

Kibble-Zurek mechanism in a pattern forming secondary bifurcation

M. A. Miranda^{1,†}, J. Burguete¹, W. González-Viñas¹ and H. Mancini^{1,2}

¹ *Department of Physics and Applied mathematics, University of Navarra, Spain*

² *Institute of Physics, University of Navarra, Spain*

Abstract. We present new experimental results on the quenching dynamics of an extended thermo-convective system (a network array of 100 convective oscillators) through a secondary bifurcation. The topology of the new coherent structures which are fronts (domain walls), when this secondary bifurcation is crossed upwards from a basic stationary multicellular pattern towards a propagative pattern, allows us to characterize a freezing dynamics related to the Kibble-Zurek mechanism (KZM). This mechanism defines a correlation length $\xi \sim \mu^\sigma$ when the threshold has been crossed at a quench rate $\mu = \left. \frac{d\epsilon}{dt} \right|_{\epsilon=0}$ sufficiently large to freeze almost instantaneously the phase transition front. This study concerns several sequences of quenches where convective oscillators become differently correlated depending on μ . Spatio-temporal correlation analysis will allow us to determine the behavior of the KZM from the front dynamics. The novelty of these results is that the front dynamics is expected to show the freezing-out dynamics beyond the threshold, and therefore it will provide the healing length of the system depending on the strength of the quench rate. Furthermore, in our system the effect of resonant nonlinearities pins the front and diminishes its fluctuations as we increase the quench rate.

Keywords: Non-equilibrium phase transitions, pattern formation, critical phenomena, Networks, symmetry breaking bifurcations, experimental cosmology, cosmic domains

MSC 2000: 82C26, 82B27, 83F05, 94Cxx.

1. Introduction

The Kibble-Zurek mechanism (KZM) [7, 8] is applied to the out-of-equilibrium dynamics and phase transitions of condensed matter physics as well as to the very first instants after the Big-Bang. The KZM predicts freezing of the correlation length in systems with global symmetry in quenches based on

causal arguments. Thus, similarly to the rapid cooling that took place in the Big-Bang, the distribution of defects in rapid phase transitions obey the same Physics as galaxies throughout the Universe. According to this parallelism, the distribution of galaxies and other cosmic objects represent a fossil discovery of an early Universe because it is expected to keep the freezing-out dynamics of those firsts instants. This is what we expect from the fluctuating fronts that are defined in our system by the critical modes of a secondary bifurcation. The Zurek prediction, which was initially derived for a second-order phase transition, determines the healing length in the course of a symmetry breaking transition so that $\xi \sim \tau_Q^\sigma$ (with $\sigma < 0$), where τ_Q is the “quench time” defined as $\tau_Q = \mu^{-1}$. Also, the relaxation time follows $\tau \sim \tau_Q^\eta$ (with $\eta < 0$). In a quench, the dynamics of the system is far from the adiabatic evolution. In consequence, fluctuations may become frozen if the system is not able to readjust to the instantaneous control parameter value. Regarding a first-order phase transition (discrete symmetry breaking transition), Rajantie [9] holds that for fast quenches this type of transitions might be responsible for a very inhomogeneous Universe although the behavior of the correlation length at quenches is not completely understood in experiments and simulations.

Cosmology in the laboratory has become an intriguing subject since first attempts suggested by Zurek were tried on cooling helium 4 systems. We find experimental evidence on the KZM in other systems like nonlinear optical systems [4] and Bénard-Marangoni conduction-convection transition in a cylindrical cell [5, 6]. Nevertheless, these experiments have always taken primary bifurcations into account. Here, we show the quenching dynamics in a secondary bifurcation from a stationary multicellular pattern (ST) towards a oscillatory pattern throughout the presence of domains of traveling waves (TW) and a mixed pattern of counter-oscillating waves (ALT) over ST.

2. Front dynamics at quenches

We study the quench dynamics of a dissipative system at the threshold of a secondary bifurcation. The convective system is a rectangular fluid layer (Silicone oil with a kinematic viscosity of 5 cSt) with a localized heating along a line. For a fixed depth of the fluid layer, each characteristic pattern of the stability diagram [1, 2] is obtained by controlling the heating temperature or control parameter ϵ . As the temperature is increased, the system breaks more symmetries and new critical modes take part in the spatiotemporal Fourier spectra. We have focused on the secondary bifurcation that takes place from a stationary multicellular pattern (ST) with wavelength λ_s which consists of an array of 100 convective cells. When the critical point is crossed upwards in a

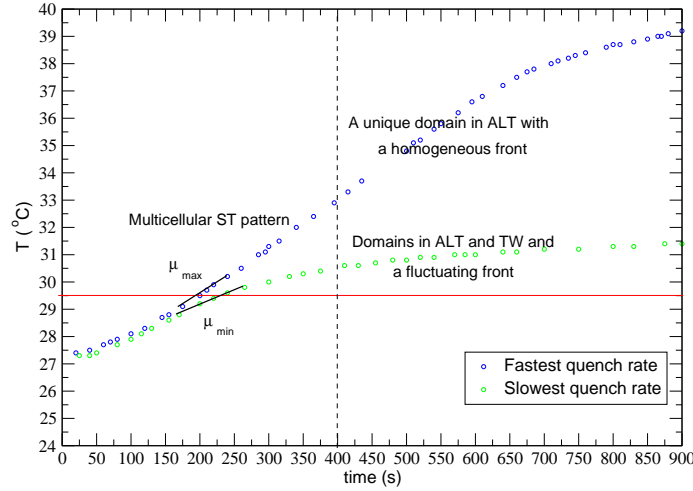


Figure 1: Evolution of the control parameter T for a maximum and a minimum quench rate μ . The horizontal red line defines the threshold in adiabatic conditions. The vertical dashed line represents the approximate transition time.

quench by increasing the temperature of the system, a 1D-front is formed. This front connects the previous ST pattern with a new mixed pattern consisting of highly unstable domains of traveling waves and mostly domains of counter-propagative waves with wavelengths $\lambda_{v\pm} \approx 2\lambda_s$. More detailed information about the bifurcations of this system can be found in Ref. [1, 2], and from the point of view of phase synchronization transitions in Networks see Ref. [3].

In order to obtain the reported results, we set our control parameters near the codimension-2 point at a fixed depth $d = 7.5$ mm, where the system bifurcates from a multicellular pattern towards the oscillatory pattern described above. For depths $d > 7.5$ mm, the system bifurcates supercritically from a homogeneous state towards traveling waves. This secondary bifurcation is weakly subcritical when it is crossed quasi-statically with a subcriticality of $\epsilon \approx -0.02$ [1].

When the convective array of cells is quenched towards the new pattern by crossing the adiabatic threshold (at 29.5 °Celsius) with different quench intensities, a front dynamics is expected to show the freezing-out dynamics that takes place near the threshold. In Fig. 1 we show two different quench rates, for a smooth crossover (μ_{min}) and for a fast one (μ_{max}).

The front profile is obtained from demodulation techniques by selecting the critical propagative modes $M_{v\pm}$. By measuring the degree of correlation

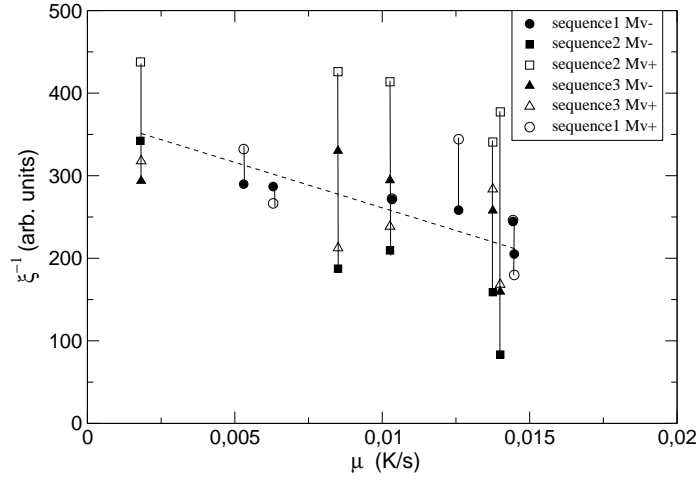


Figure 2: Inverse of the correlation length ξ vs the quench rate μ for different sequences of measurements. The dashed line is a guide to the eye.

of these fronts for different sequences, we are able to determine the behavior of the freezing mechanism for different quench intensities (see Fig. 2). Thus, the inverse of the correlation length ξ^{-1} is given by the deviation of the front. Our results show how an initial fluctuating 1D-front becomes more homogeneous as we increase the quench intensity. The stationary mode which defines the previous pattern (ST), becomes stronger above the critical point as we increase μ due to the increasing nonlinear resonance with the critical traveling modes $M_{v\pm}$.

3. Conclusions

Although weak subcriticality sets the necessary condition for a freezing-out dynamics, as we crossover a secondary bifurcation we find out an unexpected critical behavior in regard of the standard Kibble-Zurek prediction. Concerning the interest of these results, we are planning to look for further evidences and thus, more experimental and analytical effort is going to be developed soon.

Acknowledgements

This work has been partly supported by the Spanish Contract No. FIS2008-01126 and by the Gobierno of Navarra (Departamento de Educación).

References

- [1] M. A. MIRANDA AND J. BURGUETE, *Phys. Rev. E* **78**, 046305 (2008).
- [2] M. A. MIRANDA AND J. BURGUETE, *Phys. Rev. E* **79**, 046201 (2009).
- [3] M. A. MIRANDA AND J. BURGUETE, *Int. J. Bifurcation and Chaos* **20**, 835 (2010).
- [4] S. DUCCI AND P. L. RAMAZZA AND W. GONZÁLEZ-VIÑAS AND F. T. ARECCHI, *Phys. Rev. Lett.* **83**, 05210 (1999).
- [5] S. CASADO AND W. GONZÁLEZ-VIÑAS AND H. MANCINI, *Phys. Rev. E* **74**, 047101 (2006).
- [6] S. CASADO AND W. GONZÁLEZ-VIÑAS AND S. BOCCALETTI AND P. L. RAMAZZA AND H. MANCINI, *Eur. Phys. J. Special Topics* **146**, 87 (2007).
- [7] W. H. ZUREK, *Phys. Rep.* **276**, 177 (1996).
- [8] T. W. B. KIBBLE, *J. Phys. A* **9** 1387 (1976).
- [9] A. RAJANTIE, *Contemp. Phys.* **44**, 485 (2003).