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Complexity measures for multi-dimensional and chaotical sources of information

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Abstract. We apply a modified LMC complexity measure [1] to an information source. The source is modeled by two identical Takens-Bogdanov equations synchronized, bi-directionally coupled and perturbed by a harmonic signal. In this system, when the frequency of the signal is tuned the complexity of the system changes [2]. The aim of the work is to show that with a modification on the interpretation of the LMC measure, we can obtain an extensive measure that can be applied to measure the complexity of completely unknown systems. As an example, we apply this procedure to a high-dimensional system with chaotical behavior.

Keywords: Complexity, Chaos, Information Theory

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1. Introduction

Reduction of complexity in complex systems has been frequently observed in experiments devoted to control chaos when a chaotic system is low dimensional. Chaos suppression is the simplest case of complexity reduction and it is an important feature in technological contexts. It could be achieved by different techniques like introducing changes in the system variables [3], applying an external periodic forcing with a special frequency and amplitude [4], coupling two oscillators one chaotic and the other in a periodic regime [5] among others.

In a former work of the authors the results of the asymmetric coupling between two identical chaotic oscillators using different types of 3 dimensional (3D) oscillators has been presented, and chaos suppression analyzed [5].

This work displays that an adequate coupling between two identical chaotic oscillators may force their dynamics towards regular periodic oscillations. Even it could seem counter-intuitive, the two subsystems oscillate at the same frequency but following a different limit cycle. The latter means that in terms of synchronization theory they are in a generalized synchronization state (GS) [6].

In systems with higher dimensional dynamics (>3D), more recent works coupling hyperchaotic systems have been reported to obtain GS synchronized states between identical systems without chaos suppression [6],[8],[7]. Complexity control with an external signal in a complex system composed by two synchronized hyperchaotic systems was numerically observed for the first time in [9] and the reduction in complexity keeping the synchronized state and the chaotic dynamics in [2]. In [8], the complexity reduction was measured by the increase in the width of the self-correlation function in the system.

2. The Information Source

The present work is addressed to obtain a quantitative measure of complexity on the states of different dynamics obtained in the numerical system [9], [8], based on Information Theory concepts applied to LMC complexity measure.

In this work we consider one of the variables of the dynamical system as the output of an Information source. This dynamical system (DS) described in [8] is composed by two synchronized Takens-Bogdanov systems with symmetry $\mathcal{D}_4$ (TB1 and TB2) that are adequately coupled to recover for the group of equations, the symmetry of each equation system (TB1 and TB2 = 4 equations coupled). It has been shown that a small amplitude harmonic signal [9] applied to a variable of one subsystem (in this case x) can be used to change the dynamical state in this system, when the frequency of the driving signal is properly chosen.

When the signal is applied and a new state appears, this can be verified obtaining the new Fourier spectrum and the attractor in the phase space. As the system has a riddle-bassin [11],[12], a control of the output against a small change in the initial conditions is necessary to verify the independence of the global dynamics from initial conditions.

Rich dynamical behaviors of the system, riddle basin, sensitivity to the initial conditions and even a driving signal make difficult to obtain an invariant measure. Then, if we can find a way to overcome all this problems, we will obtain an valuable contribution for real world problems.

3. The complexity measurements

The LMC measure defines the complexity as the product of the entropy per the *disequilibrium* function[1], i.e., $C_{LMC} = H \cdot D$.

Entropy and disequilibrium are magnitudes related to the states of the system. According to this definition we must know which are these possible states before computing them. However, such approach is not feasible for practical implementations because most of the times we do not know which are the states.

Usually, a prior assumption in order to make these calculations is to accept the ergodicity of a system. However, for those complex systems which are not ergodic, i.e., there is more than one statistical behavior, a new definition of complexity is required.

In order to overcome this problem, this work introduce an extension of the LMC complexity to evaluate how complex is a system only observing temporal data of the output. Measuring with this method and checking the temporal sequences we observe when new information sets appear in the output. Finally we link this behavior with the positive Lyapunov exponents of the information source.

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