Granular Matter
DOI 10.1007/s10035-015-0583-z

ORIGINAL PAPER

Clogging of granular materials in narrow vertical pipes discharged at constant velocity

Alvaro Janda[1,2] · Iker Zuriguel[1] · Angel Garcimartín[1] · Diego Maza[1]

Received: 21 November 2014

Abstract We report experimental results for pipe flow of granular materials discharged through vertical narrow tubes by means of a conveyor belt placed at the bottom. When the diameter of the tube is not much larger than the particle size, the system clogs due to the development of hanging arches that are able to support the weight of the grains above them. We find that the time it takes to develop a stable clog decays exponentially, which is compatible with a clogging probability that remains constant during the discharge. From this, and making an analogy with the discharge of silos, we introduce the avalanche size, measured in terms of the number of discharged tubes before the system clogs. The mean avalanche size is found to increase as the tube diameter is enlarged, the velocity of the conveyor belt grows, and the tube tilt deviates from the vertical.

Keywords Clogging · Granular · Vertical transport · Underground mining · Pipe · Ore pass · Hang-up

1 Introduction

When a granular material is discharged by gravity from a silo, if the outlet is only a few times larger than the particles, arches might develop at the narrowing arresting the flow. In recent years, an important effort has been made in order to understand this problem [1–22]. As a result, it is well known that the distribution of avalanche sizes (number of particles discharged between two successive clogging events) displays an exponential decay [2–4,6,12]. This implies that the clogging probability is constant during the whole avalanche. From the exponential distribution, the first moment can be calculated (the average avalanche size) and used to show the strong dependence of clogging on the size ratio between the outlet (D) and the particles (d). Importantly, for spherical beads in a 3D silo, a divergence of the avalanche size was found for an outlet diameter about five times the bead diameter [3]. The existence of this critical outlet size was challenged by To [4]. In a two dimensional silo, they showed how several empirical fits agree reasonably well with the experimental data. Among these, some were compatible with the existence of a critical outlet but others were not. In any case, all the fittings proposed reveal a dramatic increase of the avalanche size as the outlet size increases which prompts us to state that, in practical terms, two regions can be distinguished: one for small outlets where clogging is frequent, and another one for sufficiently large orifices where clogging can be safely ignored [22]. Interestingly, this transition has been also demonstrated for silos discharged through inclined orifices [9,16].

Remarkably, in all the abovementioned works (and others dealing with granular suspensions flowing through pores [23,24], floating particles passing through constrictions [25,26], and live beings crowding in front of narrow doors [27,28]) the clogging arches develop at the very narrowing. Indeed, this geometrical constriction seems to be a necessary condition to allow arch stabilization as all the arches rest at the hopper walls or at the bottom of the silo. A clear evidence of the importance of the bottom walls to allow arch development and stabilization is found in [1] as the probability of clogging becomes dramatically reduced as the angle of the walls increases. Nevertheless, clogging might also develop in vertical tubes without constrictions. This is a common problem in underground mining where the ore or waste is transported

✉ Iker Zuriguel
iker@fisica.unav.es

1 Departamento de Física, Facultad de Ciencias, Universidad de Navarra, 31080 Pamplona, Spain

2 Present Address: School of Engineering, University of Edinburgh, King's Buildings, Edinburgh EH9 3JL, UK

Published online: 23 August 2015

through pipes from one level of the mine to another using the gravity driving force [29,30]. As in the case of silos, it seems that the blockage occurrence mainly depends on the ratio $\phi = D/d$, where D is the pipe diameter and d the characteristic size of the particles. In the literature, the few works that can be found about this topic show a lack of agreement for a safe value of ϕ which warrants absence of clogging [31–35]; the figures range from 3 to 10, although the most accepted limit is $\phi > 5$ [33]. One of the most extensive investigations on this aspect can be found in [34]. In this work, Hadjigeorgiou et al. used discrete element method simulations to explore how the safe value of ϕ is affected by different configurations of the system such as the shape of the particles and the inclination and shape of the pipe. Their results show the importance of these parameters on the development of clogging and they propose some guidelines for choosing the value of ϕ that ensures *free flow* which is defined when, during a maximum of 10 simulations, there was not registered any clogging event. Very recently it has been numerically shown that, for a monodisperse sample, applying a helical texture to the pipes inner wall homogenizes the flow along the entire pipe reducing the probability of clogging [35].

In this work we perform, for the first time, systematic experiments of clogging during the discharge of a narrow pipe at constant velocity. We will start by describing the experimental setup. Then, we will discuss the exponential distribution of the time that it takes the system to be clogged and we will explain the reasons that prompted us to measure the avalanche size in number of discharged tubes. From this, we will calculate the dependence of the mean avalanche size on the pipe diameter, pipe inclination, and the velocity of the conveyor belt that extracts the material from the bottom.

2 Experimental set-up

The experimental set-up consists of a long narrow pipe full of granular material which is discharged by means of a conveyor belt at the bottom that extracts the material at a constant rate (Fig. 1). The pipe was made of transparent polymethacrylate, hence allowing the observation of clogs. The supporting structure was a metal sheet tilted at an angle θ that can be changed at will to allow exploring the influence of pipe inclination on clogging. Unless stated, the inclination used for all the data presented here is $\theta = 70°$. Different tubes of inner diameter (D) 32, 36, 40 and 42 mm and length $L = 1.35$ m were used for the different trials. The belt was placed at an angle that makes around 90° with the pipe and placed at an height such as the distance between the exit of the pipe and the surface of the conveyor belt was around 2 times the inlet diameter of the pipe. This configuration avoids the formation of clogs between the exit of the pipe and the surface of the conveyor belt. The granular material formed a conical pile on the surface of the conveyor belt due to the outpouring of grains from the pipe. In all cases, the width of the conveyor belt and the distance from the outlet of the pipe to the back wall of the conveyor belt were large enough to ensure that the pile of granular material did not touch the walls. The

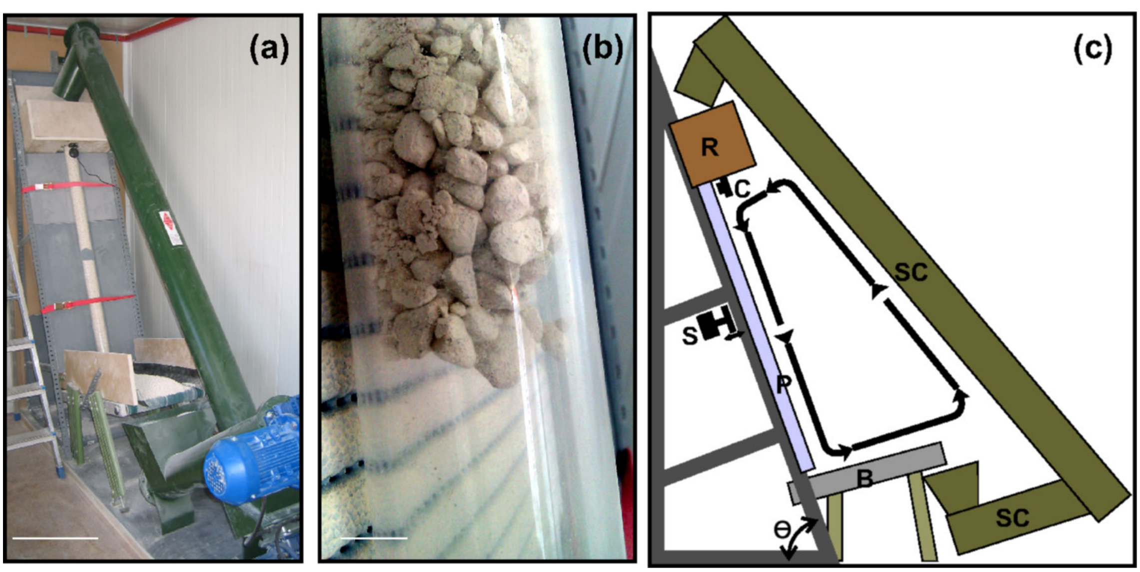

Fig. 1 **a** Picture of the experimental set-up where a blockage can be observed at the middle of the tube. The length of the *white solid line* is equivalent to 500 mm. **b** Close view of a dome that blocks the flow. The *scale bar* corresponds to 10 mm. **c** Sketch of the experimental set-up where the *arrow loop* represents the movement of the material. *B* conveyor belt, *C* camera, *P* pipe, *R* reservoir of granular material, *S* shaker, *SC* screw conveyors, θ tilt angle of the pipe

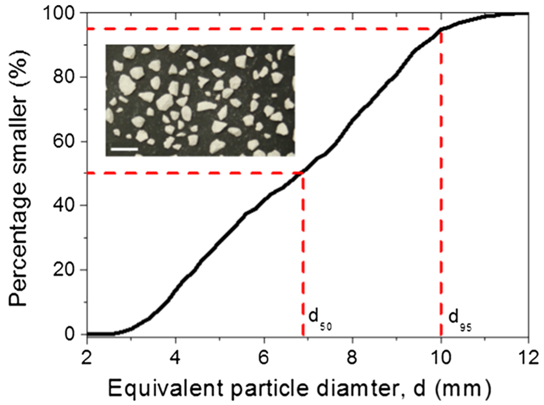

Fig. 2 Particle size distribution obtained by image analysis as explained in the text. *Dashed lines* indicate $d_{50} = 6.9$ mm and $d_{95} = 10.0$ mm. *Inset* picture of the granular material used in this work. The length of the *white solid line* is equivalent to 20 mm

velocity of the belt, which controls the discharge flow-rate, was kept constant for the duration of each trial and governed by an adjustable frequency drive. The extraction velocity for all the experiments was $v = 9.64$ mm/s and was only varied for the measurements that will be shown in Sect. 4 where the influence of this variable in the clogging probability is described. At the end of the conveyor belt, the stones are poured into a system of two connected screw conveyors that transported the grains to a reservoir placed at the top of the pipe, so that the material is refilled into the tube. The level of the granular material inside the top reservoir was always kept at least twenty times the mean diameter of the particles. This helped to damp any potential perturbation on the pipe due to the material poured by the screw conveyors. Furthermore, this protocol allowed for keeping the pipe completely full during the trials. The duty cycle of the screw conveyors was tuned to ensure that the previous criterion for the level of the material in the reservoir was met.

The granular material is a polydisperse non-spherical ballast of limestone as shown in Fig. 2. The size of the particles is defined by their equivalent diameter (d) calculated as $d = \sqrt{4A_p/\pi}$, where A_p is the projected area of the particles which was measured from pictures as the one shown in the inset of Fig. 2. The pictures were converted to gray scale and binarized using a threshold value suitable to distinguish the particles from the background. Afterwards, the projected area of each individual particle was measured counting the number of pixels of each particle in the image. The equivalent diameter of the ballasts range from 2 to 12 mm and follow the distribution shown in Fig. 2. The polydispersity of the material, defined as the coefficient of variation of the particle size distribution, is 35 % and the sizes corresponding to the 50 and 95 % of the cumulative particle size distributions are used as characteristic sizes ($d_{50} = 6.9$ mm and $d_{95} = 10.0$ mm respectively). One of the particularities of this material is that it slowly degrades. To minimize this effect, we initially conducted a characterization of the potential degradation by measuring the particle size distribution of the sample versus the number of discharged pipes. From this, we defined a safe total number of 200 discharged pipes for replacing the material. In this way, we assure a variation of <2 % in all the parameters used to characterize the particle size distribution (i.e. d_{50}, d_{95} and polydispersity).

The measuring protocol was as follows. First, the pipe was manually filled through the reservoir at the top which was also partially filled. Then, the conveyor belt was switched on and the material started to flow through the pipe by gravity. This instant was registered and defined the beginning of the avalanche (t_{start}). When eventually, a hanging arch clogged the pipe at any position, all the material above it stopped. This precise moment was detected by means of a real-time analysis of the images acquired by a web-cam placed at the very top of the pipe at 25 frames per second. When two consecutive images did not show any relative difference for at least 3 s, the time corresponding to the first two equal images was registered as the end of the 'avalanche' (t_{end}). Then, an electric vibrator placed at the back of the supporting structure was activated for a lapse of 3 s. The vibrator hit several times the back side of the metal sheet that supported the pipe. This supplied a vibration to the pipe that was strong enough to break the clog. After the vibration halted, the conveyor belt was switched on and the extraction of material started again. This time was taken as the beginning of the next 'avalanche' and the measurement loop started again. From the registered times of the beginning and the end of the avalanches we calculate their duration as: $t_{flow} = t_{end} - t_{start}$. Note that the screw conveyors kept the top reservoir partly filled with granular material during the trials so the initial manual filling of the pipe was not conducted again until the material was replaced. In order to avoid any possible influence of this initial manual filling process of the pipe, we discarded the avalanches corresponding to the discharge of the first complete pipe.

3 Avalanche size distribution and critical pipe diameter

We start by presenting in Fig. 3 the results of the avalanche duration distributions. The number of avalanches measured for each experimental condition was at least 200, except for the larger pipe diameter for which 100 avalanches were measured due to a practical time constraint (each avalanche in average could last more than 3 h and sometimes the material had to be replaced due to degradation). The survival or reliability distribution reported in Fig. 3 is computed from the avalanche duration data as the probability of getting an avalanche longer than a certain value of t_{flow}. Note that this

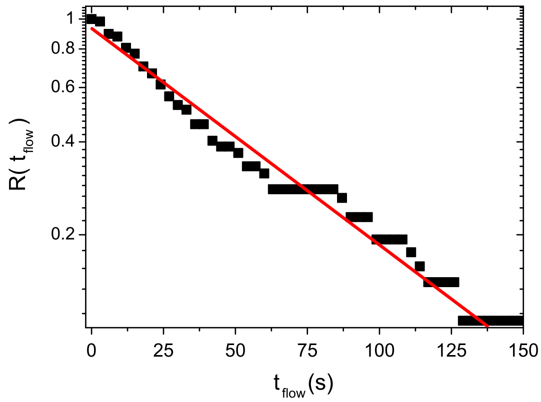

Fig. 3 Reliability distribution $R(t_{flow})$ of the avalanche durations for a pipe diameter $D = 32$ mm, velocity of belt $v = 9.64$ mm/s, and pipe inclination with respect to the horizontal $\theta = 70°$. The *squares* are the experimental data and the *solid line* is an exponential fit. Note the semilogarithmic scale

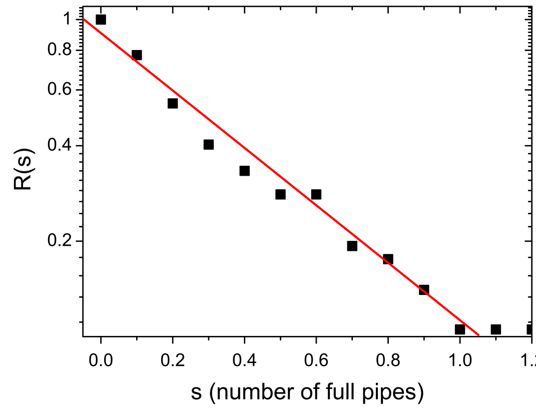

Fig. 4 Reliability distribution $R(s)$ of the avalanche sizes measured in number of pipes discharged, for the same conditions displayed in Fig. 3. The *squares* are the experimental data and the *solid line* is a exponential fit. Note the semilogarithmic scale

distribution is just the complementary of the cumulative distribution function, i.e. $1 - cfd$. The computed reliability distribution displays an exponential tail which is compatible with a constant probability of clogging during the discharge. This absence of memory in the system while it is flowing is also observed in bottlenecks, hence implying that this behavior is not related with the particular geometry of the confined system (either narrowing or pipe).

It is important to remark that t_{flow} might not be the most appropriate parameter to characterize the clogging process [21]. This is so because if the material extraction rate is changed, t_{flow} will be different while the number of locations where a clogging can develop will remain the same. Therefore, a fair comparison of the clogging probability for different velocities of the conveyor could not be conducted. A rescaling parameter that arises naturally is the time that takes for a pipe to be completely emptied t_{tube} or, what is the same, the transit time of a particle in the tube [36]. In our experiments, t_{tube} was obtained by measuring the time that a coloured particle located initially at the top of the tube takes to go along the whole length of the pipe. This measurement was repeated 30 times for each pipe diameter and t_{tube} was defined as the arithmetic average of the measurements. Once t_{flow} and t_{tube} are known, s is calculated as $s = t_{flow}/t_{tube}$ which indeed accounts for the number of full pipes that are discharged before a hanging arch develops and clogs the system. As expected, the reliability distribution of s also displays an exponential tail as shown in Fig. 4. Of course, it has not escaped our notice that s should be strongly dependent on the pipe length (L). This is so because t_{tube} is linearly dependent on L as the extraction rate is constant. Furthermore, enlarging L should lead to a reduction of t_{flow} as the number of clogging positions increases. Having said that, the election of this parameter is convenient from an applied point of view as it straightly correlates with the average amount of material that can be discharged before the development of a clog.

Once we have defined the way in which the avalanche size is measured, we calculate the first moment of the exponential distribution to obtain the mean avalanche size $\langle s \rangle$ (in number of discharged tubes). This magnitude will be used to study the dependence of clogging on the internal diameter of the pipe D. Instead of D, we will use the adimensional ratio $\phi = D/d$, where d is the characteristic particle diameter. In this way, we take d_{50} to define $\phi_{50\%} = D/d_{50}$ and d_{95} to obtain $\phi_{95\%} = D/d_{95}$. In Fig. 5 we present the results of the dependence of mean avalanche size on ϕ. The trends of the two curves are the same as the only difference is the scaling factor d. More interestingly, it is evidenced the huge impact that the tube diameter has on the mean avalanche size. Indeed, the data can be fitted by the equation:

$$\langle s \rangle = \frac{A}{(\phi_c - \phi)^{\gamma}} \tag{1}$$

identical to the one used for the case of clogging in bottlenecks [3]. This fitting implies a divergence of mean avalanche at a critical pipe diameter; i.e. the existence of a critical pipe diameter above which clogging does not occur. As in the case of the silo [4], there are other fitting alternatives that produce non divergent behaviour; but still display a dramatic increase of the average number of pipes discharged before a clog appears when the diameter of the pipe is enlarged. This behaviour would allow to define a tube diameter for which, in practical terms, the probability that a clog appears during the whole lifespan of the pipe would be negligible. Concerning the values of ϕ_c obtained (see caption of Fig. 5) we anticipate that these will strongly depend on the properties of the material used, specially on the particle shape, and the presence of faceted particles for which rotation is frustrated. In this par-

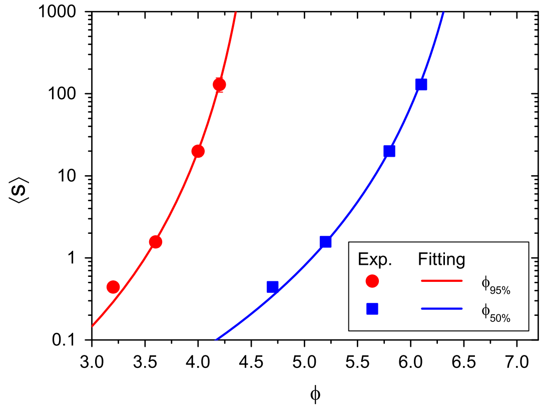

Fig. 5 Semi-logarithmic plot of the mean avalanche size $\langle s\rangle$ in number of discharged tubes as a function of ϕ for a pipe inclination with respect to the horizontal $\theta = 70°$, and a velocity of belt $v = 9.64$ mm/s. *Different symbols* are used to indicate the characteristic size of the particle granulometry (d_{50} and d_{95}) used to calculate the value of ϕ ($\phi_{50\%}$ and $\phi_{95\%}$ respectively). The *solid lines* correspond to the fitting using Eq. 1 with: $A = 21 \pm 5$, $\gamma = 5.5 \pm 0.8$ and $\phi_c = 6.8 \pm 0.1$, for the case of $\phi_{50\%}$; $A = 3 \pm 1$, $\gamma = 5.5 \pm 0.8$ and $\phi_c = 4.7 \pm 0.1$, for the case of $\phi_{95\%}$. The *error bars* of $\langle s\rangle$ are of the order of the size of the symbols

ticular case, the critical diameter for $\phi_{50\%}$ is slightly higher than the value proposed in [33] while the one obtained when using $\phi_{95\%}$ becomes closer. From an applied point of view the later value can be more useful as, in industrial processes, the maximum size of the ballast dumped into the pipe can be limited by the usage of screening infrastructures at the top of the pipe.

4 Influence of system parameters

4.1 Influence of extraction rate

In this section we perform a first check of the effect on clogging of the extraction rate (speed of the conveyor belt). To this end we use the mean avalanche size ($\langle s\rangle$) obtained for a pipe diameter corresponding to $\phi_{50\%} = 5.2$ and we explore how it is affected as the speed of the conveyor belt is increased. Clearly, the faster is the belt, the larger $\langle s\rangle$, and hence the smaller the probability of clogging (Fig. 6). This result suggests that the dynamics of the problem are important as the higher the velocity of the grains, the smaller the number of spatial configurations explored, and hence the smaller the number of these that are able to develop a clog. An alternative reasoning could be that higher extraction rates imply higher amount of the total kinetic energy of the system. Then, at high velocity the clogging probability would be lower as some of the clogging domes that would be stable at low extraction rates are unable to resist until all the energy within the system is dissipated. Note that a similar effect (but at a much smaller scale) was obtained in a silo [21].

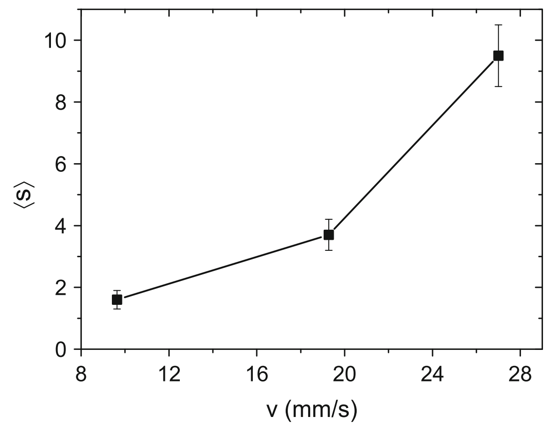

Fig. 6 Mean avalanche size $\langle s\rangle$ as function of the velocity of the conveyor belt for $\phi_{50\%} = 5.2$ (or what is the same, $\phi_{95\%} = 3.6$) and $\theta = 70°$

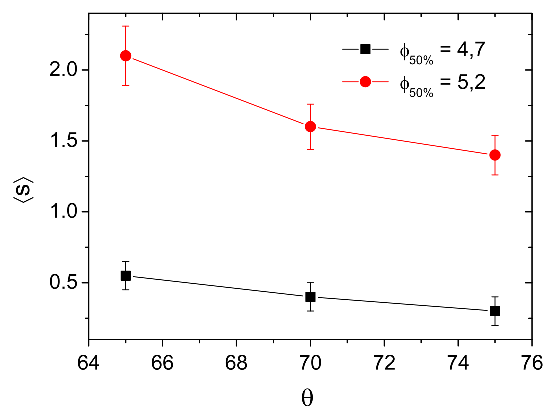

Fig. 7 Mean avalanche size $\langle s\rangle$ as function of the inclination of the pipe (θ) for $v = 9.64$ mm/s and two different internal pipe diameters as indicated by the values of $\phi_{50\%}$ in the legend

4.2 Influence of pipe inclination

Finally, we report results of the influence of the inclination of the pipe on the mean avalanche size (Fig. 7). To this end we perform experiments with two different pipe diameters, for which three different inclinations with respect to the horizontal (θ) are investigated. The velocity of the belt was kept constant at $v = 9.64$ mm/s. Even though the effect of pipe inclination is not as important as the one reported for the velocity of the belt, for both pipe diameters, the avalanche size increases (clogging decreases) as θ is reduced (and the pipe is tilted more horizontally). These results seem to contradict the ones obtained by Hadjigeorgiou and Lessard [34] who found -by means of DEM simulations- that the lower the inclination, the higher the clogging probability. Nevertheless, care should be taken when doing such comparison as the scenario explored in [34] is different from the one reported here. Indeed, Hadjigeorgiou and Lessard simulated a chute flow; i.e. a discharge of material purely driven by gravity (without a belt at the bottom extracting material at constant velocity). Hence, the inclination of the pipe controlled the discharge rate, and the lower inclination, the lower velocity of the particles inside the pipe. Therefore, we speculate that their result was probably the consequence of the alteration of the velocity of the particles provoked by the modification of the pipe inclination. In our case, as we control the discharge rate by means of the velocity of the conveyor belt, we are able to study separately the effect of the two variables: velocity and inclination. In principle, the increase of the avalanche size when reducing θ can be attributed to a increase of the projected area of the pipe section in the horizontal plane, in analogy to existent results of inclined orifices in the discharge of a silo [9,16].

5 Discussion

We have experimentally shown that the development of hanging arches of macroscopic particles might cause the clogging of a narrow vertical tube. Even though this confined geometry is very different from the typical bottleneck of a silo exit, the similarities found among these two phenomena are striking. First, the distribution of flowing times reveals an exponential decay which correlates with a constant probability of clogging during the discharge process. Second, the results of the mean avalanche size (measured in number of pipes discharged) versus the pipe diameter are compatible with the existence of a critical value of such parameter for which the mean avalanche size diverges. At least, in practical terms, there is a sharp transition from a regime where clogs appear to another one where they are absent. In addition, we prove that the probability of clogging is notably affected by the extraction rate, a behavior that stresses the importance of the dynamics of the problem. Finally, we show that the inclination of the pipe has a noticeable effect on the clogging probability as -at least for the range of inclinations explored- the more vertical the pipe is, the higher the probability of clogging (the smaller the avalanche size).

The results reported in this work represent a first step towards a deeper understanding of the process of clogging in confined geometries without constrictions. Among the many questions that remain open let us stress some. The first one is about the role of the pipe length (L) and the most appropriate way of defining the avalanche size. In principle as $s = t_{flow}/t_{tube}$ and $t_{tube} \propto L$, if clogging was equally likely along the whole tube it will follow that $t_{flow} \propto 1/L$, so s would scale with $1/L^2$. To validate this assumption experiments with several tube lengths are necessary and, moreover, it should be checked that the clogging locations are uniform along the length of the pipe. The second question to address concerns the existence of a critical outlet size above which jamming does not occur. In this sense, let us state that -for small pipe diameters- we have observed flow intermittencies that seem to vanish for large enough diameters where clogging is not observed. This behavior is, again, similar to that already reported in the silo [37] and deserves further investigation. The clear advantage of the pipe geometry is that, in principle, it prevents the development of very important spatial gradients of all magnitudes existing in the silo, specially in the bottleneck proximities. Another issue that remains unsolved is if the behavior observed in this work, where the particles are faceted, holds for homogeneous spherical beads as suggested in [34,38]. We know that elongated particles align when they are sheared [39]. This alignment has been also observed for faceted particles during the silo discharge, leading to the formation of very stable structures [40–42]. Then, in agreement with the numerical prediction made in [34], we anticipate that if hanging arches of spherical particles are able to cause clogging, the size ratio necessary to observe them would be considerably smaller than the one obtained with faceted particles. Finally, from a fundamental point of view, it is crucial to understand the role of the belt velocity when this is lowered to a value where a quasistatic regime is attained. Unfortunately this has not been possible with present experimental setup due to a limitation in the engine gear of the conveyor belt as for very small velocities the conveyor belt engine did not have enough power to drag the material.

Acknowledgments We want to thank the contribution of referees. We are also grateful to J. Bienzobas and L. Quero for discussions and L.F. Urrea for his technical help. This work has been supported by Projects Nos. FIS2011-26675 and FIS2014-57325 Ministerio de Economa y Competitividad (Spanish Government) and partially funded by LKS-INGENIERIA S. COOP.

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