac{\partial IF}{\partial \rho_y} < 0$ for π high. As a result, an increase in α will increase the costs of choosing influence contracts since both αy and $(\mathbf{w}^{\iota})^{\top} \mathbf{P}^{\iota}_1$ increase in α but also $(\mathbf{w}^{f})^{\top} \mathbf{P}_1$ decrease in α since the influence-restriction becomes looser as α increases.

- We conclude that there exists a level $\alpha_f \in, (0, 1]$ above which the principal will always choose to design influence-free contracts. Indeed, for the upper bound $\alpha = 1$ we know that influence-free contracts are the only solution since then the principal obtains no revenues from the agent.

- Also, as the ability of the worker increase the only part of the inequation that is affected is αy so that there exists a level of ability y_f above which the principal will decide to design influence-free contracts.

- Concerning π, there exist two opposite effects. First an increase in π rises the costs of implementing influence contracts but at the same time it tends to render more attractive the influence activity so that $\frac{\partial IF}{\partial \pi} < 0$ meaning that influence-free contracts become more costly as π rises. ■

Proof of Corollary 10. We need to solve the following optimization problem.

$$\begin{cases} (1)\ W^* = \min\limits_{\mathsf{w}_i \in \mathbb{R}^4} \mathbf{w}^\top \mathbf{P}_1 \\ (2)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \bar{u} & \textbf{IR} \\ (3)\ \mathbf{u}(\mathbf{w})^\top \mathbf{P}_1 - C \geq \mathbf{u}(\mathbf{w})^\top \mathbf{P}_0 & \textbf{IC} \\ (4)\ \mathbf{u}\left(\mathbf{w}^f\right)^\top \mathbf{P}_1 \geq \mathbf{u}\left(\mathbf{w}^f\right)^\top \mathbf{P}_1^\iota & \textbf{IF} \end{cases}$$

We get the following first order conditions, where δ is the non-negative Lagrange multiplier associated to restriction **IF**. It is easy to see that $\lambda > 0$, $\mu > 0$ and $\delta > 0$ as long as $\mathbf{w}^{**}$ is not a solution to the optimization problem.

$$\begin{cases} (1_{1G})\ u'\left(w_{1G}^f\right) = \frac{\rho_y \rho_v}{\lambda \rho_y \rho_v + \mu(\rho_y + \rho_v - 1) + \delta\left(\rho_y \rho_v - (1-\alpha)\rho_y \rho_v - \pi(1-\alpha)\rho_y(1-\rho_v)\right)} \\ (1_{1B})\ u'\left(w_{1B}^f\right) = \frac{(1-\rho_v)\rho_y}{\lambda(1-\rho_v)\rho_y + \mu\left(\rho_y - \rho_v\right) + \delta \rho_y (1-\rho_v)(1-(1-\alpha)(1-\pi))} \\ (1_{0G})\ u'\left(w_{0G}^f\right) = \frac{\left(1-\rho_y\right)\rho_v}{\lambda\left(1-\rho_y\right)\rho_v + \mu\left(\rho_v - \rho_y\right) + \delta\left(\left(1-\rho_y\right)\rho_v - \left(1-(1-\alpha)\rho_y\right)(\rho_v + \pi(1-\rho_v))\right)} \\ (1_{0B})\ u'\left(w_{0B}^f\right) = \frac{\left(1-\rho_y\right)(1-\rho_v)}{\lambda\left(1-\rho_y\right)(1-\rho_v) + \mu(1-\rho_y-\rho_v) + \delta(1-\rho_v)\left(1-\rho_y - \left(1-(1-\alpha)\rho_y\right)(1-\pi)\right)} \end{cases}$$

We conclude after some algebra that:

$$\begin{cases} (1_{1G})\ \frac{\partial w_{1G}^F}{\partial \alpha} > 0 \text{ for } \alpha > \alpha_1, \text{ where } \alpha_1 = \frac{\pi(1-\rho_v)}{(1-\pi)\rho_v + \pi}. \\ (1_{1B})\ \frac{\partial w_{1,B}^F}{\partial \alpha} > 0 \text{ for any } \alpha > 0. \\ (1_{0G})\ \frac{\partial w_{0,G}^F}{\partial \alpha} < 0 \text{ for any } \alpha > 0. \\ (1_{0B})\ \frac{\partial w_{0B}^F}{\partial \alpha} < 0 \text{ for any } \alpha > \alpha_0, \text{ where } \alpha_0 = \frac{\pi\left(1-\rho_y\right)}{(1-\pi)\rho_y}. \end{cases}$$

- This is a summary of our results on influence-free contracts, wage compression and responsiveness

- Given $\alpha^+ \leq \alpha_F$

i) For $\alpha < \alpha_F$, there is wage compression for both hard and soft signals for both influence parameters α or π.

ii) For $\alpha \geq \alpha_F$, there is wage expansion for the hard signal for the influence parameter α.

- Given $\alpha_F < \alpha_0 < \alpha_1 < 1$ [$\alpha_F < \alpha_1 < \alpha_0 < 1$]

i) For $\alpha < \alpha_F$, there is wage compression for both hard and soft signals for both influence parameters α or π.

ii) For $\alpha_F \leq \alpha < \alpha_0$ [$\alpha_F \leq \alpha < \alpha_1$], there is wage compression for the soft signal for both influence parameters α or π.

iii) For $\alpha_0 \leq \alpha < \alpha_1$ [$\alpha_1 \leq \alpha < \alpha_0$], there is wage compression for the soft signal when $y = 1$ [$y = 0$] and wage expansion for the hard signal when $v = B$ $[v = G]$ for the influence parameter α.

iv) For $\alpha \geq \alpha_1$ $[\alpha \geq \alpha_0]$, there is wage expansion for the hard signal for the influence parameter α.

- Given $\alpha_0 < \alpha_F < \alpha_1 < 1$ $[\alpha_1 < \alpha_F < \alpha_0 < 1]$

i) For $\alpha < \alpha_F$, there is wage compression for both hard and soft signals for both influence parameters α or π.

ii) For $\alpha_F \leq \alpha < \alpha_1$ $[\alpha_F \leq \alpha < \alpha_0]$, there is wage compression for the soft signal when $y = 1$ $[y = 0]$ and wage expansion for the hard signal when $v = B$ $[v = G]$ for the influence parameter α.

iii) For $\alpha \geq \alpha_1$ $[\alpha \geq \alpha_0]$, there is wage expansion for the hard signal for the influence parameter α. ■

Proof of Corollary 11. It follows from the last proposition since for any $\alpha \geq \alpha^+$, there exists a level of productivity $\bar{R} \equiv R(\alpha)$ such that high-productivity agents $(R \geq \bar{R})$ gets an influence-free contract whereas low-productivity agents $(R < \bar{R})$ get an influence contract. Regarding the variance of wages one can see the wage scheme as a mixed Bernoulli distribution with parameter ζ so that the variance of wages $\sigma^2(w)$ in that case is such that: $\sigma^2(w) = \zeta\sigma^2(B_G) + (1-\zeta)\,\sigma^2(B_B) + \zeta\,(1-\zeta)\,[E(B_G) - E(B_G)]^2$ where B_G $[B_B]$ is the Bernoulli distribution that takes values w_{1G} and w_{1B} $[w_{0G}$ and $w_{0B}]$ with probability ρ_y and $\left(1-\rho_y\right)$ respectively. To show that $\sigma^2(w)$ increase in α we are left to demonstrate that $\frac{\partial}{\partial\alpha}[E(B_G) - E(B_G)] \geq 0$, that is to show that $\rho_y\,(w_{1G} - w_{1B}) + \left(1-\rho_y\right)(w_{0G} - w_{0B})$ is increasing in α. We know that as α increases the (IF) constraint is relaxed since costs of influence increase for the agent and at the same time the power of incentives in the hard signal increases in α as we have shown in the previous proposition. As a result, for (IF) to be binding in equilibrium (it has to be the case since $\delta > 0$) it has to be that the benefits associated to influence rise to compensate an increase in costs associated to the influence activity previously mentioned. That is, the power of incentives in the soft signal has to increase with regard to α. This implies that both $(w_{1G} - w_{1B})$ and $(w_{0G} - w_{0B})$ cannot decrease in α. This completes the proof that $\sigma^2(w)$ is increasing in α. ■

Proof of Corollary 12. Free supervision may be detrimental for the principal as long as $\mathbf{w}^{*\top}\mathbf{P}_1 \leq \min\{\left(\mathbf{w}^I\right)^\top \mathbf{P}_1^\iota; \left(\mathbf{w}^f\right)^\top \mathbf{P}_1\}$. In particular, for $\pi = 1$ we know that $\mathbf{w}^{*\top}\mathbf{P}_1 = \mathbf{w}^{**\top}\mathbf{P}_1$ and $\mathbf{w}^{**\top}\mathbf{P}_1 \leq \left(\mathbf{w}^f\right)^\top \mathbf{P}_1$. Also, for $\alpha \geq \alpha_f$ we know that $\left(\mathbf{w}^f\right)^\top \mathbf{P}_1 = \arg\min\{(\mathbf{w}^\iota)^\top \mathbf{P}_1^I; \left(\mathbf{w}^f\right)^\top \mathbf{P}_1\}$. As a result, $\mathbf{w}^{*\top}\mathbf{P}_1 \leq \min\{(\mathbf{w}^\iota)^\top \mathbf{P}_1^\iota; \left(\mathbf{w}^f\right)^\top \mathbf{P}_1\}$ for any $\alpha \geq \alpha_f$ and for any $\pi \geq \pi_0$, where π_0 is such that $\mathbf{w}^{*\top}\mathbf{P}_1 - \mathbf{w}^{**\top}\mathbf{P}_1 = \left(\mathbf{w}^f\right)^\top \mathbf{P}_1 - \mathbf{w}^{**\top}\mathbf{P}_1$.

For $\alpha < \alpha_f$ we know that $(\mathbf{w}^\iota)^\top \mathbf{P}_1^\iota = \arg\min\{(\mathbf{w}^\iota)^\top \mathbf{P}_1^\iota; \left(\mathbf{w}^f\right)^\top \mathbf{P}_1\}$.

We know that $\left(\mathbf{w}^{\iota}\right)^{\top}\mathbf{P}_1^{\iota} > \mathbf{w}^{*\top}\mathbf{P}_1$ for any $\pi \geq \pi_1$, where π_1 is such that $\left(\mathbf{w}^{\iota}\right)^{\top}\mathbf{P}_1^{\iota} = \mathbf{w}^{*\top}\mathbf{P}_1$. ■

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