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Keywords: Vertical differentiation; third-degree price discrimination; welfare.

JEL codes: D43; D60; .

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1 Introduction

In many industries, competition takes place between firms that operate in a particular geographic market and firms that are active in many different geographic markets. For instance, the retailing industry is characterized by competition between local firms and chain stores, the latter being active in a large number of different markets. Other industries where this pattern appears are hotels, restaurants, hairdressers, or auto repair shops. Frequently, local stores charge a higher price and actually manage to survive suggests that consumers value their products more. This higher valuation is consistent with their products being of higher quality, not necessarily in terms of product characteristics, but for instance because of more proximate location of local stores, or the provision of additional services.

A question naturally arises about the welfare implications of the pricing policies adopted by the firm active in several markets, especially price discrimination. This pricing policy is widely used. For instance, Cooper (2003) reports empirical evidence from the UK supermarket industry that pricing policies vary across supermarket chains, with seven supermarket groups pricing according to local conditions, a strategy known as price flexing. These pricing policies are sensitive to income and the presence of discount retailers. The UK Competition Commission is concerned about some of the chains engaging in price flexing for reasons not just attributable to local operating costs. The Commission considered that this practice was anticompetitive, although no action was taken. We analyze the welfare implications of price discrimination in the presence of differences in quality among competitors. For instance, Matsa (2011) defines a supermarket's product as the shopping experience it provides its customers. He argues that product availability constitutes an important aspect of product quality in the supermarket industry. Using data collected by the US Bureau of Labor Statistics, he finds that supermarkets that face more intense competition have less frequent stockouts.

Price discrimination has typically been regarded as welfare-reducing unless it leads to an increase in output. In this paper, we show that in the presence of quality differences among firms, this may not be the case. In our model, we consider two markets and three firms. In each market, there is one local firm, which operates exclusively in that market. Additionally, there is a third firm that operates in both markets, which we refer to as the global firm. The global firm sells a product whose quality is lower than those of local firms. We compare the cases of the global firm choosing a uniform price and being allowed to price discriminate across markets. We find that it may be the case that, even though price discrimination leads to an aggregate output decrease, it may give rise to a welfare increase in the two markets combined. This is because price discrimination brings about a positive allocation effect that more than offsets the negative output effect. We believe that this result should be taken into account by the competition authority when evaluating the potential welfare implications of third-degree price discrimination.

Our goal, therefore is to study whether price discrimination carried out by firms that operate across different markets, for instance retail chains, is beneficial for society. A naïve approach would be that third-degree price discrimination is welfare increasing as long as output increases, whether the market total output or merely that of the retail chains. However, if quality differences are present, we show that this is not necessarily the case. In particular, and using linear demand functions, total output may decrease while welfare increases with price discrimination.

The study of the relationship between the output and misallocation effects constitutes the traditional approach to the analysis of the welfare implications of price discrimination. According to this approach, if total output does not vary, there is a misallocation effect that brings about a welfare loss. Of course, if output increases, other effects on welfare are present. Among the classical contributions to the study of this issue we find Robinson (1933), or Schmalensee (1981), that conclude that an output expansion is a necessary condition for welfare to increase with price discrimination, assuming firms operating with constant marginal costs. Varian (1985) generalizes this result to interdependent markets and increasing marginal costs, and Schwartz (1990) proves this result for decreasing marginal costs. Aguirre et al. (2010) discuss conditions under which welfare increases or decreases with price discrimination when all markets are served, and considering general demand functions, see also Cowan (2007) for an analysis assuming non-linear demands. In an oligopoly setting, Stole (2007) argues that the basic result goes through in imperfect competition, provided that the firms are equally efficient at production and the number of firms is fixed. In this line, Dastidar (2006) extends the basic analysis to a symmetric cost duopoly to provide conditions under which total output decreases with price discrimination, and welfare may either increase or decrease with price discrimination.

In addition to the well-known output and misallocation effects, an additional effect of price discrimination, namely a cost effect, may be considered. That is, the total cost of producing the same output level may be different under price discrimination than with a uniform price, and this cost effect may be positive or negative. For instance, Galera and Zaratiegui (2006) show that in a Cournot duopoly where firms have different costs, the cost effect my more than offset the output and misallocation effects, so that the total effect of price discrimination is positive, even though total output decreases. On the other hand, Galera et al. (2014) prove that the cost effect may also arise in a monopoly with increasing marginal costs and in the presence of demand uncertainty. This cost effect may make total welfare to increase, although output decreases with price discrimination.

There are a number of contributions that depart from the assumption of product homogeneity, introducing either horizontal or vertical product differentiation. For instance, Jorge and Pires (2013) take into account the role of price discrimination on industry structure with two geographically different markets and potential entry by a producer of a horizontally differentiated product. The effect on welfare depends on the degree of product substitutability and entry costs. The product quality dimension has recently been considered in the analysis of price discrimination, with quality typically being endogenously chosen by competing firms. In a vertical differentiation setting, Ikeda and Toshimitsu (2010) show that if a monopolist facing linear demands and simultaneously choosing product quality and price, price discrimination brings about a quality effect that always dominates the misallocation effect –output effects are absent in this model. Quality is introduced in the analysis as a proportional increase in willingness to pay. The monopolist's incentives to invest in quality increase with price discrimination, which increases welfare via an increase in consumer surplus. A similar result is obtained in Alexandrov and Deb (2012), namely that the increase in the investment in quality when price discrimination is allowed relative to a uniform price may dominate the misallocation effect. This model introduces different preferences for quality in the markets among which there is price discrimination, and the result is extended to the case of a Bertrand duopoly. Nguyen (2014) uses instead a model with variable quality costs to find that monopolistic third-degree price discrimination is always welfare-reducing, regardless of whether quality levels are endogenous or exogenously given, a result that contrasts with that in Ikeda and Toshimitsu (2010).

Price discrimination in input markets, as opposed to final goods markets, has also received a considerable attention from researchers. Indeed, according to O'Brien (2014), the Robinson-Patman Act arose from concerns that large downstream firms were harming smaller rivals by negotiating larger discounts with suppliers. Yoshida (2000), assuming that downstream producers differ in their efficiency levels, finds that an increase in total output of the final good is a sufficient condition for welfare to decrease with price discrimination. Adachi and Matsushima (2014) consider an oligopoly of differentiated producers to derive a necessary and sufficient condition for price discrimination to increase welfare, namely the degree of substitution being sufficiently greater in the strong market than in the weak market.

At an empirical level, Shepard (1991) analyzes data on the gasoline retailing market in Massachusetts and finds evidence consistent with firms engaging in price discrimination in an oligopolistic environment where service quality varies. Analyzing the Swedish newspaper industry, Asplund et al. (2008) find that newspapers use price discrimination as a way to attract subscribers from rival newspapers. Focusing on the supermarket industry, Basker (2011) finds empirical evidence of Wal-Mart selling inferior goods, since its revenues increase during downturns, and Dobson and Waterson (2005) analyze a chain-store's decision on whether to use uniform prices or prices that are adapted to the characteristics of local markets to find that price discrimination is not always optimal from the perspective of the chain store.

Our paper differs from the previous contributions to the literature in a number of aspects. Specifically, in our model, the global firm offers an exogenously-given quality level that is different than those offered by local firms. We find that the misallocation effect might go in the opposite way as it typically does, absent quality effects and cost effects. In particular, the fact that there are two qualities in each market may make the misallocation effect positive, so that price discrimination may have a positive effect on welfare, even though total output remains constant. In our model, firms do not choose quality levels, and all costs are set to zero. This way, the quality and cost effects do not play any role in the basic result of our model. The basic result is obtained only if the quality of the global firm's product is lower than that of the local firms. If the global firm offers a higher-quality product than the local firms, the sign of the misallocation effect is negative, as expected by the standard theory.

The remainder of the paper is organized as follows. Section 2 will describe the theoretical model that is used in the analysis. Section 3 discusses the conditions that must hold for price discrimination to be the global firm's optimal strategy, increase welfare, and decrease total output. Finally, section 4 presents some concluding comments.

2 The model

2.1 Basic setup

Assume that there are two geographically different markets, with a continuum of consumers in each market. As in Shaked and Sutton (1982) or Shaked and Sutton (1983), consumers in each market differ in their willingness to pay for quality. In particular, the willingness to pay for the low and high quality goods by a consumer in market i characterized by θ is given by $a_i - \theta$ and $s_i(a_i - \theta)$, respectively, with $s_i > 1$ being the quality level of the high-quality good in market i. Furthermore, we assume that consumers' willingness to pay for quality follows a uniform distribution in the $[0, a_i]$ support. Under this assumption, markets differ in their total sizes. Without loss of generality, let us assume that $s_2 \geq s_1$, that is, the quality increase of the high-quality product is valued more in market 2 than in market 1.

In each of the two markets, there is a local firm, which operates in that market only, and a global firm, which operates in both markets. The local firm in market i offers a product with quality $s_i > 1$. Hence, local firms offer products that are of superior quality relative to that of the global firm. As it is usually assumed in this type of models, every consumer consumes at most one unit of one of the two goods offered in each market, whether the high- or the low-quality good. Firms engage in a price-setting game, whose stages are as follows:

1. The global firm decides whether to introduce a uniform price or to price discriminate.
2. Firms simultaneously choose prices, and quantities an profits are realized.

As it is usually done in this type of games, we will analyze the final stage first and proceed backwards. This task is undertaken in the next subsection.

2.2 Market outcome

In the final stage of the game, firms simultaneously choose prices for their products. Let $p^k_{i,j}$ denote the price posted by firm j in market i under pricing regime k, where $j \in \{L, G\}$, with L denoting the local firm and G denoting the global firm. Additionally, $k \in \{U, D\}$, where U denotes a uniform price, whereas D denotes price discrimination. Given prices, we find that the consumer in market i that is indifferent between purchasing from the local and from the global firm, given pricing regime k, has willingness to pay x_i such that

$$s_i(a_i - x_i) - p^k_{iL} = a_i - x_i - p^k_{iG} \Leftrightarrow x_i = a_i - \frac{p^k_{iL} - p^k_{iG}}{s_i - 1}. \tag{1}$$

Since the difference between the surpluses from buying from the local firm and buying from the global firm, $(s_i - 1)(a_i - x_i) - p^k_{iL} + p^k_{iG}$, grows when x_i decreases, all consumers characterized by $\theta \in [0, x_i]$ will purchase from the local firm, whereas the remaining consumers will opt for the global firm. This way, and since consumers' willingness to pay for quality follows a uniform distribution, the demand for the local and the global firm will be

$$x_{iL} = a_i - \frac{p^k_{iL} - p^k_{iG}}{s_i - 1}, \quad x_{iG} = a_i - x_i - p^k_{iG}. \tag{2}$$

In the following two subsections we present the computation of the equilibrium outcome, that is prices, quantities, and profits for the two alternative pricing policies that the global firm may adopt, namely price discrimination or a uniform price.

2.2.1 Price discrimination

With price discrimination, the global firm will choose market-specific prices, p_{1G}^D and p_{1G}^D, whereas the local firms will choose prices p_{1L}^D and p_{2L}^D, respectively. This way, the expression for profits as a function of prices is given by

$$\Pi_{1L}^D = p_{1L}^D \left(a_1 - \frac{p_{1L}^D - p_{1G}^D}{s_1 - 1} \right); \ \Pi_{2L}^D = p_{2L}^D \left(a_2 - \frac{p_{2L}^D - p_{2G}^D}{s_2 - 1} \right); \tag{3a}$$

$$\Pi_G^D = p_{1G}^D \left(\frac{p_{1L}^D - p_{1G}^D}{s_1 - 1} - p_{1G}^D \right) + p_{2G}^D \left(\frac{p_{2L}^D - p_{2G}^D}{s_2 - 1} - p_{2G}^D \right). \tag{3b}$$

Given that firms simultaneously choose the prices of their products, the equilibrium prices are given by

$$p_{1L}^D = \frac{2a_1 s_1 (s_1 - 1)}{4s_1 - 1}, \qquad p_{2L}^D = \frac{2a_2 s_2 (s_2 - 1)}{4s_2 - 1} \tag{4a}$$

$$p_{1G}^D = \frac{a_1 (s_1 - 1)}{4s_1 - 1}, \qquad p_{2G}^D = \frac{a_2 (s_2 - 1)}{4s_2 - 1}. \tag{4b}$$

Once the equilibrium prices have been obtained, output levels in equilibrium are

$$q_{1L}^D = \frac{2a_1 s_1}{4s_1 - 1}, q_{2L}^D = \frac{2a_2 s_2}{4s_2 - 1}, q_{1G}^D = \frac{a_1 s_1}{4s_1 - 1}, q_{2G}^D = \frac{a_2 s_2}{4s_2 - 1}. \tag{5}$$

2.2.2 Uniform price

In this case, the global firm is constrained to posting a uniform price, p_G^U, whereas local firms choose prices p_{iL}^U. Then, profits may be written, as a function of these prices, as

$$\Pi_{1L}^U = p_{1L}^U \left(a_1 - \frac{p_{1L}^U - p_G^U}{s_1 - 1} \right); \;\; \Pi_{2L}^U = p_{2L}^U \left(a_2 - \frac{p_{2L}^U - p_G^U}{s_2 - 1} \right); \tag{6a}$$

$$\Pi_G^U = p_G^U \left(\frac{p_{1L}^U - p_G^U}{s_1 - 1} - p_G^U \right) + p_G^U \left(\frac{p_{2L}^U - p_G^U}{s_2 - 1} - p_G^U \right). \tag{6b}$$

Therefore, solving the first-order conditions of these three maximization problems, $\frac{\partial \Pi_{1L}^U}{\partial p_{1L}^U} = 0$, $\frac{\partial \Pi_{2L}^U}{\partial p_{2L}^U} = 0$ and $\frac{\partial \Pi_G^U}{\partial p_G^U} = 0$, the equilibrium prices chosen by the global and the local firms are:

$$p_{1L}^U = \frac{(s_1 - 1)a_1}{2} + \frac{(a_1 + a_2)(s_1 - 1)(s_2 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}, \tag{7a}$$

$$p_{2L}^U = \frac{(s_2 - 1)a_2}{2} + \frac{(a_1 + a_2)(s_1 - 1)(s_2 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}, \tag{7b}$$

$$p_G^U = \frac{(a_1 + a_2)(s_1 - 1)(s_2 - 1)}{8s_1s_2 - 5(s_1 + s_2) + 2}. \tag{7c}$$

These three equations, together with equation (2) allow us to compute the equilibrium quantities. These are:

$$q_{1L}^U = \frac{a_1}{2} + \frac{(a_2 + a_1)(s_2 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}, \tag{8a}$$

$$q_{2L}^U = \frac{a_2}{2} + \frac{(a_2 + a_1)(s_1 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}, \tag{8b}$$

$$q_{1G}^U = \frac{a_1}{2} - \frac{(a_2 + a_1)(2s_1 - 1)(s_2 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}, \tag{8c}$$

$$q_{2G}^U = \frac{a_2}{2} - \frac{(a_2 + a_1)(s_1 - 1)(2s_2 - 1)}{2(8s_1s_2 - 5(s_1 + s_2) + 2)}. \tag{8d}$$

Notice that $8s_1s_2 - 5s_2 - 5s_1 + 2$ is positive for s_1, $s_2 > 1$. to see this, simply substitute $s_1 = 1 + x$ and $s_2 = 1 + y$, with $x, y > 0$. The global firm will produce in both markets under a uniform price as long as q_{1G}^U, $q_{2G}^U > 0$, which translates into:

$$\frac{(2s_1 - 1)(s_2 - 1)}{6s_1s_2 - 4s_2 - 3s_1 + 1} < \frac{a_1}{a_2} < \frac{6s_1s_2 - 3s_2 - 4s_1 + 1}{(s_1 - 1)(2s_2 - 1)}.$$

Also notice that the total local quantity is proportional to total quantity produced by the global firm, in whatever regime, $K = U$, D, because

$$\left(q_{1L}^K + q_{2L}^K\right) = 2\left(q_{1G}^K + q_{2G}^K\right).$$

This way, the sign of the total output change is the same as that of the change in the global firm's output.

3 Welfare analysis

In this section, we present the main results of the paper. Given the equilibrium outcomes that we computed int he previous section, we are interested in verifying whether there are parameter values such that the following three results hold: i) total output decreases with price discrimination; ii) total welfare increases with price discrimination; and iii) the global firm's profits increase with price discrimination. If this is the case, then, the global firm finds it profitable to engage in price discrimination, total output decreases, and yet welfare decreases. We will analyze each of these conditions in turn.

3.1 Total output

If the global firm is allowed to price discriminate, it will raise its price in one of the two markets and lower it in the other market, relative to uniform pricing. Following Robinson (1933) and Holmes (1989), we refer to the strong market as that in which the global firm raises its price with price discrimination, and to the weak market that in which the firm lowers its price. The following proposition considers how the quantities moves with a price regime change.

Proposition 1 *If the quality level offered by local firms is the same in both markets, that is if $s_2 = s_1$, then both the total output and the quantity produced by the global firm remains unchanged with the change in price regime. If the quality level of the local firm is higher in the strong market, then the global firm reduces its total production under price discrimination. The op-*

posite occurs when the valuation is higher in the weak market.

Proof. Remember that we have introduced the assumption $s_2 \geq s_1$.

Now, let us fix the parameters s_1, s_2 and a_2 in any value, and let a_1 be a variable. Let us define the critical value

$$a_1^* = \frac{a_2(s_2-1)(4s_1-1)}{(s_1-1)(4s_2-1)}. \tag{9}$$

This is the cutoff value that determines which one is the strong market. Specifically, from equations (24b) and (27b), when $a_1 = a_1^*$, then $p_{1G}^D = p_{2G}^D = p_G^U$. In consequence, all the quantities are the same in both price regimes. When $a_1 > a_1^*$, then the strong market is Market 1, because $p_{1G}^D > p_{2G}^D$.

Now we calculate the derivative of the change in total quantity produced by the global firm with respect to a_1. Let us call

$$\Delta Q_G = q_{1G}^D + q_{2G}^D - (q_{1G}^U + q_{2G}^U). \tag{10}$$

Notice that ΔQ_G is a linear function of a_1. Then

$$\begin{aligned} \frac{\partial \Delta Q_G}{\partial a_1} &= \frac{s_1}{4s_1-1} - \frac{2s_1s_2 - s_1 - s_2}{8s_1s_2 - 5s_1 - 5s_2 + 2} = \\ &= \frac{(s_1-1)(s_2-s_1)}{(4s_1-1)(8s_1s_2 - 5s_2 - 5s_1 + 2)} \geq 0 \end{aligned} \tag{11}$$

Let us see first the case $s_2 = s_1$. By equation (11), ΔQ_G does not change when a_1 changes. But, from equation (24b), we know that when $a_1 = a_1^*$, all the quantities under the different regimes are the same. So, if $s_2 = s_1$, then

$\Delta Q_G = 0$ for any value of the parameters. Therefore, the first part of the proposition is proved.

Let us now assume that $s_2 > s_1$. In this case, by equation (11), ΔQ_G increases when a_1 increases. We know that for $a_1 = a_1^*$, $\Delta Q_G = 0$. Then, for $a_1 > a_1^*$, total quantity increases when switching from uniform price to price discrimination. ■

This proposition implies that when $s_2 > s_1$ the global firm's output will decrease with price discrimination for values $a_1 < a_1^*$. Hence, it is left for us to study what is the behavior of total welfare and the global firm's profits when $a_1 < a_1^*$. We intend to verify whether there are values of $a_1 < a_1^*$ such that the global firm's profits increase and yet total welfare increases.

3.2 Welfare

We now proceed to characterize the total welfare function, defined as the sum of consumer surplus and firm's profits. We will compare the two relevant cases, namely when the global firm is constrained to posting a uniform price and when it is allowed to price discriminate across markets.

Specifically, given equations (5) and (8), we can compute welfare for $K = \{U, D\}$

$$
\begin{aligned}
W^K =& s_1 a_1 q_{1L}^K - s_1 \frac{\left(q_{1L}^K\right)^2}{2} + s_2 a_2 q_{2L}^K - s_2 \frac{\left(q_{2L}^K\right)^2}{2} + \\
&+ q_{1G}^K \left(a_1 - q_{1L}^K - \frac{q_{1G}^K}{2}\right) + q_{2G}^K \left(a_2 - q_{2L}^K - \frac{q_{2G}^K}{2}\right).
\end{aligned} \tag{12}
$$

We now define the function

$$
\Delta W = W^D - W^U \tag{13}
$$

The following proposition states under what circumstances total welfare increases with price discrimination.

Proposition 2 *The following claims are true:*

(1) The function ΔW may be expressed as $a_1^2 A + a_2^2 B - 2a_1 a_2 C$, where A, B and C are functions of s_1 and s_2. Furthermore, $A(s_1, s_2) = B(s_2, s_1)$ holds.

(2) If s_1, $s_2 > 1$, then $A > 0$.

(3) The second-degree polynomial $\Delta W(a_1)$ has two positive roots. One of them is $r_1 = a_1^*$. The other root, call it $r_2 = w^*$, satisfies (under the assumption $s_2 > s_1$) $w^* < a_1^*$.

Proof. Claim (1) can be easily seen because all the expressions for quantities are linear functions of a_1 and a_2, and welfare is obtained as a sum of products of at most two quantities. This is robust to changes in the market subindices, hence the function ΔW is symmetric with respect to a change in the subindex.

Regarding claim (2), in order to compute A, let us compute first $\Delta W(a_1, a_2)$ for $a_2 = 0$. That is, let us assume that $a_2 = 0$ in equations (5) and (8). Of course, we are aware of the fact that when $a_2 = 0$ output is zero in market 2. We are merely interested in determining the sign of the A coefficient. Using equation (12), we obtain

$$W^D(a_1, 0) = a_1^2 \frac{s_1(12s_1^2 - s_1 - 2)}{2(4s_1 - 1)^2}.$$

And under the assumption of a uniform price, we have,

$$W^U(a_1, 0) = a_1^2 \left(\frac{3s_1 + 1}{8} + \frac{(s_1 - 1)(s_2 - 1)(8s_1s_2 - 3s_2 - 3s_1 - 2)}{8(8s_1s_2 - 5s_2 - 5s_1 + 2)^2} \right).$$

After some algebra, one can see that the difference can be written as

$$A = \Delta W(a_1, 0) = a_1^2 \frac{(s_1 - 1)^2 R(s_1, s_2)}{8(4s_1 - 1)^2 (8s_1s_2 - 5s_2 - 5s_1 + 2)^2}. \tag{14}$$

where

$$R(s_1, s_2) = 128{s_1}^2 s_2^2 - 16s_1s_2^2 - 144s_1^2 s_2 + 52s_1^2 - 28s_2^2 - 11s_1 + 21s_2 - 2. \tag{15}$$

It is easy to see that $R > 0$, since when replacing $s_1 = 1 + x$ and $s_2 = 1 + y$, the expression R simplifies to

$$R(x, y) = 128x^2y^2 + 112x^2y + 240xy^2 + 36x^2 + 84y^2 + 192xy + 45x + 45y,$$

which is positive as long as x, $y > 0$ or, equivalently s_1, $s_2 > 1$. This proves claim (2).

Regarding claim (3) we know that the polynomial in a_1, $\Delta W(a_1)$, has a root in $r_1 = a_1^*$. This is because we defined a_1^* in equation (9), in such a way that $p_{1G}^D = p_{2G}^D = p_G^U$. This implies that for $a_1 = a_1^*$, the rest of the prices and quantities are the same in the uniform price and in the price discrimination regimes. Hence, it is clear that $\Delta W(a_1^*) = 0$.

Since this polynomial has a real root, it must have another real root, call it w^*. Using this, we know that

$$\Delta W(a_1) = A(a_1 - a_1^*)(a_1 - w^*) = Aa_1^2 - a_1 A(a_1^* + w^*) + a_1^* w^* A.$$

We verified in claim (1) that $\Delta W(a_1) = a_1^2 A + a_2^2 B - 2a_1 a_2 C$, therefore, $a_1^* w^* A = a_2^2 B$. But from claim (2), we know that $B(s_1, s_2) = A(s_2, s_1)$. Therefore, the other root of the polynomial is

$$w^* = \frac{a_2^2}{a_1^*} \frac{A(s_2, s_1)}{A(s_1, s_2)}. \tag{16}$$

Therefore, $w^* > 0$. We still have to prove that $w^* < a_1^*$, or, equivalently,

$$a_2^2 \frac{A(s_2, s_1)}{A(s_1, s_2)} < (a_1^*)^2 .$$

From equation (14), we have that

$$\frac{A(s_2,s_1)}{A(s_1,s_2)} = \frac{\frac{(s_2-1)^2 R(s_2,s_1)}{8(4s_2-1)^2(8s_1s_2-5s_2-5s_1+2)^2}}{\frac{(s_1-1)^2 R(s_1,s_2)}{8(4s_1-1)^2(8s_1s_2-5s_2-5s_1+2)^2}} = \frac{(s_2-1)^2(4s_1-1)^2 R(s_2,s_1)}{(s_1-1)^2(4s_2-1)^2 R(s_1,s_2)}.$$

Recall the definition of a_1^* in equation (9). From this, it may be seen that in order for us to prove this point, we need to show that

$$1 < s_1 < s_2 \Rightarrow R(s_2,s_1) < R(s_1,s_2).$$

But applying equation (15)

$$R(s_1,s_2) - R(s_2,s_1) = 16(s_2-s_1)(8s_1s_2-5s_2-5s_1+2) > 0.$$

This completes the proof. ■

What proposition (2) shows is that total welfare is a quadratic function of a_1, with a positive coefficient on a_1. Moreover, since at a_1^* the outcome is identical in both regimes, it must be the case that $\Delta W = 0$ at a_1^*. Then the other point at which ΔW intersects the horizontal axis is to the left of a_1^*. Hence, welfare increases with price discrimination to the left of this other root and to the right of a_1^*.

3.3 The global firm's profits

We now turn to the analysis of the global firm's profits so as to determine whether the global firm engages in price discrimination, and compare the restrictions on the parameters with those from the analysis of total output and welfare. If the global firm was a monopoly, it is easy to see that the level of π if maximizing over H will never exceed the level of π if the choice variables are from $J \supset H$. Therefore, since a uniform price is a subset of two prices, in principle profits with differentiated prices should exceed those with a uniform price.

However, we are now in an oligopoly setting, and the global firm's profits do not depend on its own prices *only*, but also on the reaction to those prices by the rest of the firms. This –as it is well known–, may make profits be larger if there is a commitment to uniform pricing. In our setting, prices are strategic complements. Relative to uniform pricing, price discrimination softens competition in the weak market, and increases it in the strong market. If $s_1 = s_2$, there are parameter values such that the global firm's profits may actually decrease with price discrimination.

In order to compute the global firm's profits, we will make use of equations (24b), (5), (27b) and (8). Therefore, since marginal costs are zero, we can express the global firm's profits in both regimes, price discrimination and uniform prices as

$$\Pi_G^D = p_{1G}^D q_{1G}^D + p_{2G}^D q_{2G}^D; \quad \Pi_G^U = p_G^U \left(q_{1G}^U + q_{2G}^U \right). \tag{17}$$

With these profit levels at hand, let us define the function that describes the change in the global firm's profits when moving from uniform pricing to price discrimination as

$$\Delta\Pi = \Pi_G^D - \Pi_G^U \tag{18}$$

Once the $\Delta\Pi$ function has been defined, we may now formulate the following proposition, which is analogous with the previous one, which dealt with total welfare.

Proposition 3 *The following claims are true:*

(1) The function $\Delta\Pi$ can be expressed as $a_1^2 M + a_2^2 N - 2a_1 a_2 H$, where M, N and H are functions of s_1 and s_2. Furthermore, it is the case that $M(s_1, s_2) = N(s_2, s_1)$.

(2) If s_1, $s_2 > 1$, then $M > 0$.

(3) The second-degree polynomial $\Delta M(a_1)$ has two positive roots. One of them is $r_1 = a_1^*$. The other root, call it $r_2 = \pi^*$, is such that (assuming that $s_1 < s_2$) $w^* < \pi^* < a_1^*$, where w^* was computed in proposition 2.

Proof. Claim (1) may be proved identically as proposition 2.

Regarding claim (2), in order to obtain the expression for M, we will compute $\Delta\Pi(a_1, a_2)$ for $a_2 = 0$. Therefore, we will assume that $a_2 = 0$ in equations (24b), (5), (27b) and (8). Using equation 17, we obtain

$$\Pi^D(a_1, 0) = a_1^2 \frac{s_1(s_1 - 1)}{(4s_1 - 1)^2}.$$

Under the assumption of uniform prices, we obtain

$$\Pi^U(a_1, 0) = a_1^2 \frac{(s_1 - 1)(s_2 - 1)(2s_1 s_2 - s_1 - s_2)}{(8s_1 s_2 - 5s_1 - 5s_2 + 2)^2}.$$

After some algebra, it may be seen that the difference is

$$M = \Delta\Pi(a_1, 0) = a_1^2 \frac{(s_1 - 1)^2 T(s_1, s_2)}{(4s_1 - 1)^2 (8s_1 s_2 - 5s_2 - 5s_1 + 2)^2}. \tag{19}$$

where

$$T(s_1, s_2) = 32s_1^2 s_2^2 - 32s_1^2 s_2 - 16s_1 s_2^2 + 9s_1^2 - s_2^2 + 10s_1 s_2 - 3s_1 + s_2. \tag{20}$$

It may be readily verified that $T > 0$, since when substituting $s_1 = 1 + x$ and $s_2 = 1 + y$, the expression for T boils down to

$$T(x, y) = 32x^2 y^2 + 32x^2 y + 48xy^2 + 9x^2 + 15y^2 + 42xy + 9x + 9y,$$

which is positive as long as x, $y > 0$, or, equivalently s_1, $s_2 > 1$. This completes the proof of claim (2).

Finally, regarding claim (3) we know that the polynomial in a_1, $\Delta\Pi(a_1)$, has one root in $r_1 = a_1^*$. This is the case because we have defined a_1^* in equation (9), so that $p_{1G}^D = p_{2G}^D = p_G^U$. This implies that for $a_1 = a_1^*$, all remaining prices and quantities are the same in both regimes. Therefore, it is clearly the case that $\Delta\Pi(a_1^*) = 0$.

Since this polynomial has one real root, it must have another root, call it π^*. This allows us to conclude that

$$\Delta\Pi(a_1) = M(a_1 - a_1^*)(a_1 - \pi^*) = Ma_1^2 - a_1 M(a_1^* + \pi^*) + Ma_1^*\pi^*.$$

We have seen in the proof of claim (1) that $\Delta\Pi(a_1) = a_1^2 M + a_2^2 N - 2a_1a_2H$, hence, $Ma_1^*\pi^* = a_2^2 N$. But from the proof of claim (2), we know that $N(s_1, s_2) = M(s_2, s_1)$. Therefore, the other root of the polynomial is

$$\pi^* = \frac{a_2^2}{a_1^*}\frac{M(s_2, s_1)}{M(s_1, s_2)}.$$

And therefore, $\pi^* > 0$. We are still left with proving that $w^* < \pi^* < a_1^*$. Recalling te definition of w^*, from equation (16), we have to prove that

$$\frac{a_2^2}{a_1^*}\frac{A(s_2, s_1)}{A(s_1, s_2)} < \frac{a_2^2}{a_1^*}\frac{M(s_2, s_1)}{M(s_1, s_2)} < a_1^*.$$

The proof for $\pi^* < a_1^*$ is analogous to that in proposition 2. From equation (19), we have that

$$\frac{M(s_2, s_1)}{M(s_1, s_2)} = \frac{\frac{(s_2-1)^2 T(s_2,s_1)}{(4s_2-1)^2(8s_1s_2-5s_2-5s_1+2)^2}}{\frac{(s_1-1)^2 T(s_1,s_2)}{(4s_1-1)^2(8s_1s_2-5s_2-5s_1+2)^2}} = \frac{(s_2-1)^2(4s_1-1)^2 T(s_2, s_1)}{(s_1-1)^2(4s_2-1)^2 T(s_1, s_2)}.$$

Recall the definition of a_1^* in equation (9). From this that we conclude that in order to prove this claim, we need to show that

$$1 < s_1 < s_2 \Rightarrow T(s_2, s_1) < T(s_1, s_2).$$

However, applying equation (15)

$$T(s_1, s_2) - T(s_2, s_1) = 2(s_2 - s_1)(8s_1s_2 - 5s_2 - 5s_1 + 2) > 0.$$

In order to see that $w^* < \pi^*$, we simply have to check that

$$\frac{R(s_2, s_1)}{R(s_1, s_2)} < \frac{T(s_2, s_1)}{T(s_1, s_2)}.$$

Since

$$R(s_1, s_2)T(s_2, s_1) - R(s_2, s_1)T(s_1, s_2) =$$

$$= 2(4s_1 - 1)(s_2 - s_1)(4s_2 - 1)(8s_1s_2 - 5s_2 - 5s_1 + 2)^2,$$

This completes the proof. ■

3.4 Price discrimination and welfare

The previous three subsections analyzed conditions such that total output decreases, welfare increases, and the global firm's profits increase with price discrimination. We now combine these results and summarize them in the following corollaries. Furthermore, we illustrate the intuition by means of some numerical examples.

Corollary 4 *If $s_2 = s_1$, then total output is the same under price discrimi-*

nation as under a uniform price. Total welfare and the profits of the global firm are greater under price discrimination, except in the case $a_1 = a_2$, when they are equal.

If $s_1 = s_2$, total quantity does not change when the global firm engages in price discrimination, although welfare increases. Moreover, the quality level of the products produced by the two local firms are the same, both being greater than the quality of the product produced by the global firm. Notice that if $s_1 = s_2$, then the critical value $a_1^* = a_2$, and thus market 2 will be the strong market as long as $a_2 > a_1$. Without loss of generality, assume that this is the case. Then, relative to uniform pricing, the global firm raises its price in the strong market (market 2) and lowers it in the weak market (market 1).

As a consequence of the global firm's engaging in price discrimination, total output increases in the weak market, and decreases in the strong market, but the addition across markets remains constant. Furthermore, by examining equations (5) and (8) we immediately verify that the local firm in the strong market raises its output if the global firm price discriminates, whereas the local firm lowers its output under price discrimination. Not only that, but the absolute value of the change in sales by the two local firms is the same. Hence, since the global firm's output remains constant and the local firm's output in the strong market increases by the same amount as the decrease in the local firm output in the weak market, welfare increases with price discrimination, even though total output remains constant. If $a_1 = a_2$

then the two markets are identical, and price discrimination has no effect on welfare.

In order to illustrate these ideas, let us consider some numerical examples. In all cases, let us fix $a_2 = 1$ and consider different realizations of the rest of the parameters. For instance, if $s_1 = s_2 = 1.1$, if $a_1 = \frac{1}{2}$, then market 1 is the weak market (and market 2 is the strong market). In fact, when the global firm is allowed to price discriminate, it optimally chooses $p_{1G}^D = 0.0147$ and $p_{2G}^D = 0.0294$ in contrast with a uniform price $p_G^U = 0.022$. The effect of price discrimination is that the local firm's output increases in the strong market and decreases in the weak market, with total output remaining constant relative to the case of uniform pricing. Specifically, in the strong market, the local firm increases its output from $q_{2L}^U = 0.6103$ to $q_{2L}^D = 0.647$, whereas in the weak market the local firm reduces its output from $q_{1L}^U = 0.3602$ to $q_{1L}^D = 0.3235$. All this leads to a 0.11% increase in welfare.

Now, if $a_1 = 2$ then market 2 is the weak market. In this case, the local firm increases its output in the strong market from $q_{1L}^U = 1.22$ to $q_{1L}^D = 1.2921$, whereas in the weak market, the local firm reduces its output from $q_{2L}^U = 0.7206$ to $q_{2L}^D = 0.647$. As in the previous case, total output remains constant, but welfare increases by 0.108%.

Finally, throughout this paper, we have assumed that the level of quality of the global firm's product is lower than those of the local firms. A natural question arises on the welfare implications of the global firm's product having a higher quality level than those of the local firms. We show in the appendix that, for the case $s_1 = s_2$, the results are reversed relative to the case that we consider in the paper.

The next corollary deals with the case of different quality levels in the two markets.

Corollary 5 *If $s_2 > s_1$, there is a threshold value of a_1, such that for values of a_1 that do not exceed that threshold value, welfare increases while total output decreases with price discrimination.*

If $s_1 < s_2$ the level of quality of the product that the local firm sells in market 2 exceeds that of the product sold by the local firm in market 1. Unlike the case $s_1 = s_2$, and as seen in the previous section, total output is not always constant. Hence, whether market 1 is the strong market depends on the comparison between a_1 and a_1^*, where a_1^* does not equal a_2, as it was the case when $s_1 = s_2$. While there are some substitution effects involved in both markets, since total output is not constant as in the case $s_1 = s_2$, the welfare comparison is not as straightforward. Then we may come up with four cases, which are listed as:

1. If $a_1 < w^*$ price discrimination decreases quantity, but welfare increases.

2. If $w^* < a_1 < \pi^*$, price discrimination decreases both quantity and welfare.

3. If $\pi^* < a_1 < a_1^*$, the global firm is better off with a uniform price.

4. If $a_1^* < a_1$, price discrimination increases both quantity and welfare.

Figure 1 illustrates graphically these cases, by plotting the ΔW, $\Delta\Pi_G$, and ΔQ_G functions against a_1. Recall that the functions are defined as the differences between the cases of price discrimination and uniform price. This means, for example, that whenever the $\Delta\Pi_G$ function is above the horizontal axis, then the global firm's profits increase with price discrimination. This allows us to see for what values of a_1 total welfare, total output, and the global firm's profits increase or decrease with price discrimination. In particular, we can immediately verify that for values $a_1 < a_1^*$, output decreases and yet welfare increases with price discrimination, while the global firm being better off with price discrimination.

The ΔQ_G function is a linear function of a_1. Recall that total output is proportional to the global firm's output, both under uniform price and under price discrimination. Therefore, the sign of ΔQ_G is the same as the sign of the change in total output. In the graph we see that when $s_1 < s_2$ total output increases when the strong market is market 1, that is, when $a_1 > a_1^*$. Furthermore, when that is the case, both the global firm's profits and welfare increase with price discrimination.

In contrast, when market 1 is the weak market (provided that $s_1 < s_2$) then there is an interval of values of a_1 such that the global firm's profits decrease with price discrimination and another interval such that total welfare decrease with price discrimination. These intervals are $[\pi^*, a_1^*]$ and $[w^*, a_1^*]$ respectively. Hence, if $a_1 \in [\pi^*, a_1^*]$ then the global firm does not engage in price discrimination. However, if $a_1 \in [w^*, \pi^*]$ then the global firm optimally uses price discrimination, and welfare decreases. Finally, if $a_1 < w^*$ the global firm price discriminates, total output decreases and welfare increases.

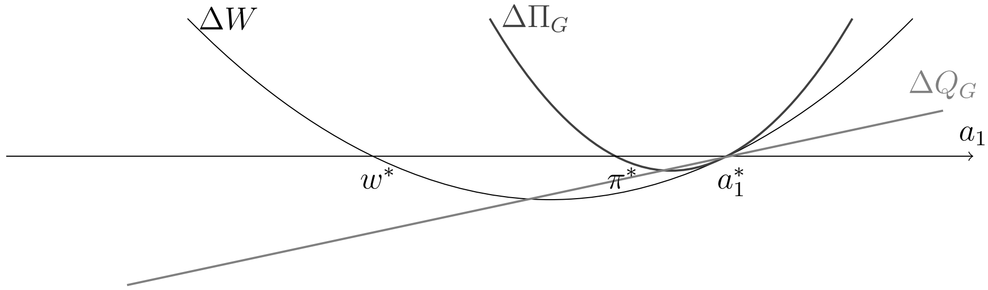

Figure 1: Output, profits, and welfare change as a function of a_1

In order to assess how wide these intervals are, figure 2 plots the values of a_1^*, w^* and π^* as a function of s_2, for $s_1 = 1.1$. For every value of s_2, if a_1 is greater than the maximum of a_1^*, w^* and π^*, then price discrimination increases both output and welfare. If it is below the minimum of a_1^*, w^* and π^*, output decreases, but welfare increases.

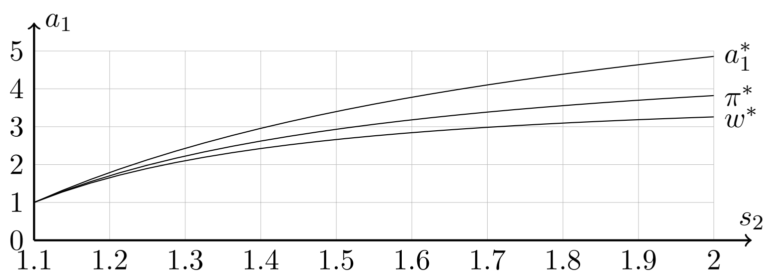

Figure 2: a_1^*, π^* and w^* as a function of s_2, given that $s_1 = 1.1$ and $a_2 = 1$.

For instance, if $s_1 = 1$ and $s_2 = 1.5$ (with $a_2 = 1$), then $a_1^* = 3.4$. Hence, if $a_1 = 2$, then market 1 is the weak market, since $2 < a_1^*$. In this case, total output decreases by 0.784%, although total welfare increases by 0.047%. In the strong market, the local firm raises its output with price discrimination from $q_{2L}^U = 0.5682$ to $q_{2L}^U = 0.6$, whereas in the weak market the local firm decreases its output from $q_{1L}^U = 1.3409$ to $q_{1L}^D = 1.2941$. While total output decreases, the fact that the local firm produces more in the strong market makes welfare to increase.

In contrast, if $a_1 = 2.8$ and the rest of the parameters remain constant, welfare decreases and the global firm's profits increase with price discrimination. Hence, in the case, the global firm has the incentive to introduce price discrimination, this pricing policy being detrimental to welfare. If $a_1 = 3.1$, the global firm is better off setting a uniform price, which is better in terms of welfare than price discrimination. Finally, if $a_1 = 4$, then market 2 is now the strong market and total output, welfare, and the global firm's profits all increase with price discrimination.

4 Concluding comments

This paper revisits the question of the welfare implications of third-degree price discrimination, incorporating a vertical differentiation component. We propose a theoretical model with two markets in which a firm that is present in both markets, call it the global firm, competes against two firms active in one of the markets only, call them the local firms. These are assumed to offer a higher-quality product than the global firm. We find that there are parameter values such that, although price discrimination leads to a decrease in total output, total welfare may increase. This is because the positive allocation effect in the strong market more than offsets the negative misallocation effect in the weak market.

We believe that our results have important policy implications. While we do not argue that this inverse relationship between total output and welfare must hold in all cases, our contribution tries to highlight the idea that the competition authority can not take the basic result that an output expansion is a necessary condition for welfare to increase with price discrimination at face value. There are instances in which this result may fail to hold, for instance when there are differences in quality and interactions among markets, as we show in our paper.

References

Adachi, T. and N. Matsushima (2014). The welfare effects of third-degree price discrimination in a differentiated oligopoly. *Economic Inquiry 52*(3), 1231 – 1244.

Aguirre, I., S. Cowan, and J. Vickers (2010). Monopoly price discrimination and demand curvature. *American Economic Review 100*(4), 1601 – 1615.

Alexandrov, A. and J. Deb (2012). Price discrimination and investment incentives. *International Journal of Industrial Organization 30*(6), 615 – 623.

Asplund, M., R. Eriksson, and N. Strand (2008). Price discrimination in oligopoly: Evidence from regional newspapers. *Journal of Industrial Economics 56*(2), 333 – 346.

Basker, E. (2011). Does Wal-Mart sell inferior goods?. *Economic Inquiry 49*(4), 973 – 981.

Cooper, D. (2003). Findings from the competition commission's inquiry into supermarkets. *Journal of Agricultural Economics 54*(1), 127 – 143.

Cowan, S. (2007). The welfare effects of third-degree price discrimination with nonlinear demand functions. *RAND Journal of Economics 38*(2), 419 – 428.

Dastidar, K. G. (2006). On third-degree price discrimination in oligopoly. *Manchester School 74*(2), 231 – 250.

Dobson, P. W. and M. Waterson (2005). Chain-store pricing across local markets. *Journal of Economics and Management Strategy 14*(1), 93 – 119.

Galera, F., M. Kinateder, and P. Mendi (2014). The cost effect in third-degree price discrimination. *Journal of Economics (Zeitschrift fur Nationalokonomie) 113*(2), 175 – 186.

Galera, F. and J. M. Zaratiegui (2006). Welfare and output in third-degree price discrimination: A note. *International Journal of Industrial Organization 24*(3), 605 – 611.

Holmes, T. J. (1989). The effects of third-degree price discrimination in oligopoly. *American Economic Review 79*(1), 244 – 250.

Ikeda, T. and T. Toshimitsu (2010). Third-degree price discrimination, quality choice, and welfare. *Economics Letters 106*(1), 54 – 56.

Jorge, S. F. and C. P. Pires (2013). Does banning price discrimination promote entry and increase welfare? a model of differentiated-product duopoly with asymmetric markets. *Manchester School 81*(4), 660 – 681.

Matsa, D. A. (2011). Competition and product quality in the supermarket industry. *Quarterly Journal of Economics 126*(3), 1539 – 1591.

Nguyen, X. (2014). Monopolistic third-degree price discrimination under vertical product differentiation. *Economics Letters 125*(2), 153 – 155.

O'Brien, D. P. (2014). The welfare effects of third-degree price discrimination in intermediate good markets: The case of bargaining. *RAND Journal of Economics 45*(1), 92 – 115.

Robinson, J. (1933). *The economics of imperfect competition* (1 , reprint ed.). London: Macmillan and Co. by Joan Robinson; ; 22 cm.

Schmalensee, R. (1981). Output and welfare implications of monopolistic third-degree price discrimination. *American Economic Review 71*(1), 242 – 247.

Schwartz, M. (1990). Third-degree price discrimination and output: Generalizing a welfare result. *American Economic Review 80*(5), 1259 – 1262.

Shaked, A. and J. Sutton (1982). Relaxing price competition through product differentiation. *Review of Economic Studies 49*(1), 3 – 13.

Shaked, A. and J. Sutton (1983). Natural oligopolies. *Econometrica 51*(5), 1469 – 1483.

Shepard, A. (1991). Price discrimination and retail configuration. *Journal of Political Economy 99*(1), 30 – 53.

Stole, L. (2007). Price discrimination and competition. In M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization* (1 ed.), Volume 3, Chapter 34, pp. 2221–2299. Elsevier.

Varian, H. R. (1985). Price discrimination and social welfare. *American Economic Review 75*(4), 870 – 875.

Yoshida, Y. (2000). Third-degree price discrimination in input markets: Output and welfare. *American Economic Review 90*(1), 240 – 246.

A The global firm's product being of higher quality

If the product sold by the global firm is of higher quality than those sold by local firms then equations 1 y 2 become:

$$s_i(a_i - x_i) - p^k_{iG} = a_i - x_i - p^k_{iL} \Leftrightarrow x_i = a_i - \frac{p^k_{iG} - p^k_{iL}}{s_i - 1}. \tag{21}$$

$$x_{iG} = a_i - \frac{p^k_{iG} - p^k_{iL}}{s_i - 1}, \quad x_{iL} = a_i - x_i - p^k_{iL}. \tag{22}$$

We now proceed to compute the market outcome, assuming that $s_1 = s_2 = s$, in order to simplify the analysis. We will verify that the sign of the welfare change is reversed relative to the case of the local firm's products being of superior quality.

A.1 Market outcome: Price discrimination

Given the expression for the demand functions, profits under price discrimination are:

$$\Pi^D_L = p^D_{1G}\left(a_1 - \frac{p^D_{1G} - p^D_{1L}}{s - 1}\right) + p^D_{2G}\left(a_2 - \frac{p^D_{2G} - p^D_{2L}}{s - 1}\right); \tag{23a}$$

$$\Pi^D_{1L} = p^D_{1L}\left(\frac{p^D_{1G} - p^D_{1L}}{s - 1} - p^D_{1L}\right); \;\; \Pi^D_{2L} = p^D_{2L}\left(\frac{p^D_{2G} - p^D_{2L}}{s - 1} - p^D_{2L}\right), \tag{23b}$$

and equilibrium prices are:

$$p_{1G}^D = \frac{2a_1 s(s-1)}{4s-1}, \qquad p_{2G}^D = \frac{2a_2 s(s-1)}{4s-1} \tag{24a}$$

$$p_{1L}^D = \frac{a_1(s-1)}{4s-1}, \qquad p_{2L}^D = \frac{a_2(s-1)}{4s-1}. \tag{24b}$$

Once the equilibrium prices have been obtained, output levels in equilibrium are:

$$q_{1G}^D = \frac{2a_1 s}{4s-1}, q_{2G}^D = \frac{2a_2 s}{4s-1}, q_{1L}^D = \frac{a_1 s}{4s-1}, q_{2L}^D = \frac{a_2 s}{4s-1}. \tag{25}$$

A.2 Market outcome: Uniform price

As it was done in section 2.2.2, the global firm is constrained to posting a uniform price, p_G^U, whereas local firms choose prices p_{iL}^U. Then, profits may be written, as a function of these prices, as

$$\Pi_{1L}^U = p_{1L}^U\left(\frac{p_G^U - p_{1L}^U}{s-1} - p_{1L}^U\right); \ \Pi_{2L}^U = p_{2L}^U\left(\frac{p_G^U - p_{2L}^U}{s-1} - p_{2L}^U\right); \tag{26a}$$

$$\Pi_G^U = p_G^U\left(a_1 - \frac{p_G^U - p_{1L}^U}{s-1}\right) + p_G^U\left(a_2 - \frac{p_G^U - p_{2L}^U}{s-1}\right). \tag{26b}$$

Solving the first-order conditions of these three maximization problems, $\frac{\partial \Pi_{1L}^U}{\partial p_{1L}^U} = 0$, $\frac{\partial \Pi_{2L}^U}{\partial p_{2L}^U} = 0$ and $\frac{\partial \Pi_G^U}{\partial p_G^U} = 0$, the equilibrium prices chosen by the global and the local firms are:

$$p_{1L}^U = p_{2L}^U = \frac{(a_1 + a_2)(s - 1)}{2(4s - 1)} \tag{27a}$$

$$p_G^U = \frac{(a_1 + a_2)s(s - 1)}{4s - 1}. \tag{27b}$$

These three equations, together with equation (22) allow us to compute the equilibrium quantities. These are:

$$q_{1L}^U = q_{2L}^U = \frac{s(a_1 + a_2)}{2(4s - 1)}, \tag{28a}$$

$$q_{1G}^U = \frac{2s(a_1 + a_2) + a_1 - a_2}{2(4s - 1)}, \tag{28b}$$

$$q_{2G}^U = \frac{2s(a_1 + a_2) + a_2 - a_1}{2(4s - 1)}. \tag{28c}$$

A.3 Welfare

By comparing equations (25) and (28), it is easy to see that if $s_1 = s_2 = s$, total output produced by the local firms and by the global firm is the same under both regimes. Regarding welfare, this is given by the following expressions:

$$W^D = \frac{(a_1^2 + a_2^2)s(12s^2 - s - 2)}{2(4s - 1)^2}$$

and

$$W^U = \frac{(a_1^2 + a_2^2)s(28s^2 - 9s - 1) - 2a_1a_2(s-1)s(4s-3)}{4(4s-1)^2}$$

Computing the difference between the two welfare levels, we see that whenever $a_1 \neq a_2$, welfare will be higher under a uniform price:

$$W^D - W^U = -\frac{(a_1 - a_2)^2(s-1)s(4s-3)}{4(4s-1)^2} < 0$$