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ABSTRACT

It has been proved that in an homogeneous product industry, price over marginal costs, business stealing, set up costs and free entry imply excess entry from the welfare point of view. The proof assumes identical firms. We show by example that with non-identical firms, those conditions are compatible with insufficient entry. Besides, we provide a criterium to evaluate excess entry in industries with non-identical firms and externalities.

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Free Entry and Welfare with Different Firms

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Abstract

It has been proved that in an homogeneous product industry, price over marginal costs, business stealing, set up costs and free entry imply excess entry from the welfare point of view. The proof assumes identical firms. We show by example that with non-identical firms, those conditions are compatible with insufficient entry. Besides, we provide a criterium to evaluate excess entry in industries with non-identical firms and externalities.

Keywords:
Free entry, Social welfare, Oligopoly
D24, D43, L13

1. Introduction

Is free entry convenient for society? It is well-known that for competitive markets the answer is positive. It has also long been known that for imperfectly competitive markets entry may be socially wasteful. The reason is that although the entry of an additional firm expands the market, the cost of this market expansion may be higher than its value, because the new firm gets some of its market share by stealing from existing businesses. This result has been proved with different degrees of generality for homogeneous markets by von Weizsaker (1980), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). All these papers base their conclusions on models with identical firms.

The approach of Mankiw and Whinston (1986) is quite general. They state that in any homogeneous market with free entry, price over marginal

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cost, set up costs, business stealing and *identical firms*, entry is excessive. This result is valid whatever the way the firms compete among themselves, cooperative or not. Considering that the number of firms must be an integer, this result is subject to some usually minor qualifications. With product diversity they find that the welfare of free entry is ambiguous.

In real markets, firms in the same industry tend to be different. Will the fact that there are different firms in the market affect the result of excess entry? Suppose an observer from a competition authority examines an industry –that seems to be in equilibrium; this industry shows price over marginal costs, significant fixed cost, business stealing and free entry; could this observer be sure that there is excess entry if the firms happen to be different? The main purpose of this note is to provide some illustration to this question.

This paper relies on two main ideas. The first one is that business stealing could be beneficial if the "robber" has lower costs than the "victim." The second one is that with non-identical firms, the equilibrium market structure will probably allow for firms that break even, together with firms with positive profits. In this structure there will be presumably less firms. Entry, if successful, will tend to replace high cost firms rather than lower cost firms; and that could be beneficial, even if business stealing is present.

Business stealing has established itself as a widely accepted and used concept in the literature. But it is usually considered as harmful to society. However, business stealing is a essential part of competition, perfect or imperfect. If the degree of competition is measured only with the number of identical firms in a market, it is easy to agree to that negative view. But when competition is seen more as a kind of biological process where the best are the successful ones, then more firms in the market probably means more incentives for the incumbents to improve, striving to being different from the other firms, and entry is also a good antidote against X-inefficiency. From this point of view, business stealing might be considered beneficial. However, in this paper we will rely on the static view of welfare.

With different firms there is more freedom for models than with identical firms. There is probably no single model that captures all the features of different firms. We present in this paper four models that show that insufficient entry is compatible with business stealing, free entry, price over marginal costs and fixed costs.

This paper proceeds as follows. Section 2 will show how, when firms does not share the same marginal costs, business stealing does not necessarily indicate excess entry. Section 3 will provide similar results for the case of firms with identical marginal costs, but with an unequal market share. Section 4 will propose a simple alternative that is better suited than business steal-

ing to determine the welfare effects of entry, when firms are different and there are externalities: the evolution of average costs with entry. Section 5 concludes.

2. Different marginal costs

In this section we present two models, a Cournot and a Stackelberg one. There are two types of firms with constant marginal cost, and the demand is of the constant elasticity type. In an identical model, without fixed costs and unspecified demand, Ghosh and Saha (2007) show that there is always excess entry, while we find insufficient entry for some cases. The reason for the inconsistency between their paper and ours is that they assume that the demand function, $p(Q)$, always fulfills $p'(Q) + Qp''(Q) \leq 0$. In this way they exclude from their analysis the constant elasticity type of demand.

We begin our analysis considering the welfare as a function of the number of firms.

Let us consider a market with demand $p = p(Q)$ and two types of firms. There are r type 1 firms with marginal cost c_1, and free entry of type 2 firms with marginal cost $c_2 > c_1$. Type 2 firms have a fixed cost of F, and type 1 firms have a fixed cost of G.

Welfare in this market with n high cost entrants is

$$W = \int_0^Q p(s)ds - rc_1q_1 - nc_2q_2 - nF - rG \qquad (1)$$

Deriving, we get

$$\frac{\partial W}{\partial n} = p(r\frac{\partial q_1}{\partial n} + n\frac{\partial q_2}{\partial n} + q_2) - rc_1\frac{\partial q_1}{\partial n} - c_2q_2 - nc_2\frac{\partial q_2}{\partial n} - F. \qquad (2)$$

With free entry, n^e firms will enter so that the profits of all high cost firms be zero. Evaluating equation 2 with free entry, and rearranging we get

$$\left.\frac{\partial W}{\partial n}\right|_{n=n^e} = r(p - c_1)\frac{\partial q_1}{\partial n} + n(p - c_2)\frac{\partial q_2}{\partial n} \qquad (3)$$

Even if we suppose that there is business stealing, i.e., $\frac{\partial Q}{\partial n} < q_2$, still $\frac{\partial q_1}{\partial n}$ could be positive. In this case, there could be insufficient entry with business stealing.

2.1. A Cournot Model

Let us consider a Cournot oligopoly with demand $p = (a/Q)^{1/\epsilon}$ and two types of firms. There are r firms of type 1 with marginal cost $c_1 = 1$,[2] and free entry of firms of type 2 with marginal cost $c_2 = c > 1$. As before, type 2 firms have a fixed cost of F, and type 1 firms have a fixed cost of G.

Profit in any Cournot oligopoly is $\Pi = p(Q)q - c(q)$; where Q is the quantity produced in the industry, and q is the quantity produced by the firm. Deriving Π, we get $\frac{\partial \Pi}{\partial q} = p'(Q)q + p - c'(q) = 0$. Rearranging this expression we get the well-known condition, where s_i is the market share, $s_i = q_i/Q$:

$$\frac{p - c_i}{p} = \frac{s_i}{\epsilon}. \tag{4}$$

The second derivative of profit, $\Pi(x) = x \left(\frac{a}{Q+x} \right)^{1/\epsilon} - cx$, for a firm producing x at a marginal cost c, while the others produce Q, is:

$$\frac{\partial^2 \Pi}{\partial x^2} = -\frac{(2\epsilon Q + \epsilon x - x) \left(\frac{a}{Q+x} \right)^{\frac{1}{\epsilon}}}{\epsilon^2 (Q + x)^2}.$$

If $\epsilon \geq 1$, the second derivative of profit is always negative. If $\epsilon < 1$, then this expression is negative whenever $x < 2Q\epsilon/(1 - \epsilon)$. We restrict ourselves to these situations.

Remember that there are r firms with marginal cost 1, and n firms with marginal cost $n > 1$. Let us call $s_2 = s$ to the market share of the high cost firms. Then, the market share of type 1 firms is $s_1 = (1 - ns)/r$. Applying equation 4, yields,

$$\frac{p - 1}{p} = \frac{1 - ns}{r\epsilon}, \text{ and } \frac{p - c}{p} = \frac{s}{\epsilon}. \tag{5}$$

From here the equilibrium price and quantity in the market is

$$p = \frac{r + nc}{r + n - 1/\epsilon}, \text{ and } Q = a \left(\frac{r + n - 1/\epsilon}{r + nc} \right)^{\epsilon}, \tag{6}$$

and the market shares are

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[2]This assumption represent no loss of generality, because if the marginal cost were, for instance, $t = 3.5\$$, we will redefine our monetary unit to a unit whose value is exactly 3.5\$.

$$s_1 = \frac{1 + n\epsilon(c-1)}{r + nc}, \text{ and } s_2 = s = \frac{c - r\epsilon(c-1)}{r + nc} \qquad (7)$$

We can calculate equation 3:

$$\left.\frac{\partial W}{\partial n}\right|_{n=n^e} = \frac{Qs(An^2 + Bn + C)}{(r + nc)(r\epsilon + n\epsilon - 1)^2}$$

with

$$A = \epsilon\left((c-1)^2(\epsilon^2 + \epsilon)r - c^2\right).$$
$$B = (c-1)^2(\epsilon^2 + \epsilon)r(r\epsilon - 2) + c\epsilon(c - 2r) + c^2.$$
$$C = r(\epsilon(1-r) + 1). \qquad (8)$$

Equation 8 provides the main result of this subsection.

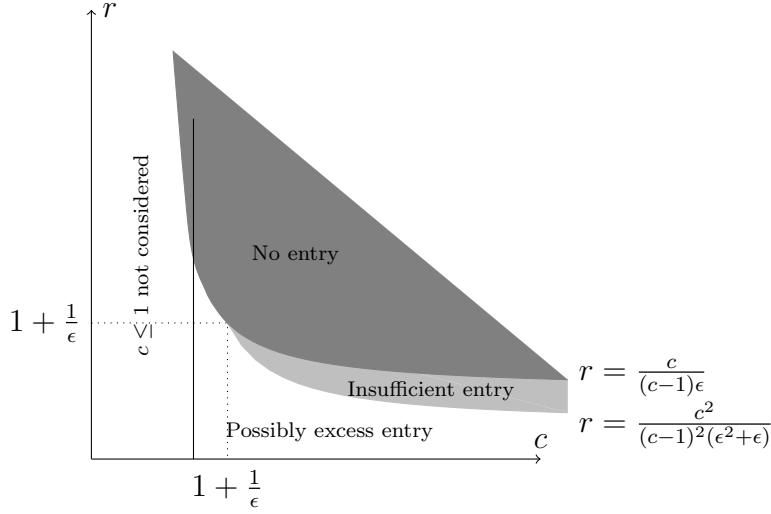**Proposition 1.** *In a Cournot market with constant elasticity of demand, ϵ, r incumbents with constant marginal cost 1, and free entry of firms with fixed entry costs and constant marginal costs $c > 1$, the entry will be insufficient whenever $\frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)} < r < \frac{c}{(c-1)\epsilon}$.*

PROOF. We will prove that $\frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)} < r < \frac{c}{(c-1)\epsilon}$ implies $A, B, C, s > 0$.

(1) Note that $\frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)} < \frac{c}{(c-1)\epsilon}$ is equivalent to $c > 1 + \frac{1}{\epsilon}$. Consequently, $r < \frac{c}{(c-1)\epsilon}$ implies (because $c/(c-1)$ is a decreasing function for $c > 1$) $r < 1 + \frac{1}{\epsilon}$, and then $C > 0$.

(2) The expression $\frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)} < r$ is equivalent to $A > 0$.

(3) The expression $r < \frac{c}{(c-1)\epsilon}$ is equivalent to $s > 0$.

(4) In B, we substitute r by its lower bound $\frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)}$ in all ocurrences but in $(c - 2r)$, where we substitute by its upper bound $\frac{c}{(c-1)\epsilon}$. It yields $B > \frac{c^2(\epsilon c - \epsilon - 1)^2}{(\epsilon+1)(c-1)^2} > 0$.

With $A, B, C, s > 0$, it is clear that entry is insufficient. Naturally, there could be other values with insufficient entry.

We show in the adjunct figure the areas of excess and insufficient entry. Note that when $c = r = 1 + 1/\epsilon$, then $A = s = B = 0$.∎

The figure shows a plot with axis r (vertical) and c (horizontal). Regions labeled "No entry", "Insufficient entry", "Possibly excess entry", with $c \leq 1$ not considered. Marked values $1 + \frac{1}{\epsilon}$ on both axes, and curves $r = \frac{c}{(c-1)\epsilon}$ and $r = \frac{c^2}{(c-1)^2(\epsilon^2+\epsilon)}$.

Numerical examples. We begin with the specific case $r = \epsilon = 1$. From equations 6 and 7 we find

$$p = c + \frac{1}{n}, \quad Q = \frac{na}{1+nc}, \quad q_1 = \frac{na(1+n(c-1))}{(1+nc)^2}, \quad q_2 = \frac{na}{(1+nc)^2}. \quad (9)$$

The equilibrium number of firms, n^e, can be found easily:

$$\Pi = (p-c)q_2 - F = \frac{a}{(1+n^e c)^2} - F = 0 \Rightarrow n^e = \frac{1}{c}\left(\sqrt{\frac{a}{F}} - 1\right)$$

Let $a = 625$, $c = 12$ and $F = 1$. Then $n^e = 2$. What is the optimal number of firms, n^*? We will evaluate $W = a \ln Q - (q_1 + ncq_2 + nF)$; or,

$$625 \ln\left(\frac{625n}{1+12n}\right) - \left(\frac{625n(1+11n)}{(1+12n)^2} + \frac{12 \cdot 625n^2}{(1+12n)^2} + n\right).$$

The maximum is found at $n^* = 6.28$. In this case only two firms enter, while the optimal number of firms is at least 6. With a more elastic demand, we can find insufficient entry with c not that far from 1. For example, with $\epsilon = 5$ and $r = 1$. Then we will find insufficient entry for $c = 1.224$.

2.2. A Stackelberg Model

In a Stackelberg market with demand $p = a/Q$, the leader has constant marginal cost $t = 1$. The followers have constant marginal cost $c > 1$ and a fixed cost F. The leader set a quantity, K, knowing that there will be free entry of followers.

Demand for followers is $p = a/(Q+K)$. Any entrant has profit $\Pi = (p-c)q - F$. To maximize, any follower solves $p - c - qa/(Q+K)^2 = 0$.

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Suppose that the number of entrants is n. Adding up all this equations for the n followers, we get $n(p - c) - Qa/(Q + K)^2 = 0$. Using the demand equation, and rearranging yields the demand of the leader when there are n followers:

$$K = \frac{anc - a(n-1)p}{p^2}.$$

The leader's profit, $\Pi_L = (p-1)K$, is maximized with a price and quantity:

$$p = \frac{2nc}{n + nc - 1}, \text{ and } K = a\frac{c^2n^2 - (n-1)^2}{4nc}.$$

It is easy to see that the price p is decreasing with n while the quantity of the leader K is increasing with n. The quantity produced by the followers is

$$Q = \frac{a}{p} - K = \frac{a\left(n^2 - (nc-1)^2\right)}{4nc}.$$

From this expression, it is clear that when $c \geq 1 + 1/n$, the leader will not allow any firm entering the market. We suppose that $c < 1 + 1/n$.

In this market there is business stealing when

$$\frac{Q}{n} - \frac{\partial(Q + K)}{\partial n} = a\frac{(1 - c^2)n^2 + 2cn - 3}{4cn^2} > 0.$$

This happens when

$$\frac{c - \sqrt{3 - 2c^2}}{c^2 - 1} < n < \frac{c + \sqrt{3 - 2c^2}}{c^2 - 1}.$$

For example, for $c = 1.1$, then $1.6 < n < 8.8$; for $c = 1.05$, then $1.5 < n < 18.9$. In any case we restrict ourselves to cases where business stealing is present.

The profit of the followers is

$$\Pi = (p - c)\frac{Q}{n} - F = \frac{a(n + 1 - nc)^2}{4n^2} - F.$$

Consequently, the number of firms with free entry is

$$n^e = \frac{1}{2\sqrt{\frac{F}{a} + c - 1}}.$$

Applying equation 3, or deriving directly we find, calling $x = \sqrt{F/a}$,

$$\left.\frac{\partial W}{\partial n}\right|_{n=n^e} = \frac{a(2(1+2c)x^3 - 2(2-c^2)x^2 + 2(c-1)^2 x + c(c-1)^2)}{2c(1-x)} \quad (10)$$

With equation 10, we can state the following

Proposition 2. *In a Stackelberg market with demand $p = a/Q$, only one leader with constant marginal cost 1, and free entry of firms with fixed entry costs and constant marginal costs $c > 1$, entry could be insufficient.*

PROOF. We suppose that $x = \sqrt{F/a} < 1$.
   Looking only at the coefficients of x^2 and x in the numerator of equation 10, it can be seen that $-2(2-c^2)x^2 + 2(c-1)^2 x > 0$ is equivalent to $x < \frac{(c-1)^2}{2-c^2}$. Then, for whatever $c \in (1, \frac{1+\sqrt{3}}{2})$, we can find $x < 1$ so that equation 10 is positive, and entry is insufficient. Naturally x could be higher because we have not considered the two additional positive monomials.∎

   Curiously, in the Stackelberg model is easier to find examples when the cost of the entrants is similar to that of the incumbent. For example, with demand $p = 1000/Q$, marginal cost $c = 1.1$, and fixed cost $F = 5$. The number of firms will be $n^e = 4$. The optimal number of firms is $n^* = 10$, but remember that there is business stealing until $n = 8.8$. Increasing F to 5.1, the optimal number of firms is $n^* = 8.89$.
   When $a = 700$, $F = 0.88$ and $c = 1.05$, then the number of entrants is $n^e = 8.27$, while $n^* = 18.638$.
   To evaluate the optimal number of entrants we have used the derivative of welfare, $W = a\ln(Q+K) - (K + cnq + nF)$, with respect to n.

$$\frac{\partial W}{\partial n} = a\frac{(c^2-1)^2 n^3 - (c-1)^2(c+1)n^2 - (c-1)^2 n + c + 1}{4cn^2(nc+n-1)} - F.$$

   The examples in this section should be viewed as counterexamples, in the sense that it cannot be assured that, when firms are not identical, free entry is always excessive. The fact that most common models show that there is excess entry does not imply that in reality it must be so. It depends on the demand and on the way firms compete among themselves.

3. The same marginal costs

Both models in the past section have firms with different marginal costs. In this section we present two models, where both firms share the same marginal cost. The first model of this section has in common with the two preceding models that upon entry some firm expand its output. However, in the second example of this section, all firms reduce its output upon entry, all have the same marginal cost, and, nevertheless, we find insufficient entry.

3.1. Some differentiation

It is well known that, with product differentiation, the welfare of free entry depends on the balance between business stealing and the value added by the variety of new entrants. In this model there is product differentiation, but entrants do not add to variety, because entrants are in an homogeneous market. However, the price in the homogeneous market influence the price in the differentiated markets.

Consider several interrelated markets, in one of them outcomes are determined through Cournot competition with fixed entry costs, while the other sets prices depending on the first market's outcome.

A number of examples can be used to illustrate this type of connection. Consider, for example, the market for name-brand clothes and that of their generic counterparts so that, if similarly priced, consumers will prefer the brand product. If competition in the first market results in a certain equilibrium price p, the producer in the second market will charge a price of $p + t$, where t is the maximum amount consumers are willing to pay to acquire the brand-name product. Another example would entail identical goods produced in an oligopoly market and a monopolistic market separated by a certain distance. If the monopolist charges a price that exceeds the oligopoly price plus the cost of transportation, every consumer will purchase the product from the oligopoly market, even if it means paying the transportation cost. Consider that there is excess entry in the radio broadcasting market. The lower price for advertisers in radio may attract advertisers from TV or other media, and consequently the price in these other media may be lowered.

Under these circumstances, the social welfare effects of entry in the first market (the one that sets prices independently) cannot be accurately measured by considering only the direct effects. Additional entry in the first market will reduce not only the price prevalent in that market, but also the price charged in the second (dependent) market. If the price reduction in the second market is large enough, society may benefit from additional entry in the first market, even if the direct benefits are not large.

The model. In a market with demand $Q = 1 - p$, there are m differentiated firms and free entry for generic firms. Demand is divided in $Q = a(1 - p)$ for the generic product, and $Q_i = s_i(1 - a)(1 - p)$ for the differentiated firms, $i = 1, \ldots, m$, with $\sum s_i = 1$. The technology to provide this good is a fixed cost, with no variable cost. The fixed cost for the generic product is F, and for the differentiated goods, F_i. In the generic market, firms play Cournot with free entry, while in the differentiated markets each firm is a monopoly, so that no one can sell there except the incumbents. However, clients of the differentiated good can buy the generic, but with a transportation cost, t. If p is the generic price, then the price of all the differentiated products is going to be $p + t - \epsilon$ (with $\epsilon > 0$ very small), because, with a higher price, the differentiated firms would have no clients. We rule out competition among differentiated producers to simplify the model.

In a market with linear demand, $p = r - sQ$, willingness to pay as a function of price is $(r^2 - p^2)/(2s)$. In the situation above described welfare is

$$W = \frac{1 - a}{2} \left(1 - (p + t)^2\right) + \frac{a}{2} \left(1 - p^2\right) - nF - \sum_{i=1}^{m} F_i.$$

Deriving,

$$\frac{\partial W}{\partial n} = -\left(p + (1 - a)t\right) \frac{\partial p}{\partial n} - F.$$

In the generic market with n firms, price will be $p = 1/(1 + n)$; consequently, $\partial p / \partial n = -p^2$. And the profit will be zero when $ap^2 = F$. Then, welfare is

$$\left. \frac{\partial W}{\partial n} \right|_{n = n^e} = p^2(p + (1 - a)t - a). \tag{11}$$

From equation 11, whenever $t > a/(1 - a)$, free entry is always insufficient, whatever the number of firms. But, as $p = 1/(1 + n)$, it can be also insufficient when $n^e < \frac{1}{a - (1 - a)t} - 1$.

Total output in this model is $Q = n/(1 + n) - (1 - a)t$, and each entrant produces $q = a/(1 + n)$. There is business stealing when $dQ/dn < q$, or $1/a < n + 1$. Consequently, there is insufficient entry with business stealing when $1/a < n^e + 1 < \frac{1}{a - (1 - a)t}$.

For example if $a = 0.2$ and $t = 0.1$, entry will be insufficient with business stealing when $4 < n^e \leq 7.3$.

3.2. *Different outputs*

Suppose that a group of identical firms in an industry agree on a price p. Is it sure that all firms will sell the same quantity? No; clients are indifferent

among firms, because all firms sell an homogeneous product with the same price. Suppose that there is some kind of a structure on the demand; for example there are some groups of different size, and each group prefer –if the price of the product is the same across firms– to buy all of them from the same firm. Some firms can get a big market share, while other firms have less clients. If a firm gets a small market share, it could low the price, but then the agreement on price is broken, and competition begins. It is not unlikely that identical firms sell different quantities in a cooperative oligopoly with an agreement on prices.

If there is an agreement on prices and all firms sell the same quantity with free entry, then the number of firms is at a maximum. Suppose that all firms sell q. Any firm that sell less than q will exit the market. If the distribution is uneven, any different firm will sell more than q; consequently, with different firms, there are less firms.

Before we proceed with the model, it is advisable to remember the purpose of this note. Mankiw and Whinston (1986) show that in any oligopoly with identical firms, cooperative or not, set up costs, price over marginal cost and free entry imply excess entry. The question we want to address is the following: will there be always excess entry if technologically identical firms sell different quantities? Our answer is no, because the number of firms depends on how output is distributed among them.

The model. Consider an oligopoly with demand $p = a/Q$, and free entry of identical firms, all with fixed cost $F = 1$ and no variable costs. There is an agreement among the firms on a price, depending on the number of firms, $p = a/(n+1)$. Clients are indifferent among firms. But, by the characteristics of demand, the clients distribute themselves in the following way

$$q_i = \frac{2i}{n},$$

where i represents the order of entry $(i = 1, \ldots, n)$ of all the firms *in* the market. The oldest firm in the market will be firm $i = 1$, and the newest $i = n$. Total production is

$$Q = \sum_{i=1}^{n} \frac{2i}{n} = n + 1.$$

As firms continue to enter, the benefit for the oldest one decreases, until it reaches zero. At this point, any additional entry will cause the oldest firm to exit, and the oldest firm *in* the market has now index $i = 1$.

In this market there is business stealing, because the output of any entrant is 2, while total output increases in 1. Besides, all firms reduce their output

with entry. When a new firm enters and none of the older ones leaves the market, firms keep their order, but the total number of firms in the market, n, increases so that each q_i is reduced. If a firm leaves as another enters the market, the actual number of firms, n, would not change, but, since i will decrease for all firms, q_i is also reduced. This is true for any number of firms in the market.

Suppose that n^e is the equilibrium number of firms. We want to calculate the optimal number of firms, n^*. If n^e is an equilibrium, then the first firm has no profits: $p^e q_1 - F = 0$. This means that $p^e = n^e/2$. We know that total output is $n^e + 1$. Then, using the demand function, $p^e = a/(n^e + 1)$, yields $a = n^e(n^e + 1)/2$.

If the number of firms increases by 1, then the market expansion is 1. The cost of the new entrant is also $F = 1$. So until $p = 1$ there is room to beneficial entry. If $a = n^e(n^e + 1)/2$, then $p = 1$ means $1 = a/(n^* + 1)$. In consequence $n^* = a - 1$, or $n^* = n^e(n^e + 1)/2 - 1$. For example, if $a = 10$, then $n^e = 4$ and $n^* = 9$; if $a = 210$, then $n^e = 20$ and $n^* = 209$.

Suppose that in equilibrium, with $a = 210$ and $n^e = 20$, a new firm enters producing $q = 1.11$, with a market expansion of 0.11 and a business stealing of 1, while the first firm, with losses, remains in the market. The value of the market expansion, $0.11 \cdot 210/21.11 > 1$, indicates that the entry is convenient for society. This example describes an equilibrium with free entry, where a new firm enters with a tiny market expansion –less than a 0.6%–, most of the output of this firm comes from business stealing, but welfare increases with entry.

There is a big insufficient entry in this market –a cooperative oligopoly with zero marginal costs, identical firms and fixed costs. This is admittedly an ad-hoc example, but shows that without the hypothesis of identical output, the result by Mankiw and Whinston (1986) is no longer valid for cooperative oligopolies agreeing on price. We need to know how output is distributed among firms to see if there is excessive entry or not.

4. Average costs

We show in this paper that, with non identical firms, the presence of business stealing in a market with free entry, price over marginal cost and set up cost does not necessarily imply excess entry. We now want to provide a criterium to judge excess entry, that is useful when firms are different or there are externalities among firms. This criterium is the average cost in the industry as a function of the number of firms.

Consider a market with demand $p = P(Q)$, and n firms with, possibly, different cost functions $C_i(q_i, n)$; the variable n in the cost function means

that we allow for externalities depending on the number of firms. For this market we represent the production of each firm as $q_i(n)$, $i = 1, \ldots n$; thus, $\sum_{i=1}^{n} q_i(n) = Q(n)$.

For each set of $q_i(n)$ in the market we define the average cost function:[3]

$$AC(n) = \frac{1}{Q(n)} \sum_{i=1}^{n} C_i(q_i(n), n).$$

In this market welfare as a function of the number of firms is:

$$W(n) = \int_0^Q P(s)ds - Q(n)AC(n).$$

Deriving with respect to n, we get:

$$W'(n) = p\frac{dQ}{dn} - AC\frac{dQ}{dn} - Q\frac{dAC}{dn} = (p - AC)\frac{dQ}{dn} - Q\frac{dAC}{dn}. \qquad (12)$$

From equation 12 is clear that if all firms in the industry have almost zero profits, or if new firms barely expand the market, the excess entry problem depends on the sign of $\frac{dAC}{dn}$. Without externalities, this is the result of Mankiw and Whinston (1986). However, there could be positive or negative externalities among firms. Hsieh and Moretti (2003) provide an example of negative externalities that imply excess entry. Yet, if some firms have positive profits in equilibrium and there is some market expansion with entry, then $(p - AC)dQ/dn > 0$ may counterbalance the possible negative sign of $\frac{dAC}{dn}$. Naturally, if entrants in the market have lower costs than incumbents, $\frac{dAC}{dn}$ could be negative, and entry would be insufficient.

Equation 12 deals with externalities and different firms. We provide now an example, with all requirements to excess entry, surmounted by positive externalities. Let's consider a market where firms have costs taking the form $C(q) = F + cq$, $c < 1$, and firms agree on a unitary price. To simplify, assume that total output is $Q = 1$ and that all firms produce the same amount. It is then obvious that each new firm steals its business from the previously existing ones, only adding cost, and that entry is excessive. However, if there are cost-reducing positive externalities, the cost function may take the form: $C(q, n) = F(n) + cq$, with each identical firm producing $q = 1/n$. The average cost in the industry will then be $nF(n) + c$. To judge the welfare effects of

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[3]The numbers i and n must be integers. However, continuous functions that interpolate among the integers may be allowed for calculations. In this case, the function $Q(n)$ and $AC(n)$ are defined, and make sense, only when n is an integer, but they can be used when n is not an integer.

entry, we apply equation 12 to this case to get $W'(n) = -nF'(n) - F(n)$. The change in society's welfare will then depend on the exact cost function of firms, and we can easily find examples where entry can be insufficient even though business stealing is still present (take, for example, $F(n) = 1/n^r$, with $r > 1$).

5. Final remarks

There is a consensus among economists on the bias toward excessive entry in homogeneous markets with imperfect competition and business stealing effect. This opinion is based on models with identical firms. However, when different firms are allowed, a plethora of new possibilities is opened, and in this paper we show with examples that insufficient entry may appear where, with identical firms, only excess entry could happen. This is not a positive result, but it is important to do robustness tests to the ideas that may influence public policy.

In most real world industries, firms do not have the same market share. Our examples show that the possibility of insufficient entry cannot be excluded only by the theory based on identical firms. With a more minute research of each industry it is possible to arrive at the existence of excess entry. Unfortunately, our paper shows that in most cases this investigation is needed in order to reach this conclusion.

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