Universidad de Navarra

Facultad de Ciencias Económicas y Empresariales

Working Paper nº 02/13

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Tommaso Trani
University of Navarra

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Working Paper No.02/13
March 2013

ABSTRACT

In this paper, I study the international transmission of shocks when assets traded across borders are differently suitable as collateral for borrowing (i.e., pledgeability). Under financial integration, differences in pledgeability have implications for the demand for assets. For instance, if a shock makes it more difficult to pledge the assets of the country receiving the shock, agents expect these assets to yield a relatively higher premium than foreign assets in the near future. I develop an approach to determine the optimal portfolio allocations, as existing methods cannot be directly applied to capture differences in asset pledgeability. In this case of heterogeneously pledgeable assets, financial shocks are transmitted from one country to another because the same asset is held by residents of different countries. Valuation effects arise as a consequence of the reaction of asset returns in different countries. In contrast, a standard model cannot generate any of these implications when assets have the same degree of pledgeability. Indeed, when assets have the same degree of pledgeability, financial shocks are country-specific and hinder the access to credit only for the residents of the country hit by the shock.

* The external appendix with the methodological details is available upon request.

Tommaso Trani
School of Economics and Business Administration
University of Navarra
ttrani@unav.es

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Tommaso Trani†

This draft: February 20, 2013

Abstract

In this paper, I study the international transmission of shocks when assets traded across borders are differently suitable as collateral for borrowing (i.e., pledgeability). Under financial integration, differences in pledgeability have implications for the demand for assets. For instance, if a shock makes it more difficult to pledge the assets of the country receiving the shock, agents expect these assets to yield a relatively higher premium than foreign assets in the near future. I develop an approach to determine the optimal portfolio allocations, as existing methods cannot be directly applied to capture differences in asset pledgeability. In this case of heterogeneously pledgeable assets, financial shocks are transmitted from one country to another because the same asset is held by residents of different countries. Valuation effects arise as a consequence of the reaction of asset returns in different countries. In contrast, a standard model cannot generate any of these implications when assets have the same degree of pledgeability. Indeed, when assets have the same degree of pledgeability, financial shocks are country-specific and hinder the access to credit only for the residents of the country hit by the shock.

JEL classification: E44, F32, F41, G11, G15

Keywords: international portfolio choice, riskiness of pledged collateral, return differentials, macroeconomic interdependence

*The current draft is a revision of the version of February 2012. I wish to thank Cédric Tille, Rahul Mukherjee, Pierpaolo Benigno and Nicola Borri for comments and suggestions. I am also grateful to Christopher A. Gohrband and Kristy L. Howell, for generously sharing their data, to Mirko Abbritti and Michael Burda, for occasional discussions, and finally to seminar and conference participants at IHEID, the 13th Spanish Conference on International Economics, the 37th Spanish Simposio and the 11th Workshop on Macroeconomic Dynamics in the Bank of Italy. All remaining errors are exclusively mine.

†Address: School of Economics and Business Administration, Universidad de Navarra, Campus Universitario, 31009 Pamplona, Navarra (Spain). E-mail: ttrani@unav.es

1 Introduction

The recent financial crisis has highlighted the global dimension of frictions in credit markets, with borrowing constraints playing an important role because investors take advantage of international market integration and purchase securities of different countries. These cross-border positions form the total wealth of investors, and this wealth is simultaneously the collateral to pledge as a guarantee for the loans available in the credit market. This internationally diversified collateral is a channel for the transmission of shocks from one country to another (Devereux and Yetman, 2010)[1]. This transmission mechanism has been at work during the financial debacle started in the US in 2007. In the first stage of the crisis, borrowing constraints tightened worldwide, because European banks had invested in the U.S. and U.S. agents had purchased assets in Europe.

In examining this mechanism, the literature so far has not looked into the possibility that investors of a given country can borrow more or less against local collateral assets than against foreign assets. Specifically, the standard assumption is that assets of different countries constitute a unique collateral (i.e., have the same degree of pledgeability). It follows that a country-specific real shock drives a wedge between the relative return on local versus foreign assets, but the fact that there is no difference in the way investors can use all of these assets as collateral remains unaltered. Most importantly, also financial shocks are country-specific, as they reduce the access to the credit market for all of the investors of a given country regardless of the assets that they pledge for borrowing. This is an important case, but it is arguably not so appropriate for understanding situations like the one observed at the onset of the recent crisis, when European banks could not borrow pledging assets connected with the U.S. housing sector.

In this paper, I introduce asset-specific pledgeability, so that - under idiosyncratic shocks - assets display different degrees of pledgeability across the border. Two are the contributions of this analysis. First, I develop an approach for the solution of the international portfolios under heterogeneous pledgeability. This approach extends the methods developed by Devereux and Sutherland (2011a) and Tille and Van Wincoop (2010), which are otherwise not directly applicable to the case I analyze. Second, I study the international transmission of asset-specific financial shocks as opposed to the transmission of country-specific financial shocks. Understanding the different mechanisms is nowadays crucial, since productivity shocks fall short of fully explaining gross financial flows in times of distress (Broner, Didier, Erce and Schmukler, 2013).

There is ample evidence that lenders may have a preference for some collateral assets as opposed to other alternatives. For example, in 2008-2009 it was easier for borrowers to obtain funds

[1]See also Dedola and Lombardo (2012), Devereux and Sutherland (2011b) and Trani (2012)

pledging non-subprime bonds than pledging subprime securities (Gorton and Metrick, 2010)[2]. Garleanu and Pedersen (2012) show that differences in asset liquidity can justify price-differences between assets that otherwise yield the same income stream as well as between investment-grade and high-yield bonds.

This paper analyzes how these types of differences in the pledgeability of assets can improve our understanding of the interlinkages between financially integrated countries. For example, BEA data show that in recent years the financial flows between the U.S. and the rest of the world were markedly influenced by the change in the quality of U.S. private securities. In 2003-2007, there had been a massive increase in the external demand for the debt securities issued by the U.S. private sector, including asset-backed securities. According to BEA data (Gohrband and Howell, 2010), the investment in private debt had increased by 19%, which is bigger than the 16% average increase in the net purchase of all types of U.S. assets for those four years[3]. But after 2007, previously good U.S. private assets lost their rating, affecting negatively the U.S. capital inflows. In fact, over the short-run the international investment positions of countries is strongly affected by flows involving private assets. Figure 1 decomposes the net foreign assets of the U.S. in the net position in private assets and the net position in Treasury securities. While the latter are persistently safe and explain the downward trend, the volatility of the overall net foreign assets is to be attributed to private assets.

Thus, using a two-country model with endogenous portfolio choice and borrowing constraints, I let the pledgeability of assets from each country to react to productivity and financial shocks in that country. The rest of the model is fully in line with previous studies, which can improve comparability. The goal is to study the implications of the new assumption for the international transmission of shocks and put forth an approach to solve the model numerically.

I develop a solution strategy because the endogenous portfolio choice is not straightforward to solve once the assumption that assets have the same pledgeability is relaxed. My approach is based on the fact that the heterogeneous degree of asset pledgeability generates an expected return differential between local and foreign assets. And to use this mechanism for determining the country portfolios, I take advantage of the fact that credit spreads tend to be equalized across countries when markets are integrated[4].

Specifically, the mechanism works as follows. A shock in one country implies that foreign assets constitute a relatively safer collateral in this contingency. In turn, investors formulate expectations that they will earn relatively higher returns on the assets of the country hit by

[2]This is shown in Figure 2 of Gorton and Metrick's article. See also their Figures 3-4.

[3]See also Bertaut and Pounder (2009).

[4]See Dedola and Lombardo (2012).

the shock. In other words, these assets must pay a relatively higher equity premium to induce investors of any country to hold them in equilibrium.

To facilitate the interpretation of the resulting portfolio positions, I study how differences in the degree of asset pledgeability modify the incentive to diversify risk across borders. I find that investors allocate their wealth among international assets taking into account the combined effect of asset returns and asset pledgeability. Home bias, a well-known empirical regularity[5], tends to be slightly lower under heterogeneous pledgeability than in the standard case of homogeneous asset pledgeability. The slight decrease in home bias shows that agents are concerned about the fact that domestic assets can lose their pledgeability when also domestic consumption is low. Similar concerns cannot arise when assets have the same degree of pledgeability; there is no difference in the way local and foreign assets can be pledged as collateral. Actually, in the framework used as the benchmark this result is even stronger, since the financial frictions do not affect portfolio choice at all[6].

Also the international transmission of financial shocks changes remarkably with the type of model considered. When assets have different degree of pledgeability, financial shocks are asset-specific. A shock of this type restrains borrowing because it lowers the pledgeability of the assets of the country that receives the shock. The assets of this country are part of the portfolios of local and foreign investors, so the shock affects all investors at the same time. This leads to valuation effects, which are justified by the expected impact of changes in actual asset pledgeability on future asset returns across borders. In contrast, in models with homogeneously pledgeable assets financial shocks are country-specific. The investors of the country hit by such a shock lose access to the credit market, but foreign investors can continue to borrow. The shock is transmitted across borders inasmuch as those investors subject to the shock have positions in both local and foreign financial markets. And importantly, when assets have the same degree of pledgeability, the model cannot generate valuation effects.

I focus on short-run differences in the relative safety of collateral assets and short-run return differentials across countries. Therefore, my analysis does not extend to the relevant question of global imbalances, whose origins are to some extent connected with the idea examined here. The

[5]In the light of the home equity bias commonly observed in the data, I interpret my results looking at the share of local assets in total portfolios. Since the analysis here is not about reproducing home bias endogenously (which has been addressed by other studies), I generate home bias exogenously with a technique well-established in the literature.

[6]It may be possible to build alternative models where borrowing constraints affect asset allocation to some extent. Note however that only in models with heterogeneous pledgeability agents face a trade-off between the safety of collateral assets from one country and that of collateral from other countries, and this trade-off has implications for future asset returns and risk premiums.

literature points out that these imbalances are caused by the heterogeneity between financial sectors across countries[7]. My only contribution in this regard is to show that introducing structural differences in asset pledgeability and in the return on assets across different countries requires a more challenging solution method for international portfolios. In addition, one should more carefully think about the key mechanism. There is evidence that cross-border return differentials are short-run phenomena as in the present paper (Curcuru, Thomas and Warnock, 2012), while the global imbalances are more properly due to a "composition puzzle", with the U.S. being long in risky assets and short in inherently riskless assets (Mendoza, Quadrini and Rios-Rull, 2009)[8].

This paper continues with a brief literature review (section 2) and a brief description of the model (section 3). In section 4, I analyze the equity premiums of pledgeable traded assets. Section 5 is then devoted to the analysis of endogenous portfolios and describes the solution strategy. A numerical application of this approach is discussed in Section 6. Section 7 concludes the paper. There is an appendix at the bottom of the paper and a separate one for further details.

2 Literature

This paper is clearly connected with the literature on the determination of international portfolios in open economy macro-models: Devereux and Saito (2006), Devereux and Sutherland (2011a), Evans and Hnatkovska (2011), Pavlova and Rigobon (2010) and Tille and Van Wincoop (2010). In these models, the return on home and foreign assets is generally symmetric, and this symmetry is a condition for studying the equilibrium portfolios. In contrast, my analysis is based on an asymmetry between asset returns across countries. This asymmetry is the consequence of the different degree of pledgeability between assets. My work is mostly connected with Devereux and Sutherland (2011a) and Tille and Van Wincoop (2010).

The model I use is the one proposed by Devereux and Yetman (2010), in which I introduce a different degree of pledgeability between assets. The model is suitable for understanding the international transmission of shocks under borrowing constraints. Studies that use similar frameworks are those of Devereux and Sutherland (2011b) and Trani (2012). The novel aspect

[7]For instance, the global imbalances observed in the data have been justified as the consequence of a superior ability of the U.S. to produce safe assets (Caballero, Fahri and Gourinchas, 2008) or, alternatively, as the result of the successful development of the U.S. financial system relatively to the financial system of other countries (Mendoza, Quadrini and Rios-Rull, 2009).

[8]On this, see also Curcuru, Dvorak and Warnock (2008). As for the first important contributions on the theme, see Gourinchas and Rey (2007) and Lane and Milesi-Ferretti (2005).

here is that assets contribute differently to the tightness of the borrowing constraints because they are differently pledgeable as collateral due to real and financial factors. The different degree of pledgeability is modeled building on a brief argument made by Aiyagari and Gertler (1999) in their paper on the volatility of asset prices under financial frictions.

Two papers that focus especially on the transmission of financial shocks are Perri and Quadrini (2012) and Dedola and Lombardo (2012). Perri and Quadrini (2012), showing that financial disturbances are endogenously global, propose a theory on the synchronization of business cycles. Financial shocks produce high cross-country correlation in my model as well. With respect to the studies mentioned so far, Dedola and Lombardo (2012) do not model financial accelerator effects with borrowing constraints but with monitoring costs. These costs generate credit spreads, which tend to be equalized across countries through the process of financial integration. I use this property of integrated markets to solve the portfolio choice problem.

For modeling the financial shocks, I refer to the work of Jermann and Quadrini (2012). Here, financial shocks are shocks to the pledgeability of a given asset traded in international markets. The interpretation of these shocks is more in line with recent financial literature. The idea is that agents who lend to leveraged borrowers look closely at the quality of any collateral asset in a given state of the economy, adjusting the size of the loan in this sense (Brunnermeier and Pedersen, 2009, and Gorton and Metrick, 2010). Based on similar observations, Garleanu and Pedersen (2012) amend the CAPM with differences in margin requirements between different types of assets. There is a clear connection with the differences in asset pledgeability. But while I focus on international portfolio choice and transmission of shocks, their interesting contribution is important for capturing how an asset can outperform other assets in a financial, closed-economy setup.

Finally, international macro-models that focus on the quality of financial assets are Cao and Gete (2011) and Blengini (2011). Both have a different goal than mine. The first is not a portfolio model and studies long-run effects of inherently safe assets (i.e., U.S. Treasuries). The second is an international portfolio model, which addresses the question of rebalancing under uncertainty shocks.

3 Model

The model used is the one developed by Devereux and Yetman (2010). In this section I sketch the main blocks of their framework as well as the main modifications I introduce, remanding the interested reader to their original work.

Consider a two-county model. Each country is populated by two types of agents and produce a single good which is internationally traded. One class of agents are the investors, with overall size equal to n; the other category of agents is represented by the savers, with overall size $1-n$. Investors have the skills to trade in international markets and select the composition of their equity portfolio choosing between home stocks and foreign stocks. Stocks are shares on the fixed capital used by production firms. There are only consumption good firms in the model.

Investors pledge their equity holdings as collateral to borrow from savers. This leveraged investment strategy is modeled assuming that investors are more impatient consumers than savers. The markets for credit is unique for both countries.

The two economies are perfectly symmetric, except for the different degree of pledgeability of their equities.

3.1 Investors

Investors in the home country work for a fixed amount of hours at the real wage w_t and purchase home and foreign equities. For this purpose, in addition to their own savings, they can borrow selling a riskless bond. Labour hours are normalized to one, and the objective of investors is to maximize their utility in order to satisfy budget and collateral constraints:

$$\max_{\left\{c_t^I, b_t^I, k_{it}^I\right\}_{i=H,F}} \quad U_0^I = E_0 \sum_{t=0}^{\infty} \vartheta_t^I u\left(c_t^I\right)$$

$$\text{s.t.} \quad c_t^I - b_t^I + q_{Ht}^e k_{Ht}^I + q_{Ft}^e k_{Ft}^I = w_t - R_{t-1} b_{t-1}^I + \left(q_{Ht}^e + d_{Ht}\right) k_{Ht-1}^I + \left(q_{Ft}^e + d_{Ft}\right) k_{Ft-1}^I \quad (1)$$

$$b_t^I \leq \kappa_{Ht} q_{Ht}^e k_{Ht}^I + \kappa_{Ft} q_{Ft}^e k_{Ft}^I \quad (2)$$

where $\vartheta_{t+1}^I = \beta\left(C_t^I\right)\vartheta_t^I$, denotes the discount factor, which depends endogenously on the average consumption across investors, C_t^I. The other variables are: the bond traded on the global debt market, b_t^I; the riskless rate of interest on this debt, R_t; the purchased quantity of home and foreign equities, k_{Ht}^I and k_{Ft}^I. Equity $i = H, F$ trades on the market at price q_{it}^e and pays the dividend d_{it}. Pledging home equities as collateral, investors can borrow a fraction $\kappa_{Ht} < 1$ of their holdings of these securities at market value. Pledging foreign equity they can instead borrow a fraction $\kappa_{Ft} < 1$ of their holdings.

The efficiency conditions of this dynamic problem are

$$\lambda_t^I - \mu_t = \beta\left(c_t^I\right) E_t \lambda_{t+1}^I R_t \quad (3)$$

$$\lambda_t^I - \mu_t \kappa_{Ht} = \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{Ht+1} \quad (4)$$

$$\lambda_t^I - \mu_t \kappa_{Ft} = \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{Ft+1} \quad (5)$$

where $r_{it} = \left(q^e_{it} + d_{it}\right) / q^e_{it-1}$ is the rate of return on equity $i = H, F$, $\lambda^I_t = u'\left(c^I_t\right)$ and μ_t is the marginal value of borrowing against collateral.

Foreign investors have similar preferences and face a similar problem as home investors. In this case, variables are denoted by a "star" ($\lambda^{*I}_t = u'\left(c^{*I}_t\right)$ is their marginal utility of consumption, and μ^*_t the shadow value of their leveraged borrowing). The collateral constraints that foreign investors face is

$$b^{*I}_t \leq \kappa_{Ht} q^e_{Ht} k^{*I}_{Ht} + \kappa_{Ft} q^e_{Ft} k^{*I}_{Ft} \tag{6}$$

Borrowing constraints. The specification of the collateral constraints is the main departure from the original model of Devereux and Yetman (2010). Equations (2) and (6) show that lenders regard home and foreign equities as possibly heterogeneous collateral assets. In other words, any asset $i = h, F$ has a specific pledgeability κ_{it} for all of the investors that own that asset.

The alternative assumption, the one that has been generally used in the literature, is that investors can pledge all of their assets in the same way. Home and foreign equities have a common degree of pledgeability, and only the total wealth of each investor matters. The financial constraints can differ across countries solely when lenders are more or less willing to finance investors in the home country than to finance foreign investors:

$$b^I_t \leq \kappa_t \left(q^e_{Ht} k^I_{Ht} + q^e_{Ft} k^I_{Ft}\right) \qquad b^{*I}_t \leq \kappa^*_t \left(q^e_{Ht} k^{*I}_{Ht} + q^e_{Ft} k^{*I}_{Ft}\right) \tag{7}$$

where κ_t and κ^*_t vary over time and are agent-specific[9]. If we abstract from these cross-country differences in borrowing constraints or the synchronization studied by Perri and Quadrini (2012) applies, then there is a unique κ_t for all the agents trading in international capital markets:

$$b^I_t \leq \kappa_t \left(q^e_{Ht} k^I_{Ht} + q^e_{Ft} k^I_{Ft}\right) \qquad b^{*I}_t \leq \kappa_t \left(q^e_{Ht} k^{*I}_{Ht} + q^e_{Ft} k^{*I}_{Ft}\right) \tag{8}$$

Table 1 summarizes the alternative assumptions that one can make. I shall compare the features and implications of these alternatives throughout the paper.

3.2 Savers

Savers in the home country have a lower propensity to consume than investors: $\beta\left(C^S_t\right) > \beta\left(C^I_t\right)$. They are willing to sacrifice their consumption and purchase the bonds sold by investors. Similarly to investors, savers work inelastically in the domestic firms, earning the real wage w_t. In addition, savers run their own production using a fraction of the domestic capital as factor of

[9] See, for example, Devereux and Sutherland (2011b) and Trani (2012).

production. But they do not have access to foreign capital, and their product is not sold in any good market[10]. Their dynamic programming problem is as follows:

$$\max_{\{c_t^S, b_t^S, k_{Ht}^S\}} U_0^S = E_0 \sum_{t=0}^{\infty} \vartheta_t^S u\left(c_t^S\right)$$

$$\text{s.t.} \quad c_t^S + q_{Ht}^e \left(k_{Ht}^S - k_{Ht-1}^S\right) - b_t^S = w_t + z\left(k_{Ht-1}^S\right)^{\nu} - R_{t-1} b_{t-1}^S \tag{9}$$

where $\vartheta_{t+1}^S = \beta\left(C_t^S\right)\vartheta_t^S$ denotes the endogenous discount factor and k_{Ht}^S is the quantity of the domestic capital stock that savers use to run their "backyard" production. Their production technology is $z\left(k_{Ht-1}^S\right)^{\nu}$, with $\nu < 1$.

Foreign investors have similar preferences and face a similar problem as home savers. So the demand for bonds of home savers and that of foreign savers satisfy the following fist order conditions:

$$\lambda_t^S = \beta\left(c_t^S\right) E_t \lambda_{t+1}^S R_t \qquad \lambda_t^{*S} = \beta\left(c_t^{*S}\right) E_t \lambda_{t+1}^{*S} R_t \tag{10}$$

where $\lambda_t^S = u'\left(c_t^S\right)$. These conditions imply perfect risk-sharing between savers of different countries.

3.3 Production

Home firms produce a homogeneous good which is traded across countries using a standard, concave production function: $Y_{Ht} = A_t F\left(K_{Ht-1}\right)$. Home productivity, A_t, is a stationary stochastic process, and K_{Ht} is the stock of capital purchased by firms. Firms finance their productive capital issuing equities that are sold to home and foreign investors, so $K_{Ht} = n\chi_{Ht}$. The variable $\chi_{Ht} = k_{Ht}^I + k_{Ht}^{*I}$ is the total amount of outstanding shares normalized by the number of shareholders.

Firms are competitive, so dividends on their stocks are $d_{Ht} = A_t F'\left(K_{Ht-1}\right)$, and the wage rate they pay is $w_t = Y_{Ht} - A_t F'\left(K_{Ht-1}\right) K_{Ht-1}$.

Foreign firms produce and maximize profits in the same way as home firms. Their productive capital is $K_{Ft} = n\chi_{Ft}$, and the marginal product of this capital is the dividend $d_{Ft} = A_t^* F'\left(K_{Ft-1}\right)$.

[10]Together with that on discount factors, the assumption that savers use domestic capital to produce a non-traded good distinguishes the economic role of these agents from that of investors. Savers demand domestic capital just for their own use: they are neither leveraged nor expert enough to trade stocks in international markets. Nonetheless, their demand for domestic capital is useful to determine the total amount of capital stock in the steady state. See Kiyotaki and Moore (1997) and Devereux and Yetman (2010).

3.4 Competitive Equilibrium

The clearing conditions for the good, bond, and capital markets are, respectively, as follows:

$$n\left(c_t^I+c_t^{*I}\right)+(1-n)\left(c_t^S+c_t^{*S}\right) = Y_{Ht}+Y_{Ft}+(1-n)\left[z\left(k_{Ht-1}^S\right)^\nu+z\left(k_{Ft-1}^{*S}\right)^\nu\right] \quad (11)$$

$$n\left(b_t^I+b_t^{*I}\right)+(1-n)\left(b_t^S+b_t^{*S}\right) = 0 \quad (12)$$

$$n\chi_{it}+(1-n)\,k_{it}^S = 1, \quad \text{for } i=H,F \quad (13)$$

On each date $t=0,...,\infty$, the competitive equilibrium is a vector of allocations (c_t^I, c_t^{*I}, c_t^S, c_t^{*S}, b_t^I, b_t^{*I}, b_t^S, b_t^{*S}, k_{Ht}^I, k_{Ht}^{*I}, k_{Ht}^S, k_{Ft}^I, k_{Ft}^{*I}, k_{Ft}^{*S}) and a vector of prices (R_t, q_{Ht}^e, q_{Ft}^e, w_t, w_t^*, d_{Ht}, d_{Ft}) such that: (a) the representative investor maximizes her lifetime utility subject to the budget and collateral constraints; (b) the representative saver maximizes her lifetime utility subject to the budget constraint; (c) firms purchase capital in order to maximize profits; (d) all markets clear.

4 Pledgeability and Equity Premiums

According to the demand functions, investors purchase asset $i=H,F$ for two reasons. One reason is the expected return on that asset, r_{it+1}. The other is the possibility to pledge the asset in order to borrow a fraction κ_{it} of its value from savers. So a difference between the pledgeability of home versus foreign equities translates into a return differential, with opposite sign.

In fact, a simple combination of equations (4)-(5) yields

$$\frac{\kappa_{Ht}}{\kappa_{Ft}}=\frac{1-E_t\Lambda_{t,t+1}^I r_{Ht+1}}{1-E_t\Lambda_{t,t+1}^I r_{Ft+1}} \quad (14)$$

where $\Lambda_{t,t+1}^I=\beta\left(c_t^I\right)\lambda_{t+1}^I/\lambda_t^I$ is the stochastic discount factor of home investors. Revising the argument made by Aiyagari and Gertler (1999), the interpretation of this relation is that assets with higher degree of pledgeability are expected to pay lower returns in the future. Assume for instance that, at time t, savers prefer to lend against foreign equities as opposed to home equities: $\kappa_{Ht}/\kappa_{Ft}<1$. In this sense, foreign equities are relatively safer than home equities. In comparison to foreign firms, home firms need to promise higher returns in order to compensate investors for purchasing their stocks, which are temporarily not so useful as collateral. This is reflected in investors' expectations about future returns, so $E_t\Lambda_{t,t+1}^I r_{Ht+1}>E_t\Lambda_{t,t+1}^I r_{Ft+1}$.

In this example, foreign equities represent reliable assets for leveraged investors. With respect to home assets, foreign equities "relax" the borrowing constraints because their pledgeability as collateral is superior in periods of bad economic conditions. This sort of hedging property is *per se* an important motive for investing in foreign equities, which is alternative to the incentive given by the future payoff on these assets.

Equations (3)-(5) imply that the risk premium on any equity $i = H, F$ is

$$E_t r_{it+1} - R_t = \varrho_{it} + \frac{\mu_t}{\beta\left(c_t^I\right) E_t \lambda_{t+1}^I} \left(1 - \kappa_{it}\right) \tag{15}$$

where the first component of the risk premium is the standard comovement between the stochastic discount factor and the return on equity:

$$\varrho_{it} \equiv -\frac{cov_t\left(\Lambda_{t,t+1}^I, r_{it+1}\right)}{E_t \Lambda_{t,t+1}^I}$$

For future reference, I conveniently denote the equity premium defined in equation (15) as $EP_{it} \equiv E_t r_{it+1} - R_{t+1}$. Equation (15) shows that the risk premium of asset i does not only depend on how its rate of return comoves with the stochastic discount factor of investors, which is captured by ϱ_{it}. The premium is also affected by the tightness of the borrowing constraint, in general, and the pledgeability of asset i, in particular. The effect of the borrowing constraint is measured by $\mu_t / \left[\beta\left(c_t^I\right) E_t \lambda_{t+1}^I\right]$. This is the spread that investors must pay to borrow and guarantee a risk-free return to savers. That is, the spread is a consequence of the fact that constraints bind in equilibrium. The specific effect of the pledgeability of asset i is given by the term $1 - \kappa_{it}$. In periods in which κ_{it} is low, this term is high and asset i must pay a high equity premium[11].

On the cross-sectional dimension, the different degree of pledgeability between assets of different countries imply an expected return differential. This differential is reflected in the relative risk premium of home versus foreign equities, which is

$$EP_{Ht} - EP_{Ft} = \varrho_{Ht} - \varrho_{Ft} - \frac{\mu_t}{\beta\left(c_t^I\right) E_t \lambda_{t+1}^I} \left(\kappa_{Ht} - \kappa_{Ft}\right) \tag{16}$$

This equation shows that, other things being equal, the relative equity premium of home versus foreign equities is high if $\kappa_{Ht} < \kappa_{Ft}$. The reverse is instead true when $\kappa_{Ht} > \kappa_{Ft}$, in which case $EP_{Ht} - EP_{Ft}$ could in principle be even negative. The same line of reasoning applies to foreign investors as well.

In contrast, this is not true for any investors when assets have the same degree of pledgeability. If the form of the borrowing constraints is that of equation (7), then investors' demand functions

[11]For a related argument, see Garleanu and Pedersen (2011). They discuss extensively the effect of differences in the use of assets as collateral on rates of return. Their main point is that these differences can be used as a measure of relative performance, which allows to distinguish one security from anyone else. They show that these differences can even explain why in some periods the law of one price may fail to apply.

imply that the equity premiums on any asset i are

$$EP_{it} = \varrho_{it} + \frac{\mu_t}{\beta\left(c_t^I\right) E_t \lambda_{t+1}^I}\left(1-\kappa_t\right) \qquad EP_{it}^* = \varrho_{it}^* + \frac{\mu_t^*}{\beta\left(c_t^{*I}\right) E_t \lambda_{t+1}^{*I}}\left(1-\kappa_t^*\right)$$

And if the borrowing constraints are those of equation (8), the equity premiums are

$$EP_{it} = \varrho_{it} + \frac{\mu_t}{\beta\left(c_t^I\right) E_t \lambda_{t+1}^I}\left(1-\kappa_t\right) \qquad EP_{it}^* = \varrho_{it}^* + \frac{\mu_t^*}{\beta\left(c_t^{*I}\right) E_t \lambda_{t+1}^{*I}}\left(1-\kappa_t\right)$$

In either of these two cases, assets are equally pledgeable and the relative premiums of home versus foreign equities are simply

$$EP_{Ht} - EP_{Ft} = \varrho_{Ht} - \varrho_{Ft} \qquad EP_{Ht}^* - EP_{Ft}^* = \varrho_{Ht}^* - \varrho_{Ft}^* \tag{17}$$

Investors of different countries may face different constraints, but for anyone of them the choice between home and foreign equities is the same as if they were unconstrained investors. Indeed, (17) is a standard case in International Macroeconomics: relative equity premiums of this sort are present even in models that do not involve credit market frictions.

5 Portfolio Choice

Equations (4)-(5) govern the portfolio choice of investors in the home country, who buy stocks of local firms and of foreign firms. Similar relations govern the portfolio choice of foreign investors.

The two main methods that are available for solving this portfolio choice problem have been developed by Devereux and Sutherland (2011a) and Tille and Van Wincoop (2010). In what follows I shall draw from both methods, referring to the first as DS and to the second as TvW. Each of them has some useful elements for my purpose. In addition, the main properties of the two methods are essentially equivalent, so referring to both of them is straightforward.

The approximated relations presented below are written specifying the order of approximations as suggested by TvW[12]. The zero order component of a variable (e.g., $Z(0)$) denotes the steady state value of that variable. The first order component (e.g., $Z(1)$) corresponds to the approximated variable of a standard linearization around the steady state. And so on. However, the most of the derivations are detailed in the separate appendix.

[12] In general, any variable can be written as the sum of its order components: for example, $Z_t = Z_t(0) + Z_t(1) + Z_t(2) + \ldots$

5.1 External Wealth and Borrowing Constraints

Following DS, the model can be rewritten in such a way that the effect of portfolio choice on the buildup of wealth over time becomes explicit. As usual, this effect is captured by the budget constraint of investors, and - for the properties of general equilibrium - it is sufficient to consider the budget constraint of agents in the home country. This constraint (equation (1)) is rewritten as follows:

$$
\begin{aligned}
c_t^I + NFE_t &= w_t + d_{Ht}\chi_{Ht-1} - q_{Ht}^e\left(\chi_{Ht} - \chi_{Ht-1}\right) + b_t^I - R_{t-1}b_{t-1}^I \\
&\quad + \left(r_{Ht} - r_{Ft}\right)\omega_{t-1} + r_{Ft}NFE_{t-1}
\end{aligned} \tag{18}
$$

where $NFE_t = q_{Ft}^e k_{Ft}^I - q_{Ht}^e\left(\chi_{Ht} - k_{Ht}^I\right)$ are the net foreign equities of the home country and $\omega_t = q_{Ht}^e\left(k_{Ht}^I - \chi_{Ht}\right)$ is the portfolio share. This share is the negative of foreign investors' holdings of home equities, so the ownership of the home capital stock is internationally diversified if $\omega_t < 0$. The net foreign equities of the foreign country can be simply derived from the equilibrium in the global economy: $NFE_t + NFE_t^* = 0$.

The external wealth of investors affect also the tightness of the constraints limiting their borrowing. However, the ultimate effect depends on whether assets are equally pledgeable or differently pledgeable.

Claim 1 *The portfolio position of a country affects the borrowing constraints of its residents only when traded assets display different degree of pledgeability.*

If home and foreign equities are homogeneously pledgeable, the relevant borrowing constraints are (7)-(8). The constraint of home investors is rewritten as follows:

$$
b_t^I \leq \kappa_t NFE_t + \kappa_t q_{Ht}^e \chi_{Ht} \tag{19}
$$

The borrowing constraint is thus affected by NFE_t but not by ω_t. Also foreign investors face a similar constraint[13].

[13] With equally pledgeable assets, the model might still distinguish between the tightness of home versus foreign constraints or not. In the first case, (7) implies that the foreign borrowing constraint is

$$
b_t^{*I} \leq \kappa_t^* NFE_t^* + \kappa_t^* q_{Ft}^e \chi_{Ft}
$$

Otherwise, there is no distinction even in the tightness of home versus foreign constraints, and from (8) follows that

$$
b_t^{*I} \leq \kappa_t NFE_t^* + \kappa_t q_{Ft}^e \chi_{Ft}
$$

In contrast, if the pledgeability of assets is heterogeneous across countries, equation (2) implies that

$$b_t^I \leq (\kappa_{Ht} - \kappa_{Ft})\,\omega_t + \kappa_{Ft} NFE_t + \kappa_{Ht} q_{Ht}^e \chi_{Ht} \tag{20}$$

The effect of ω_t is evident and is proportional to the trade-off between the pledgeability of home versus foreign equities. And apart from governing b_t^I, this trade-off generates a link between budget and collateral constraints. From the above discussion, $\kappa_{Ht} - \kappa_{Ft}$ has implications for the return differential across countries, which enters into (18).

5.2 Choice with Homogeneous Pledgeability

Assume that we do not consider the specific pledgeability of home and foreign assets. This case amounts to the benchmark situation generally considered in Open Economy Models, which can be analyzed following DS and TvW straightforwardly.

For home investors, the pledgeability of both home and foreign equities is κ_t. For foreign investors it is either κ_t^* or κ_t. Respecifying (4)-(5) and their foreign counterparts using κ_t (and κ_t^*), one can derive from these equations the following portfolio choice conditions:

$$E_t \lambda_{t+1}^I (r_{Ht+1} - r_{Ft+1}) = 0 \qquad E_t \lambda_{t+1}^{*I} (r_{Ht+1} - r_{Ft+1}) = 0 \tag{21}$$

Although the borrowing constraints are binding, investors (at home as well as in the foreign country) choose their portfolios as in economies with non-binding constraints. Anyone of these investors can borrow the same amount pledging home and foreign assets. As a result, their choice between assets depends only on the comovement between stochastic discount factor and return differential. In the nearly-stochastic steady state, this differential must then be zero and returns on equities are equalized across countries: $r(0) \equiv r_H(0) = r_F(0)$. The two countries are symmetric in all the respects, so the fact that home and foreign assets are equally pledgeable preserves this symmetry.

A second order approximation of (21) is needed to pin down the steady state portfolio share $\omega(0)$. Combining the approximated relations as suggested by DS, I get

$$0 = E_t \left(\lambda_{t+1}^I(1) - \lambda_{t+1}^{*I}(1)\right) r_{xt+1}(1) \tag{22}$$

$$E_t r_{xt+1}(1) = 0 + T(2) \tag{23}$$

where $r_{xt}(j) \equiv r_{Ht}(j) - r_{Ft}(j)$ is the return differential of order $j = 0, 1, ...$ and $T(2)$ contains second order terms. The derivations, alongside with the definition of $T(2)$, are in the separate technical appendix.

Equations (22)-(23) are in line with the benchmark portfolio choice problem. The first is the portfolio choice condition. It suggests that, to determine the portfolio share $\omega(0)$, it is sufficient to take a standard linearization of the model and interact stochastic discount factors and return differential at this order of approximation. This can be referred to as the **first property** of the recent portfolio solution methods. The **second property** follows from equation (23). This shows that, under rational expectations, the predicted return differential associated to $\omega(0)$ does not have first order determinants. So, at the order of approximation which is sufficient to compute the steady state portfolio share, $r_{xt+1}(1)$ is a zero-mean shock.

An additional implication is that only the budget constraint is affected by portfolio choice and, in particular, solely by $\omega(0)$. Equation (19) shows that, when home and foreign assets are equally pledgeable, ω_t does not affect the borrowing constraint. These assets must also share the same long-run rate of return ($r_x(0) = 0$). Consequently, in the approximated model only the steady state portfolio share, $\omega(0)$, affects (18). This occurs through the term $r_{xt}(1)\,\omega(0)$.

5.3 Choice with Heterogeneous Pledgeability

When home and foreign assets have a different degree of pledgeability, equations (4)-(5) and their foreign counterparts imply the following portfolio choice conditions:

$$E_t \lambda^I_{t+1} (r_{Ht+1} - r_{Ft+1}) + M_t (\kappa_{Ht} - \kappa_{Ft}) = 0 \qquad E_t \lambda^{*I}_{t+1} (r_{Ht+1} - r_{Ft+1}) + M^*_t (\kappa_{Ht} - \kappa_{Ft}) = 0 \tag{24}$$

where I use the definitions $M_t \equiv \mu_t / \beta \left(c^I_t\right)$ and $M^*_t \equiv \mu^*_t / \beta \left(c^{*I}_t\right)$. These variables express the shadow value of the borrowing constraint of any agent in terms of her rate of time-preference. Equation (24) shows that, in case of heterogeneous pledgeability, cross-country arbitrage is not limited to the expected return on assets. The relative pledgeability of home versus foreign equities, $\kappa_{Ht} - \kappa_{Ft}$, matters as well. The effect of this difference in pledgeability depends on how costly is for an agent to invest under binding borrowing constraints: M_t, for home investors, and M^*_t, for foreign investors.

Any difference between κ_{Ht} and κ_{Ft} represents a new determinant of portfolio choice and, simultaneously, a source of asymmetry between countries. In particular, (24) suggests that the expected return differential on internationally traded assets may not be zero. There are, at least, two cases to consider.

5.3.1 Heterogeneous Pledgeability in the Steady State

If home and foreign assets have different degree of pledgeability already in the nearly-stochastic steady state, then there is always a return differential between home and foreign equities. From (24) follows that

$$r_x(0) = -\frac{\mu(0)}{\beta(c^I(0))\lambda^I(0)}\kappa_x(0) \tag{25}$$

where the term on the right of the equal sign coincides with $EP_H(0) - EP_F(0)$ (see equation (16)) and $\kappa_x(j) \equiv \kappa_H(j) - \kappa_F(j) \neq 0$ denotes the differential asset pledgeability of order $j = 0, 1, ...$ The premium on foreign equities is relatively higher if $\kappa_H(0) > \kappa_F(0)$, and viceversa if $\kappa_H(0) < \kappa_F(0)$. Investors need to take into account this heterogeneity between home assets and foreign assets when choosing their portfolios.

Claim 2 *If home and foreign assets are not equally pledgeable in the steady state equilibrium, the rates of return on home and foreign assets reflect this asymmetry and the properties of the existing portfolio solution methods are no longer satisfied.*

The portfolio choice of investors becomes more complex because the steady state return differential affects both the dynamics of wealth and their access to credit, through budget and collateral constraints. The first order approximation of (18) involves the term $\omega(0) r_{xt}(1) + r_x(0)\omega_{t-1}(1)$, and the first order approximation of (20) the term $\omega(0)\kappa_{xt}(1) + \kappa_x(0)\omega_t(1)$. That is, $\omega(0)$ is not sufficient for investors to understand how their choice between home and foreign equities will affect their future wealth and their current borrowing. Investors need to consider also the first order dynamics of their portfolio around $\omega(0)$.

Since in absence of shocks the different degree of pledgeability of home versus foreign equities is non-null but equal to $\kappa_x(0)$, a shock to one country has direct effect on the international portfolio positions of all of the economies. This effect is captured by $\kappa_x(0)\omega_t(1)$, which matters for the actual tightness of their borrowing constraints. There are dynamic implications also for the net external wealth accumulated by investors because, according to equation (25), also $r_x(0)$ is different from zero. The impact of the shock on international portfolios enters into the budget constraints because of $r_x(0)\omega_{t-1}(1)$.

On one side, the wealth effect of the steady state return differential, $r_x(0)$, must influence the way investors discount future profits. That is, there is a second order effect on their marginal utility of future consumption. On the other side, the effect of $\kappa_x(0)$ on borrowing must influence the marginal value of the borrowing constraints that bind in the present period. That is, there is a second order effect on the shadow cost of leveraging the investment in financial markets.

These second order effects appear formally in the arbitrage conditions for the selection of international portfolios. And consequently, a first order approximation of the model is not sufficient to compute the steady state portfolio share $\omega(0)$. The logic behind the methods of DF and TvW does not apply. The computation needs to be iterative, involving the solution of the model at different orders of approximation before the full determination of the steady state.

For completeness, I provide the detailed expressions in the external appendix. But note that this complex computation is not strictly needed for my purpose. Assuming $\kappa_x(0) \neq 0$ is appropriate for analyses targeted to long-run and structural differences between economies, which I do not pursue here. My focus is instead on short-run dynamics, meaning that the use of an asset as collateral changes with the prevailing economic conditions.

5.3.2 Short-Run Heterogeneous Pledgeability

In order to capture the effect of economic conditions on pledgeability, let κ_{it} be a function of the deviation of Y_{it} from its steady state, which means that the pledgeability of assets is the same in the steady state. Indeed, in the present model (as in the majority of the Open Economy Models) the two countries are symmetric, so they share the same stationary output, $Y(0)$. Formally,

$$\kappa_{it} = \Gamma\left(Y_{it} - Y(0), \varepsilon^{i}_{\kappa t}\right), \quad \text{for } i = H, F \tag{26}$$

so that only in the steady state assets have the same pledgeability:

$$\kappa(0) \equiv \kappa_H(0) = \kappa_F(0) \tag{27}$$

where $\Gamma(\cdot)$ is increasing in both of the arguments.

This assumption draws from financial studies (e.g., Gorton and Metrick, 2010, and Brunnermeier and Pedersen, 2009). This literature stresses the fact that, during adverse real and financial cycles, the pledgeability of some assets decreases because lenders become less willing to accept them as collateral. The reverse is true of expansions. In general, lenders are concerned for the value of collateral assets in a given state of the world. Also, the informational asymmetries between borrowers and lenders tend to vary with the economic conditions (e.g., informed versus uninformed savers, optimistic versus pessimistic beliefs, etc.).

I do not model these informational frictions, but assume that κ_{it} is simply a function of the output of country i and of an asset-specific shock. The deviations of output from steady state capture the real determinants that affect the pledgeability of an asset. The asset-specific shock captures factors attributable to borrowing contracts.

An alternative to output in deviation from steady state, $Y_{it} - Y(0)$, is to assume that the pledgeability of an asset is a function of its market price (again in deviation from its steady state value). But of course, the equity price depends on dividend payouts, which are in turn function of the business cycle. Furthermore, modeling κ_{it} directly as a function of the price of equity i yields qualitatively similar numerical results for portfolios[14].

Claim 3 *Under symmetric but state-dependent pledgeability, the expected return differential has a non-zero first order component. This component is inversely proportional to the different pledgeability between assets. Accounting for this first order difference, the portfolio choice problem can still satisfy the main properties of the existing solution methods.*

Approximating the conditions in (24) under assumption (26) and combining the resulting relations as suggested by DS yields

$$0 = E_t \left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1) \right) r_{xt+1}(1) + const \times \left(M_t(1) - M^*_t(1) \right) \kappa_{xt}(1) \quad (28)$$

$$E_t \Upsilon_{xt+1}(1) = 0 + T(2) \quad (29)$$

where *const* is a constant term and $\Upsilon_{xt+1}(1)$ is defined as $\Upsilon_{t+1}(1) \equiv r_{xt+1}(1) + const \times \kappa_{xt}(1)$.

From (28) follows that the first property of the portfolio solution methods is satisfied (compare it with (22)). A first order approximation of the model is sufficient to compute $\omega(0)$. To a first order, investors choose this portfolio share on the basis of two factors. One is the comovement between the return differential $r_{xt+1}(1)$ and the future marginal utility of consumption. The other is the interaction between differences in pledgeability $\kappa_{xt}(1)$ and the shadow cost of the borrowing constraints. The term *const* measures the relative weight of these two determinants. The constant is in fact connected with the loan premium, the pledgeability of equities and the equity return that prevail in the steady state equilibrium:

$$const = \frac{\mu(0)}{\beta\left(c^I(0)\right)\lambda^I(0)} \frac{\kappa(0)}{r(0)}$$

Therefore, investors choose between home and foreign assets not only for a return-seeking motive, but also for being able to borrow from savers. In other words, investors give a relative value to both $r_{xt+1}(1)$ and $\kappa_{xt}(1)$. The weighted combination of these two variables, $\Upsilon_{xt+1}(1)$, expresses the *overall differential* performance of home equities versus foreign equities. Up to a first order, this overall differential has expected value equal to zero. This result comes from

[14] That is, in one robustness check I use the price of equity in the place of aggregate output as the main driver of pledgeability. This is a test on the reliability of the numerical results that are discussed in section 6. Qualitatively, I do not find differences between alternative specifications. See the external appendix for further details. See Trani (2012) for an attempt to model the loan-to-value ratio as a function of the price of internationally traded equity.

equation (29), and imply that also the second property of the portfolio solution methods is satisfied (compare it with (23)). But notice that, since equities do not have the same pledgeability outside the steady state, the expected return differential *per se* is no longer equal to zero:

$$E_t r_{xt+1}(1) = E_t \Upsilon_{xt+1}(1) - const \times \kappa_{xt}(1) \tag{30}$$

where $-const \times \kappa_{xt}(1)$ represents the *non-zero mean* return differential.

Since the portfolio choice condition (28) can be solved with the first order behaviour of model variables, the evolution of wealth and current borrowing are influenced only by the steady state portfolio share $\omega(0)$. In the stationary equilibrium, home and foreign equities display the same $\kappa(0)$ and there is no return differential. The return on equities is unique and equal to $r(0)$ and, given equation (16), the equity premium is unique as well[15]. Therefore, a linear approximation of the budget constraint (18) only involves the steady state portfolio share through the term $r_{xt}(1)\,\omega(0)$. This is the same feature of models with equally pledgeable collateral (or with no credit constraints at all). But, according to Claim 1, now portfolio choice does not only affect the flow-of-funds, but also the collateral constraints. When one country is hit by a shock, assets cease to be equally pledgeable. And, following the same reasoning adopted for the budget constraint, only the zero order portfolio matters. Given (20), this effect is captured by $\kappa_x(1)\,\omega(0)$.

To sum up, conditions (28)-(29) recast the portfolio solution for the case of heterogeneously pledgeable assets in terms of the existing methods: DS and TvW. The heterogeneous pledgeability of assets is contingent on economic innovations, and the current approach has two specific characteristics. First, the optimal portfolio does not only depend on the expected return on assets, but also on how these assets are useful as guarantee for loans. Second, the model must be solved specifying the new variable $\Upsilon_{xt}(1)$, because international arbitrage aims at eliminating expected return differentials and differentials in pledgeability taken together.

5.4 Credit Spreads and Solution for Portfolios

Since investors choose the allocation of their portfolio on the basis of the overall differential $\Upsilon_{xt}(1)$, (28) can be transformed to make this variable explicit as follows

$$\begin{aligned} 0 \;=\;& E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)\Upsilon_{xt+1}(1) \\ & + const \times \left[\left(M_t(1) - M^*_t(1)\right) - E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)\right]\kappa_{xt}(1) \end{aligned}$$

where the term in square brackets amounts to the first order approximate behaviour of the risk premium paid by home and foreign investors on their debt ($\mu_t / \left[\beta\left(c^I_t\right) E_t \lambda^I_{t+1}\right]$, $\mu^*_t / \left[\beta\left(c^{*I}_t\right) E_t \lambda^{*I}_{t+1}\right]$).

[15] The risk premium common to all equities in the steady state is $EP(0) = -\mu(0)\,\kappa(0) / \left[\beta\left(c^I(0)\right)\lambda^I(0)\right]$.

As argued by Dedola and Lombardo (2012), financial integration tends to equalize credit spreads across home and foreign countries. Ultimately, this simplifies the solution for portfolios.

Lemma 4 *In a world with financially integrated capital and debt markets, risk premiums are equalized up to a first order:*

$$M_t(1) - E_t\lambda^I_{t+1}(1) = M^*_t(1) - E_t\lambda^{*I}_{t+1}(1)$$

Therefore, the portfolio choice condition to solve is

$$0 = E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)\Upsilon_{xt+1}(1) \tag{31}$$

Proof. See appendix A (in the back of the text). ■

Equation (31) is the condition to apply, once the first order solution of the model is available. This result relies on the forces of arbitrage under financial integration: home and foreign credit spreads are equal up to first order. This is particularly straightforward in the model under analysis, as bonds are sold in a world market. However, this simplification is not so restrictive, as debt markets have recently shown to be highly integrated. Moreover, Dedola and Lombardo (2012) show that financial integration in one market (the one for capital) is itself sufficient to generate a tendency toward the equalization of risk premiums on loans across countries. So alternative models can be amended in this sense.

In the numerical exercise that follows, I apply (31) using an iterative procedure and solve for a fixed point. This solution is useful because, to interpret the numerical results, I introduce a second order transaction cost as the one proposed by TvW. This cost affects the cross-border returns on equities and, thus, it can generate the home equity bias observed in the data. Clearly, (31) is similar to the condition derived by DS. So, apart from the transaction cost τ, also their recursive approach could be taken as reference. The details of my computation are in appendix B (in the back of the text).

6 Numerical Application

6.1 Calibration

I calibrate the model taking parameters from earlier studies on the international transmission of shocks under borrowing constraints (Devereux and Yetman, 2010, and Trani, 2012). My calibration is reported in Table 2. The population is equally divided between savers and investors, as the number of investors is $n = 0.5$. The endogenous discount factor of is $\beta(c) = \zeta(1+c)^{-\phi}$

for any individual in the population. The momentary utility function of each agent is CRRA, $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$. The production function of firms is a standard Cobb-Douglas, $Y = AK^{\alpha}$, and the capital share is $\alpha = 0.4$.

The net risk-free rate on loans is set to 4.2 percent per annum, which is close to the average annualized LIBOR in the U.K. and the U.S in recent data[16]. Leveraged investors also pay a premium as a result of the need to pledge collateral. The loan premium, $M(0)/\lambda^{I}(0)$ [17], is for simplicity equal to 100 basis points. The pledgeability of an asset in the steady state is $\kappa(0) = 0.5$[18].

The forcing variables of the system, productivity and shocks to pledgeability, are standard autoregressive processes:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{At}$$

$$\ln \kappa_{it} = \rho_\kappa \ln \kappa_{it-1} + (1-\rho_\kappa)\left[\ln \kappa(0) + \eta_Y \ln\left(Y_{it}/Y(0)\right)\right] + \varepsilon_{\kappa_i t}, \quad \text{for } i = H, F \qquad (32)$$

$$\ln \kappa_t = (1-\rho_\kappa)\ln \kappa(0) + \rho_\kappa \ln \kappa_{t-1} + \varepsilon_{\kappa t} \qquad (33)$$

where η_Y denotes the elasticity of κ_{it} to the fluctuations of GDP around its steady state. Equation (32) applies to the case of assets that have different degree of pledgeability. The pledgeability of home and foreign equities follow the business cycle of the corresponding economy and is subject to an exogenous shock. Equation (33) applies to the case of homogeneously pledgeable assets, which is used as a benchmark for comparisons. The literature so far has in fact relied on this assumption.

The parameters used for these shocks are in the bottom part of Table 2. Numerical portfolios are generally affected by such factors as the persistence of shocks and the correlation between shocks. I choose the value of these types of parameters in order to minimize their influence on the results.

The persistence of productivity shocks is $\rho_A = 0.82$, which is close to the average Solow Residual that can be computed from a sample of major OECD countries[19]. I set the persistence of financial shocks to the same level as ρ_A. In fact, according to the estimation of Jermann and Quadrini (2012), the persistence of financial shocks is rather high and close to ρ_A. In addition, their estimates show that financial shocks are more volatile than productivity shocks, so I also set $\sigma_A < \sigma_\kappa$. Nonetheless, I opt for a very small difference between the two volatilities.

To eliminate the effect of a possible comovement between shocks, I assume that shocks are

[16]See Trani (2012).
[17]$M(0)/\lambda^{I}(0) = \mu(0)/\left[\beta\left(c^{I}(0)\right)\lambda^{I}(0)\right]$
[18]See Devereux and Yetman (2010).
[19]See Trani (2012).

uncorrelated. Particularly important is the fact that shocks of the same nature are uncorrelated across countries. Setting these correlation to zero is appealing here because the goal is analytic and illustrative. Specifically, the goal is to see how the heterogeneous pledgeability between home and foreign assets affect the diversification of international portfolios.

Finally, financial shocks are calibrated, regardless of whether they affect a specific asset (equation (32)) or all the assets of the investors of a given country (equation (33)). However, in the first of these alternatives, the asset-specific pledgeability is affected by the fluctuations of output. I set the elasticity η_Y to 0.7, and I use two lower values (0.3 and 0) for robustness.

6.2 Equilibrium Portfolios

Table 3 reports the holdings of fixed capital of all the agents in the two countries. The capital holdings of savers are restricted to the domestic capital stock and depend on the allocation of this capital between the two productive sectors of each economy. The capital holdings of investors are instead diversified across borders. The optimal diversification gives rise to the equilibrium portfolio shares, the shares that satisfy the steady state of the world economy.

A stylized fact is that country portfolios (especially, the equity portfolios) are biased toward the domestic assets. I take the assumption of homogeneous pledgeability across assets as benchmark, and I calibrate the transaction cost τ so as to obtain home bias for this version of the model. This is an appropriate benchmark because it amounts to the assumption commonly made in recent studies and because it can be solved directly with the methods developed by DS and TvW. For illustration purposes, I choose a bias toward local assets equal to 65% of the overall portfolio. See the results in columns two and three of Table 3 (the bold character is used to denote that this is the benchmark case).

Introducing a different degree of pledgeability between assets, domestic equities become less appealing to investors. Under the same transaction cost τ, the share of domestic equities in country portfolios drops from 65% to 61.2%. This result is shown in columns four and five.

What is the explanation for this result? If home and foreign equities have the same pledgeability, only the return differential between these two assets matter. An investor considers the expected return of home and foreign equities in each possible future state, evaluating these returns on the basis of the marginal utility of consumption in that state. But she does not need to compare home and foreign assets for any other reasons. There is no possibility to reduce the tightness of the borrowing constraint investing more heavily in the domestic market or in the foreign market. Ultimately, an investor allocates her wealth as if she was not subject to any credit constraints. Of course, this ceases to be true when there is a marginal opportunity to relax a bit the borrowing constraint by investing more in assets that are better pledgeable.

More precisely, the relative pledgeability of home versus foreign equities depends on the state of the home country and of the foreign country, respectively. The decrease in home bias from 65% to 61.2% is thus a signal of the fact that investors want to be less exposed to equities whose pledgeability fluctuates with the domestic economic cycle. When there is a recession in a country, the residents of that country are forced to cut back their consumption and their borrowing. Also the pledgeability of that country assets will be lower, as lenders shall be more suspicious about the collateral properties of those assets. *Ceteris paribus*, the needed reduction in consumption and in borrowing is greater, the higher the exposure to stocks issued by the domestic firms.

Note that the fluctuations of output bring about a slight reduction in home bias only if the pledgeability is asset-specific. Consider again the case in which the pledgeability is different for investors of different countries but homogeneous among the assets in any investors' portfolio. Let κ_t and κ_t^* follow the same process as the one used to model asset-specific pledgeability (see equation (32)): κ_t is function of home country output; κ_t^* is function of foreign country output. Strikingly, in this case the tendency to invest more in the domestic firms as opposed to foreign firms increases to 67.6% (see the last two columns of Table 3).

Robustness. The reduction in home bias caused by the fact that assets have different degree of pledgeability across countries is clearly affected by η_Y. But qualitatively the decrease in home bias is found for various parameter values.

For $\eta_Y = 0.7$, home equity bias drops from 65% to 61.2%. As one would expect, lowering η_Y leads to a smaller reduction in home bias. I check with two alternative values: $\eta_Y = 0.3$ and $\eta_Y = 0$. The results are reported in Table 4. As expected, the reduction in home bias is smaller for this alternative calibration. For a given shock to output, the specific pledgeability of assets fluctuates less than before. The effect on the tightness of the borrowing constraint is correspondingly smaller.

Yet, a tiny reduction in home bias remains even if the pledgeability of home and foreign equities is purely exogenous ($\eta_Y = 0$). In this case, agents invest in local assets 0.2% less than in the benchmark case of homogeneous asset pledgeability. Although this is a tiny difference, note that borrowing constraints leave the international portfolios indeterminate if assets have the same degree of pledgeability (Devereux and Sutherland, 2011b). On the other hand, the tiny decrease in home bias means that investors still care for the risk posed by the heterogeneous asset pledgeability. What matters is that idiosyncratic financial shocks create a wedge between the assets in their portfolios. This wedge has important dynamic effects that are discussed below.

Finally, the bottom part of Table 4 shows how high the transaction cost τ should be to have

65% home bias also in the heterogeneous pledgeability case. For the results discussed up to now, the cost is $\tau = 2.757e^{-5}$. For any η_Y, the cost τ that is consistent with 65% home bias under heterogeneous pledgeability is higher than the benchmark value of $2.757e^{-5}$.

6.3 Dynamic Effects of Shocks

Given the steady state portfolios, one can trace the global effects of shocks in any country. The shocks I consider are unitary declines in home country productivity and in the pledgeability of home equities. Comparisons with the case of homogeneously pledgeable assets are also drawn.

Figure 2 gives a summary of the international deleveraging that these shocks generate. A shock in the home country reduces the price of home equities and, simultaneously, exacerbates the credit frictions. Borrowing constraints tighten in all countries because all investors are exposed to home equities, under financial integration. The credit spreads on loans increase for all of the investors, while the world interest rate declines because of the contraction in borrowing. The decline in borrowing leads to deleveraging of capital. Since portfolios are internationally diversified, agents need to disinvest both home and foreign capital. The shock gets transmitted, in the way described by Devereux and Yetman (2010). There is also an arbitrage effect on prices and loan premiums due to financial integration, as indicated by Dedola and Lombardo (2012). In one word, the model with heterogeneous pledgeability has similar implications as those found in the literature.

Furthermore, productivity and financial shocks have qualitatively similar effects on the world economy, although the consequences of a fall in home country productivity are quantitatively more relevant. Yet, the synchronization between countries is stronger under financial shocks. Perri and Quadrini (2012) put emphasis on this synchronization and build a theory to capture it. Here, the strong comovement is reflected in the dynamics of the trade balance. Under productivity shocks, the country hit by the shock (in this case, the home country) reduces its consumption more strongly than the other country. Yet, home country output decreases even more and the economy displays a trade balance deficit. In the case of financial shocks, consumption and output comove almost exactly in all countries. So the trade balance of the home country remains flat.

6.3.1 Financial Shocks

A different degree in the pledgeability of traded assets have implications for the mechanism underlying the financial shocks and, thus, for their interpretation. Given the numerical portfolios predicted by the model, these financial shocks modify the trade-off between home and foreign equities. Investors take into account how the pledgeability of each asset contributes to the tightness of their borrowing constraints. After a financial shock to home equities, their pledgeability falls: their usefulness as collateral assets decreases until the shock continues to produce its effects. The shock reduces consumption as well, so that investors will have a high marginal utility of income in the next periods. Equity premiums must increase, but investors require an higher premium on home assets than on foreign assets for the periods to come. Figure 3 shows that the differential premium, $EP_H - EP_F$, effectively goes up. Due to arbitrage, $EP_H - EP_F$ increases in the same way for both home country investors and foreign country investors, yet the ultimate profit of each investor depends on her initial portfolio position.

A different mechanism is instead at work when assets have the same degree of pledgeability. The response functions concerning this case are highlighted with a marker (a convention that is adopted below as well) and show that now financial shocks do not affect the variable $EP_H - EP_F$. More specifically, when there is no difference in the pledgeability of assets, a financial shock at home does not affect the overall performance of home equities versus foreign equities. In this case, the relative performance between the two asset is simply the return differential r_x, which does not react to the financial shock. When instead assets do not have the same degree of pledgeability, the relative performance between them is the overall differential Υ_x. This captures both the return on assets and their suitability as collateral. Figure 3 shows that Υ_x do indeed react to a financial shock to home equities. It falls on impact and then goes back to long-run equilibrium. However, the initial fall in Υ_x is due to a fall in the differential pledgeability of home versus foreign equities, κ_x, so the return differential r_x needs to be above its steady state level in the periods after the shock. This increase in r_x confirms agents expectations about the relative premium on home versus foreign equities: indeed, differently from r_x[20], $EP_H - EP_F$ reacts on impact to the shock.

This reaction in the return differential leads to a small but positive valuation effect for the home country, while the balance on income is basically unaffected. Also this result is reported in Figure 3, which further shows that a model with homogeneously pledgeable assets has exactly the opposite predictions. In the latter case, valuation effects are zero, while there is a decrease in the home country balance on income. This result shows that the international transmission of financial shocks is different whether assets have the same pledgeability or not. And to show this, a more precise definition of valuation effects and an analysis of how borrowing reacts to financial shocks is needed.

First, valuation effects and balance on income are defined following Devereux and Sutherland

[20]Note that, on impact, r_x declines in the same way as Υ_x and then overshoots above the steady state level. The decline on impact is not reported in the graph just for convenience.

(2010). From the dynamic equation for net foreign equities (the budget constraint (18)), one can get a more compact definition of the net foreign assets for all the agents in the home country. That is, combining (18) with (9) and (12), one can write down the following expression:

$$NFE_t - \left(b_t^I + \frac{1-n}{n} b_t^S\right) = TB_t + r_{xt}\omega_{t-1} + r_{Ft} NFE_{t-1} - R_{t-1}\left(b_{t-1}^I + \frac{1-n}{n} b_{t-1}^S\right) \quad (34)$$

where $TB_t = Y_{Ht} - c_t$[21]. This expression shows that the total net foreign assets of the economy (the term on the right side of the equal sign) is affected by international debt flows. Since the bond market is unique (equation (12)), the home country becomes a net borrower if on a given date $b_t^I + (1-n)\, b_t^S/n > 0$. This means that, at time t, the leveraged residents of the home country want to borrow more than what the domestic savers can provide. This excess demand is satisfied by the foreign country[22]. The reverse is true if $b_t^I + (1-n)\, b_t^S/n < 0$. And finally, up to a first order the definition of valuation effects and balance on income[23] is, respectively, as follows

$$VE_t\,(1) = \tilde{\omega}^I\,(0)\, r_{xt}\,(1) \quad (35)$$

$$BI_t\,(1) = (r\,(0)-1)\, NFE_{t-1}\,(1) - (R\,(0)-1)\,\frac{b^I\,(0)}{c^I\,(0)}\left(b_{t-1}^I\,(1) - b_{t-1}^S\,(1)\right) \quad (36)$$

where $NFE_{t-1}\,(1) = NFE_{t-1}/c^I\,(0)$ and $\tilde{\omega}^I\,(0) = \omega\,(0)\, r\,(0)\,/c^I\,(0)$.

Equation (35) makes it clear why financial shocks have valuation effects only when assets have different degree of pledgeability. Only in this case, r_x falls on impact (before increasing in line with the differential equity premium). Since ex ante the home country owns 65% of the home capital stock and the rest is diversified, $\tilde{\omega}^I\,(0) < 0$ and, thus, $VE_t\,(1)$ must be positive at the time of the shock. In the opposite case of equal asset pledgeability, financial shocks do not affect the return differential r_x; what matters is instead the way financial shocks affect borrowing by investors. As Figure 4 shows, the demand for credit reacts differently depending on whether assets have the same pledgeability or not. If assets have the same pledgeability, the financial shock is a shock to κ_t. Lenders are not willing to supply funds only to home country investors, regardless of the quality of the assets in their portfolio. Investors in the foreign country can

[21]Because of perfect competition, the aggregate output of goods produced by home country firms satisfies $Y_{Ht} = [1 + (1-n)\,/n]\, w_t + d_{Ht}\chi_{Ht-1}$. $c_t = nc_t^I + (1-n)\left[c_t^S - z\left(k_{Ht-1}^S\right)^{\nu}\right]$ is the amount of home production consumed by local agents.

[22]In the foreign country, $b_t^{*I} + (1-n)\, b_t^{*S}/n < 0$.

[23]See Devereux and Sutherland (2010). Here I use two of their key findings. On one side, they show that $r_x\,(1)$ is basically solely affected by movements in prices. On the other hand, the behaviour of $NFE\,(1)$ for given $r\,(0)-1$ depends mostly on dividends, which matter for the balance on income.

continue to borrow, as their access to credit remains basically unaffected. Importantly, foreign investors can continue to borrow not only from foreign savers but also from savers in the home country. There are two consequences. One is that the credit available in the world economy stops to flow to the home country and goes to the foreign country. The second is that, loosing access to external funds, only home country investors need to deleverage their foreign equities in net terms (i.e., NFE_t).

So in few words, if assets have the same pledgeability only borrowing in the home country is affected by the financial shocks. If instead assets have different degree of pledgeability, then the financial shock affects investors in all of the countries. In particular, all of the investors face tighter credit constraints, and borrowing falls both at home and in the foreign country. The reduction in borrowing is proportional to the initial exposure to home assets. As Figure 4 shows, the home equity bias of 65% implies that borrowing decreases by more in the home country. So there is still a debt outflow from the home country to the foreign country, but it is smaller than that observed in the case of homogeneously pledgeable assets. The reduction in the net foreign assets of the home country is correspondingly smaller.

The interpretation and effects of financial shocks change in accordance with the type of financial shock. If home and foreign assets have the same degree of pledgeability, a negative financial shock in the home country is a decrease in the ability of residents there to obtain funds from savers, for any given level of wealth. The shock is transmitted because home country investors have cross-border positions (through loss of access to credit market loans and deleveraging). Yet, there are no relative price effects across borders, and foreign investors continue to borrow. On the other hand, if home and foreign equities have different degree of pledgeability, the negative financial shock at home affects the assets issued by local firms. Because of financial integration in capital markets, these assets are in the portfolios of investors both in the home country and in the foreign country. And this constitutes an additional channel for the international transmission of shocks, producing valuation effects.

Robustness. The results on the consequences of financial shocks are rather robust both qualitatively and quantitatively. While the implications for portfolio choice change quantitatively with η_Y, the effects of shocks to the pledgeability of home equities are similar for various values of η_Y. Even for $\eta_Y = 0$ the impact of a financial shocks on expected return differential, valuation effects and international transmission are almost identical to those in Figures 3-4. This test can be found in the external appendix.

6.3.2 Productivity Shocks

Also the effects of productivity shocks depend on whether assets have different pledgeability. But given the structure of the model, the main consequences are produced by the fall in home country productivity itself. Consider the responses reported in Figure 5 and Figure 6.

The productivity shock has a negative impact effect on the return differential r_x. But this negative effect originates from the fact that, after the shock, home country firms pay lower dividends. This is true in any case, regardless of the pledgeability of assets. And, in both of the cases, the relative reduction in dividends and prices of home capital has direct valuation effects. These effects are large.

The possibility that assets have different degree of pledgeability influences the return differential. In this case, the pledgeability of home equities worsens together with the shock to productivity, while that of foreign equities depends on the foreign business cycle. The home country is the economy most affected by the shock, even if the shock gets transmitted to the foreign country. So the pledgeability of home equities falls relatively more than that of foreign equities, causing a relative increase of the equity premium on the former assets relative to the latter. Yet, the reaction of $EP_H - EP_F$ is smaller than under financial shocks.

Similarly, the shock already generates positive valuation effects for the home country, as the fall in productivity causes r_x to decrease. Equity premiums contribute to these valuation effects, but it is not the case that valuation effects are absent without differences in pledgeability. This was instead the case of financial shocks. As Figure 6 shows, another difference with the previous case is that the net foreign assets of the home country improves on impact. This is because the valuation effects are very strong for productivity shocks, while flows (especially debt flows) are most important under financial shocks.

Robustness. While the effects of financial shocks are robust to the choice of $\eta_Y > 0$, real shocks generate the dynamics just described only if $\eta_Y > 0$. In some sense, this is a by-product of the model at hand. Recall that, in the model under analysis, differences in pledgeability are short-term and that productivity and financial shocks are assumed to be completely uncorrelated for analytic convenience. These robustness tests are not reported for brevity.

7 Conclusion

Although the demand for an asset is principally explained by the flow of income it can generate, this paper shows that other factors can be important as well. The additional factor studied is the pledgeability of an asset as collateral for borrowing in the credit market. This captures the idea that lenders prefer to receive certain collateral assets as guarantee for their loans. In the model, the specific pledgeability of each asset is introduced in the borrowing constraints of leveraged investors.

I use borrowing constraints of this type in a two-country model recently developed for analyzing the international transmission of shocks. The model involves an endogenous portfolio choice, which is not straightforward to solve once the assumption that assets have the same pledgeability is relaxed. I propose a solution strategy that is based on both the effect of heterogeneous pledgeability on cross-border return differentials and the equalization of credit spreads under financial integration.

My numerical application shows that borrowing constraints matter for optimal portfolio choice. This is not true for the case of homogeneously pledgeable assets. Also the interpretation and effects of financial shocks are different. In fact, the shocks to the pledgeability of a single asset that is traded on international markets is naturally transmitted because of international diversification of asset holdings. This result seems to be quite relevant for the current structure of financial markets. Financial integration heightened in the last decades, and some internationally traded assets deemed to be a good collateral can suddenly loose their pledgeability.

How the approach developed here can be adopted to consider assets that are inherently safe (and the question of global imbalances) within international portfolio models is left for the future. Here I pointed out some technical issues, but further research is needed.

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A Proof Of Lemma 4

Under financial integration, loan premiums are equalized across countries up to first order. To see this, consider the optimality conditions for bonds and equities. Since focusing on just one equity is sufficient, consider equations (3)-(4). These express the demand for credit and the demand for home equity by home country investors. Similar conditions apply for foreign investors.

Since in equilibrium the marginal utility of borrowing and purchasing equities must be equal for all of the investors in the market, these conditions can be combined as follows:

$$\begin{aligned} R_{t+1} E_t \lambda^I_{t+1} + M_t &= E_t \lambda^I_{t+1} r_{Ht+1} + M_t \kappa_{Ht} \\ R_{t+1} E_t \lambda^{*I}_{t+1} + M^*_t &= E_t \lambda^{*I}_{t+1} r_{Ht+1} + M^*_t \kappa_{Ht} \end{aligned}$$

Approximating these relations to a first order yields the following expressions:

$$\left[\begin{array}{c} \lambda^I(0) R(0) \left(R_{t+1}(1) + E_t \lambda^I_{t+1}(1) \right) \\ +M(0) M_t(1) \end{array} \right] = \left[\begin{array}{c} r(0) \lambda^I(0) E_t \left(\lambda^I_{t+1}(1) + r_{Ht+1}(1) \right) \\ +\kappa(0) M(0) \left(M_t(1) + \kappa_{Ht}(1) \right) \end{array} \right] \tag{37}$$

$$\left[\begin{array}{c} \lambda^I(0) R(0) \left(R_{t+1}(1) + E_t \lambda^{*I}_{t+1}(1) \right) \\ +M(0) M^*_t(1) \end{array} \right] = \left[\begin{array}{c} r(0) \lambda^I(0) E_t \left(\lambda^{*I}_{t+1}(1) + r_{Ht+1}(1) \right) \\ +\kappa(0) M(0) \left(M^*_t(1) + \kappa_{Ht}(1) \right) \end{array} \right] \tag{38}$$

Since capital markets are integrated, every investor has a share of home equities in her portfolio. In addition, debt is issued in the international bond market. So (37)-(38) can be further combined in order to obtain a relationship between shadow cost of constrained borrowing and marginal utility of future consumption:

$$(1 - \kappa(0)) \frac{M(0)}{\lambda^I(0)} \left(M_t(1) - M^*_t(1) \right) = \left(r(0) - R(0) \right) E_t \left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1) \right) \tag{39}$$

Now recall that the loan premiums (or credit spreads) are given by

$$\frac{\mu_t}{\beta(c^I_t) E_t \lambda^I_{t+1}} = \frac{M_t}{E_t \lambda^I_{t+1}} \qquad \frac{\mu^*_t}{\beta(c^{*I}_t) E_t \lambda^{*I}_{t+1}} = \frac{M^*_t}{E_t \lambda^{*I}_{t+1}}$$

Therefore, equation (39) implies the equalization of the first order approximation of these loan premiums for suitable values of the constants of approximation, $(1 - \kappa(0)) M(0) / \lambda^I(0)$ and $(r(0) - R(0))$.

Note that, from conditions (3)-(5) and (10), the steady state return on equities is

$$r(0) = \frac{1}{\beta(c^I(0))} \left(1 - \frac{\mu(0)}{\lambda^I(0)} \bar{\kappa} \right)$$

and the premium on equities is

$$r(0) - R(0) = \frac{\beta\left(c^S(0)\right) - \beta\left(c^I(0)\right)}{\beta(c^S(0)) \beta(c^I(0))} - \frac{\mu(0)}{\beta(c^I(0)) \lambda^I(0)} \kappa(0)$$

where $\mu(0)/\left[\beta\left(c^I(0)\right)\lambda^I(0)\right] = \left(\beta\left(c^S(0)\right)-\beta\left(c^I(0)\right)\right)/\left[\beta\left(c^I(0)\right)\beta\left(c^S(0)\right)\right] > 0$. Finally, use this result with (39) and get

$$\left(M_t(1)-E_t\lambda^I_{t+1}(1)\right)=\left(M^*_t(1)-E_t\lambda^{*I}_{t+1}(1)\right) \tag{40}$$

which indeed proves that the loan premium paid by home and foreign investors is the same up to first order.

A.1 An Interpretation of Υ_x Based on Lemma 4

The proof of Lemma 4 allows for a further interpretation of Υ_x, defined in the text as the overall differential performance of home versus foreign equities.

Equation (28) is the relevant portfolio choice condition. This condition implies that the expected overall differential is a sort of risk premium:

$$E_t\Upsilon_{xt+1}(1)=-\frac{cov_t\left(\lambda^I_{t+1}(1)-\lambda^{*I}_{t+1}(1),\Upsilon_{xt+1}(1)\right)}{E_t\left(\lambda^I_{t+1}(1)-\lambda^{*I}_{t+1}(1)\right)} \tag{41}$$

That is, the expected overall differential depends on how the expected marginal utility of consumption of home agents relative to that of foreign agents comove with Υ_x. Indeed, the future marginal utility of income of any agent is affected by actual differences in asset pledgeability and by future return differentials, which are both captured by Υ_x. The expected value of this overall differential is a first order term, as it is the ratio of a second order term and a first order term. However, the numerator must be zero when the model is solved up to a first order.

Condition (29) shows that this is effectively the case. Up to first order,

$$E_t\Upsilon_{xt+1}(1)=0 \tag{42}$$

and, thus, the right hand side of (41) is equal to zero. Since $\Upsilon_{xt+1}=r_{xt+1}+const\times\kappa_{xt}$, this result confirms that the expected return differential is non-zero up to first order:

$$E_t r_{xt+1}(1)=E_t\Upsilon_{xt+1}(1)-const\times\kappa_{xt}(1) \tag{43}$$

which is equivalent to (30) in the text.

B Iterative Solution for Portfolios

The solution for the first order component of model variables is

$$\begin{aligned}\mathbf{s}_{t+1}(1) &= \mathbf{\Pi}_1\mathbf{x}_t(1)+\mathbf{\Pi}_2\mathbf{s}_t(1) &(44)\\ \mathbf{c}_t(1) &= \mathbf{\Phi}_1\mathbf{x}_t(1)+\mathbf{\Phi}_2\mathbf{s}_t(1) &(45)\end{aligned}$$

where $\mathbf{s}_{t+1}$ are the endogenous states, $\mathbf{c}_t$ are the jump variables, $\mathbf{x}_t$ are the exogenous states and, finally, $\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{\Phi}_1, \mathbf{\Phi}_2$ are matrices of numbers.

As mentioned in the text, I use an "iceberg cost" τ on financial transactions to reproduce an home equity bias similar to that observed in the data. This cost is second order, so the specific condition that I apply to determine $\omega(0)$ is actually

$$0 = 2\tau + E_t \left(\lambda_{t+1}^{I}(1) - \lambda_{t+1}^{*I}(1) \right) \Upsilon_{xt+1}(1) \tag{46}$$

The derivation of this condition is detailed in the external appendix.

In terms of procedure, (46) is applied as follows. Once the solution (44)-(45) of the model is available, it is possible to extract from this solution the following rows:

$$\begin{aligned} \Upsilon_{t+1}(1) &= \mathbf{U}\varepsilon_{t+1} \\ \lambda_{t+1}^{I}(1) - \lambda_{t+1}^{*I}(1) &= \mathbf{L}_1 \varepsilon_{t+1} + \mathbf{L}_2 \begin{pmatrix} \mathbf{x}_t(1) \\ \mathbf{s}_{t+1}(1) \end{pmatrix} \end{aligned}$$

where $\varepsilon_{t+1} = (\varepsilon_{At+1} \;\; \varepsilon_{A^*t+1} \;\; \varepsilon_{\kappa_H t+1} \;\; \varepsilon_{\kappa_F t+1})'$ is the 4×1 vector of shocks. Using these lines with (46), it is possible to define the following implicit function:

$$g(\omega(0)) \equiv 2\tau + \mathbf{U}\mathbf{\Sigma}\mathbf{L}_1' \tag{47}$$

where Σ is the variance-covariance matrix of economic innovations.

So, starting with an initial value $\omega^0(0)$, the optimal portfolio is the solution for which $g(\omega(0)) = 0$. Given the way the portfolio share is defined, there is home bias in asset holdings only if $\omega(0) < 0$. So, formally, the solution for the optimal portfolio is

$$\omega^{\star}(0) = \underset{\omega(0) \in S}{\arg\min} \left| \mathbf{g}(\omega(0)) - 0 \right| \tag{48}$$

where S is a neighbourhood of values around zero:

$$S = \{\omega(0) : \omega(0) \in [-\delta; \delta]\}$$

A possible iteration is as follows: (a) choose $\omega^0(0)$; (b) solve for the first order component of model variables; (c) extract the rows which are appropriate to compute (47) and apply it; (d) repeat steps (a)-(c) until (48) is satisfied with a certain degree of approximation. Note that the ultimate solution is conditional on a given τ.

I use a similar approach also for computing the optimal portfolio when assets have equal pledgeability. In this case, the solution methods already existing in the literature apply. The portfolio choice condition to solve is (22) and the rows of the model solution to extract are indicated by DS.

	Homogeneous Between Assets	Heterogeneous Between Assets
Home Investors	κ_t	$\kappa_{Ht} \neq \kappa_{Ft}$
Foreign Investors	κ_t or $\kappa_t^* \neq \kappa_t$	$\kappa_{Ht} \neq \kappa_{Ft}$

Table 1: Alternative Assumptions on Pledgeability

parameter	description	value
n	number of constrained investors	0.5
ϕ	discount factor parameter	0.022
σ	CRRA	2
α	capital share in income	0.4
z	fixed productivity in backyard production	1
ν	degree of homogeneity in backyard sector	0.1
$R(0)$	gross rate of interest	1.042
$M(0)/\lambda^I(0)$	guarantee premium	0.01
$A(0)$	total factor productivity	1
ρ_A	persistence of productivity shocks	0.82
σ_A	volatility of productivity shocks	0.015
$\kappa(0)$	debt-to-asset ratio	0.5
η_Y	sensitivity of debt-to-asset ratio to output gap	0.7(baseline), 0.3, 0
ρ_κ	persistence of financial shocks	0.82
$\sigma_\kappa, \sigma_{\kappa_i}$	volatility of financial shocks	0.02

Table 2: Calibration

	Homogeneous		*Heterogeneous*		Homo. dependent on GDP	
	H equity	F equity	H equity	F equity	H equity	F equity
	(2)	(3)	(4)	(5)	(6)	(7)
H Country Savers	0.792	-	0.792	-	0.792	-
H Country Investors	2.060	1.109	1.941	1.229	2.144	1.026
F Country Savers	-	0.792	-	0.792	-	0.792
F Country Investors	1.109	2.060	1.229	1.941	1.026	2.144
H Country Portfolio	**65%**	**35%**	*61.2%*	*38.8 %*	67.6%	32.4%
F Country Portfolio	**35%**	**65%**	*38.8%*	*61.2%*	32.4%	67.6%

Table 3: Portfolios under Alternative Cases of Asset Pledgeability

Note: The percentage of stocks issued by local firms in the total country portfolio depends on investors' holdings of home and foreign equities. The computation is straightforward: Holdings of local investors/(Holdings of local and foreign investors). This portfolios have been obtained with the transaction cost $\tau = 2.757e^{-5}$. This cost reproduces an home bias of 65% in a model with assets that have homogeneous pledgeability (i.e., the benchmark for comparisons). The elasticity to output assumes its baseline value ($\eta_Y = 0.7$). Finally, the last two columns show the case in which agent-specific pledgeability is modeled in the same way as the asset-specific pledgeability. That is, $\ln \kappa_t = \rho_\kappa \ln \kappa_{t-1} + (1-\rho_\kappa)\left[\ln \kappa(0) + \eta_Y \ln\left(Y_{Ht}/Y(0)\right)\right] + \varepsilon_{\kappa,t}$ and $\ln \kappa_t^* = \rho_\kappa \ln \kappa_{t-1}^* + (1-\rho_\kappa)\left[\ln \kappa(0) + \eta_Y \ln\left(Y_{Ft}/Y(0)\right)\right] + \varepsilon_{\kappa^*,t}$.

	Varying the level of η_Y		
	$\eta_Y = 0.7$ (baseline)	$\eta_Y = 0.3$	$\eta_Y = 0$
Portfolio Share of Local Assets	61.2%	63.2 %	64.8 %
Portfolio Share of External Assets	38.8%	36.8 %	35.2 %
τ needed for 65% Home Bias	$2.906e^{-5}$	$2.824e^{-5}$	$2.764e^{-5}$

Table 4: Portfolios Robustness

Note: The computation reported in the table use a transaction cost $\tau = 2.757e^{-5}$, as in Table 3. The only exception is the last row.

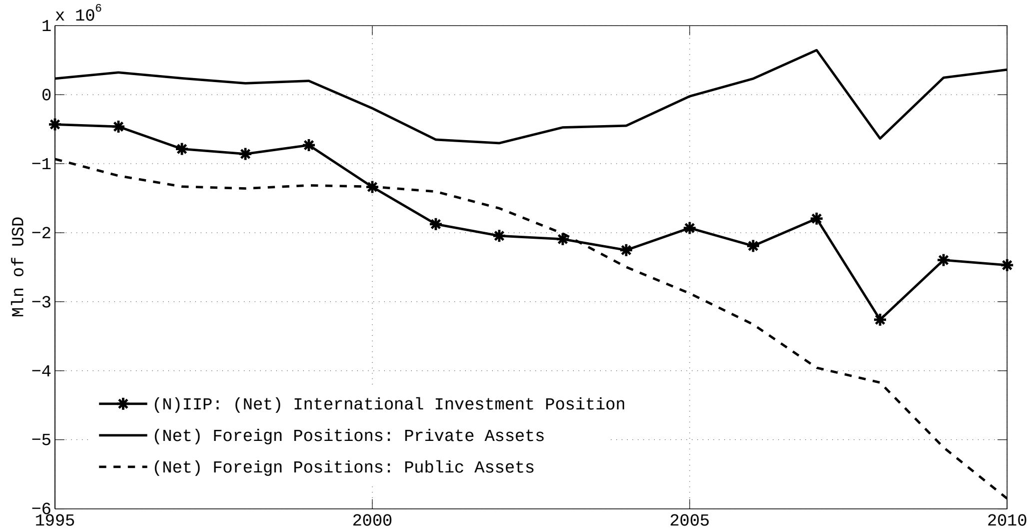

Figure 1: The International Investment Position of the U.S.

Source: International Investment Position, U.S. Department of Commerce, Bureau of Economic Analysis.

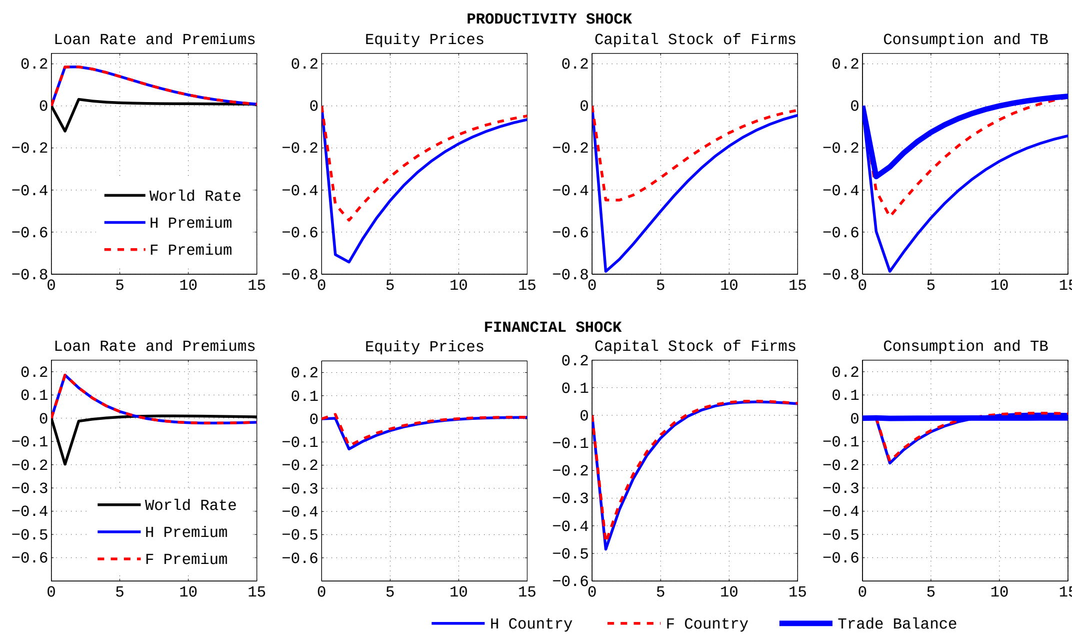

Figure 2: Summary Effects of Productivity and Financial Shocks

Note: The real shock is a unit, negative innovation to home country productivity. The financial shock is a unit, negative innovation to the pledgeability of home equity.

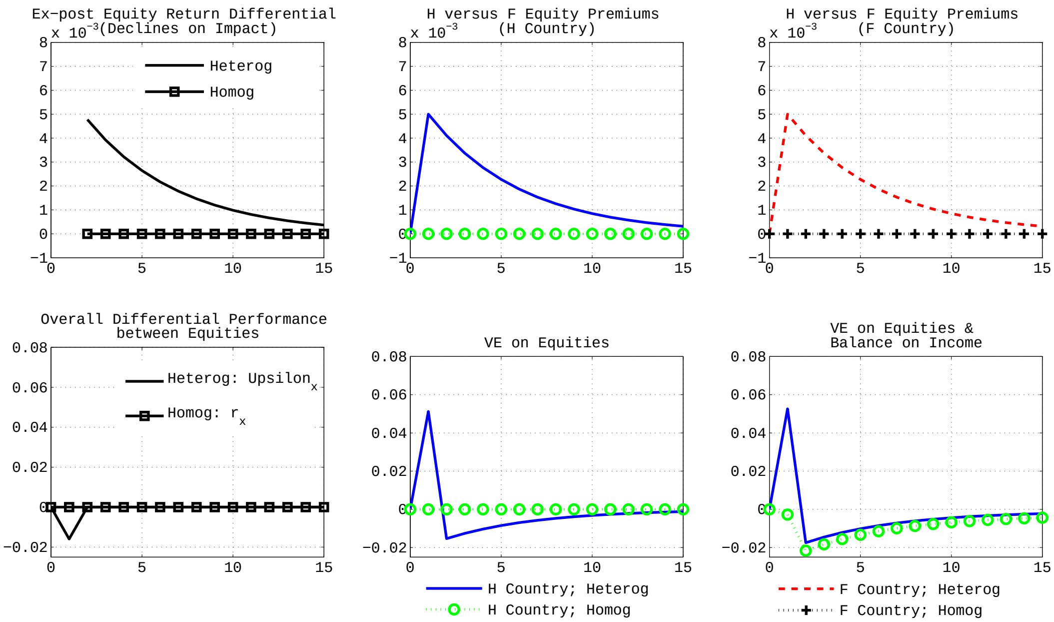

Figure 3: Effects of Financial Shocks on Asset Returns

Note: The first diagram shows the reaction of r_x only for the periods following the shock. On impact instead r_x falls, which is suggested by the fourth diagram. The differential between home equity premium and foreign equity premium is given by $EP_H - EP_F$ and reported in the diagram labeled "H versus F Equity Premiums" of the Figure.

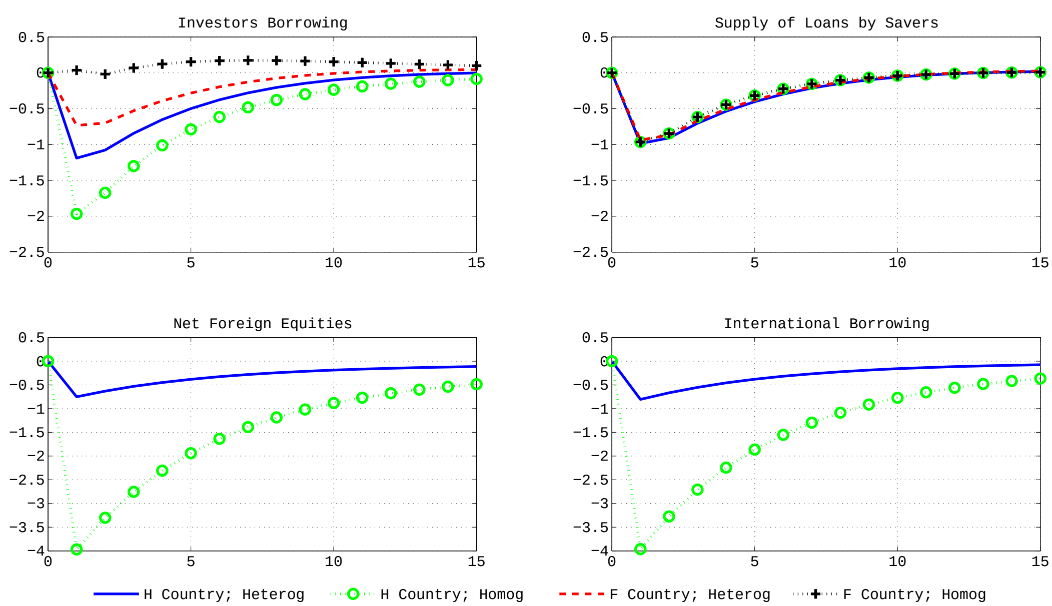

Figure 4: Effects of Financial Shocks on Borrowing and External Assets

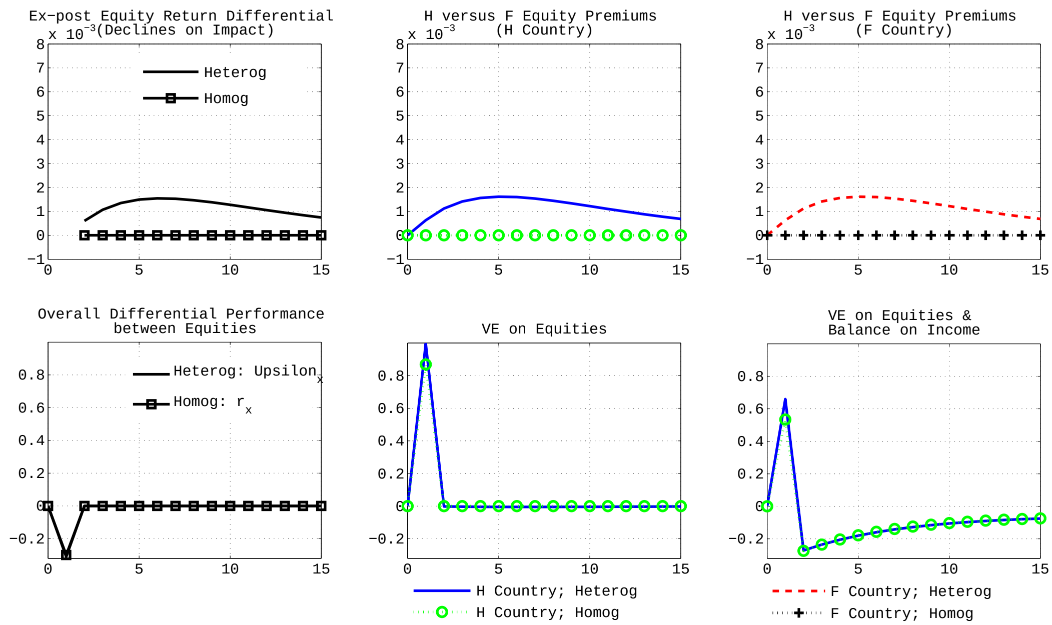

Figure 5: Effects of Productivity Shocks on Asset Returns

Note: The first diagram shows the reaction of r_x only for the periods following the shock. On impact instead r_x falls, which is suggested by the fourth diagram. The differential between home equity premium and foreign equity premium is given by $EP_H - EP_F$ and reported in the diagram labeled "H versus F Equity Premiums" of the Figure.

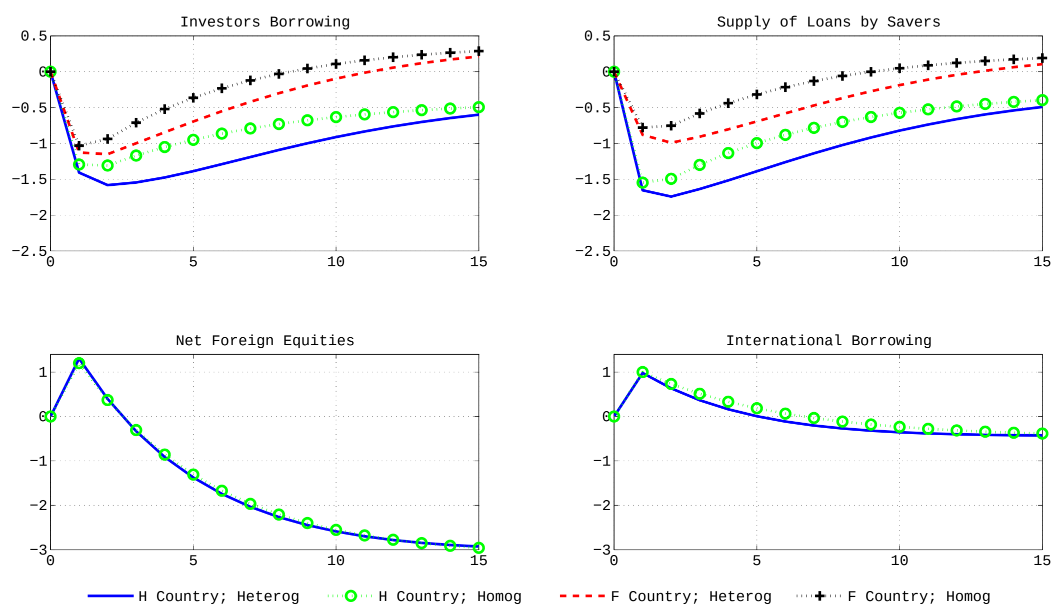

Figure 6: Effects of Productivity Shocks on Borrowing and External Assets