



## SWITCHER-RANDOM-WALKS: A COGNITIVE-INSPIRED MECHANISM FOR NETWORK EXPLORATION

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Received November 15, 2008; Revised January 27, 2009

Semantic memory is the subsystem of human memory that stores knowledge of concepts or meanings, as opposed to life specific experiences. The organization of concepts within semantic memory can be understood as a semantic network, where the concepts (nodes) are associated (linked) to others depending on perceptions, similarities, etc. Lexical access is the complementary part of this system and allows the retrieval of such organized knowledge. While conceptual information is stored under certain underlying organization (and thus gives rise to a specific topology), it is crucial to have an accurate access to any of the information units, e.g. the concepts, for efficiently retrieving semantic information for real-time need. An example of an information retrieval process occurs in verbal fluency tasks, and it is known to involve two different mechanisms: “clustering”, or generating words within a subcategory, and, when a subcategory is exhausted, “switching” to a new subcategory. We extended this approach to random-walking on a network (clustering) in combination to jumping (switching) to any node with certain probability and derived its analytical expression based on Markov chains. Results show that this dual mechanism contributes to optimize the exploration of different network models in terms of the mean first passage time. Additionally, this cognitive inspired dual mechanism opens a new framework to better understand and evaluate exploration, propagation and transport phenomena in other complex systems where switching-like phenomena are feasible.

*Keywords:* Random-walks; complex-networks; information retrieval; cognitive systems; switching-clustering.

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## 1. Introduction

Semantic memory is a distinct part of the declarative memory system [Tulving, 1978] comprising knowledge of facts, vocabulary, and concepts acquired through everyday life [Squire, 1987]. Contrary to episodic memory, which stores life experiences, semantic memory is not linked to any particular time or place. In a more restricted definition, it is responsible for the storage of semantic categories and naming of natural and artificial concepts [Budson & Price, 2005]. It is known that this memory involves distinct brain regions and its impairment in neurodegenerative diseases such as fronto-temporal dementia [Libon *et al.*, 2007], multiple sclerosis [Henry & Beatty, 2006] and Alzheimer’s disease [Rogers & Friedman, 2008] produce verbal fluency deficits. For this reason, lexical access, the cognitive information-retrieval process in charge of retrieving concepts, has been widely explored through semantic verbal fluency tasks in the context of neuropsychological evaluation [Lezak, 1995]. These tests require the generation of words corresponding to a specific semantic category, typically animals, fruits or tools, for a given time. Although the task is easy to explain, it actually results in a complex challenge where retrieving as many concepts as possible in a limited time depends more on cognitive mechanisms than on the knowledge itself. According to the two-component model proposed by Troyer [Troyer *et al.*, 1997], optimal fluency performance involves a balance between two different processes: “clustering”, or generating words within a subcategory, and, when a subcategory is exhausted, “switching” to a new subcategory. In the case of naming animals, clustering produces semantically related transitions (e.g. *lion-tiger*) and switching is a mechanism that allows to jump or shift to different semantic fields (e.g. *tiger-shark*). While the former is attached to the temporal lobe of the brain, the latter has been associated to a frontal lobe activity [Troyer *et al.*, 2002]. Evidence of the interaction between these two regions of the brain during language related tasks has led to a number of studies related to a *fronto-temporal modulation* or interaction [Poldrack *et al.*, 1999; Troyer *et al.*, 2002].

In this paper, the cognitive paradigm that consists of retrieving words from a *semantic network* [Rogers & Friedman, 2008; Thornton *et al.*, 2002] was generalized to an exploration task on a network. Clustering was modeled as a random-walker constrained to the topology of the network and

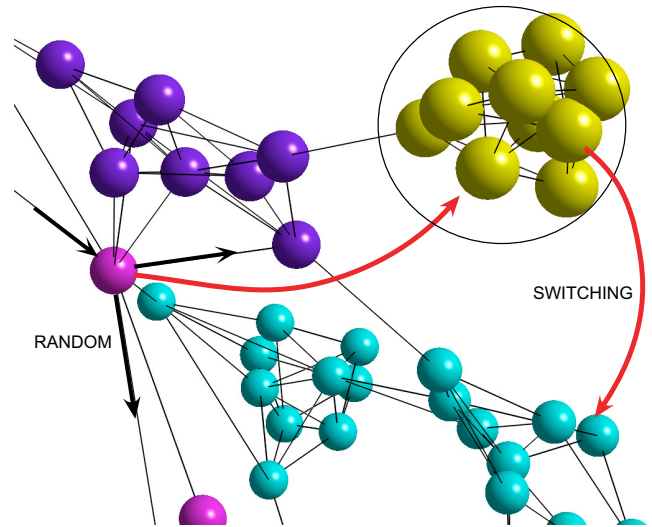


Fig. 1. Switcher random walks: transitions between nodes in a graph can occur through random movements following the edges (black arrows) but also through switches (red arrows). Switching allows a more efficient exploration, since clustered graphs might have difficulty finding rare paths between sub-graphs. Isolated modules in particular (circle) would be seldom reached and rarely abandoned through random walking.

switching as an extra-topological mechanism that is able to move from any node to another (see Fig. 1). The combination of these two processes gave rise to a dual mechanism denoted here as *switcher-random-walker* (SRW), i.e. a random-walker with the additional ability of switching. The combination of switching and clustering, i.e. free jumping and random walking, was ruled by a parameter  $q$ , which is the probability of switching at every step, and thus is the parameter that metaphorically rules the fronto-temporal modulation. Therefore, the complementary  $(1 - q)$  is the probability of clustering at every step, and can be interpreted as the strength of the *local perseverance* of the exploration before moving somewhere else within the network (specially for those networks with either high clustering coefficient or high modularity). This cognitive inspired paradigm gives rise to the following question: how does switching and its modulation affect random exploration of different network models?

Search, propagation and transport phenomena have been studied in networks [Bollt & Ben-Avraham, 2005], where it is crucial to define whether the full topology is known. When it is known, the ease to reach any node from another is measured by the *shortest path length* [Tadic & Rodgers, 2002; Watts & Strogatz, 1998]. When it remains unknown, exploration is modeled by random walks

along the network [Noh & Rieger, 2004]. This case involves retrieving concepts since the subject is not aware of his full semantic network when naming them. In this type of cases, reachability of nodes is measured with the *mean first passage time* (MFPT), i.e. the averaged number of steps needed to visit a node  $j$  for the first time, starting from a node  $i$  [Catral *et al.*, 2005; Snell, 1959]. Given its relevance in complex media, this paradigm has been recently revisited in a number of studies [Catral *et al.*, 2005; Condamin *et al.*, 2007; Noh & Rieger, 2004].

While different derivations of random-walkers have been recently used to infer the underlying topological properties of complex networks [da Fontoura Costa & Travieso, 2007; Gómez-Gardeñes & Latora, 2008; Ramezanpour, 2007], our aim was to evaluate how SRW (and in particular, the effect of different levels of switching) contributes to the exploration of network models with well-known topological properties. Different models which were not necessarily lexico-conceptual architectures were explored by a SRW and its performance was measured by the MFPT (detailed in Sec. 2.3). Going back to the cognitive paradigm, retrieving plenty of words in a semantic verbal fluency test not only depends on the number of concepts that the subject knows, but also on an equilibrium between the underlying semantic topology that organizes those concepts and the frequency of switching [Troyer *et al.*, 1997]. For example, two different studies [Boringa *et al.*, 1982; Sepulcre *et al.*, 2006] reported that their respective groups of healthy participants produced  $30.7 \pm 7.9$  and  $28.15 \pm 7.32$  animals during 90 sec. There are two remarkable aspects in these figures. First, participants obviously knew many more animals than those said and, second, there is a high heterogeneity in the number of words. Hence, even though all participants only named a low fraction of the animals they knew, some of them had much more success than others when retrieving them.

## 2. A Markov Model of SRW

As introduced in the previous section, our approach for a clustering step consists of a walker unaware of the full network moving from one node to any of its neighbors with no preferential gradients among neighbors. Such exploration task was modeled by the well-known *random-walker* (RW). Switching was implemented as a mechanism where

the walker moves to any other node following different probabilistic approaches. Summarizing, SRW can be defined as a random-walker with the capability of rendering random shifts.

### 2.1. Markov chains

A finite Markov chain is a special type of stochastic process which can be described as follows. Let

$$S = \{s_1, \dots, s_r\} \quad (1)$$

be a finite set whose members are the *states* of the system, which we label  $s_1, \dots, s_r$ . The process moves through these states in a sequence of *steps*. If at any time it is in state  $i$ , it moves to a state  $j$  on the next step with some probability,  $\Pi : S \times S \rightarrow \mathcal{M}_{S \times S}$ , where  $\mathcal{M}_{S \times S}$  is the set of  $S \times S$  matrices of non-negative entries where the sum of every row is 1. These probabilities define a square,  $r \times r$  matrix,  $\Pi$ :

$$\Pi \equiv [p_{ij}], \quad (2)$$

which we call the *matrix of transition probabilities*. The importance of matrix theory to Markov chains comes from the fact that the  $ij$ th entry of the  $n$ th power of  $\Pi$ ,  $\Pi^n = [p_{ij}^{(n)}]$  represents the probability that the process will be in state  $j$  after  $n$  steps considering that it was started in state  $i$ . The study of a general Markov chain can be reduced to the study of two special types of chains. These are *absorbing chains* and *ergodic chains* (also known as *irreducible*). The former contain at least one absorbing state, i.e. a state constituted by a proper subset of the whole by which, once entered it cannot be left, and furthermore, which is reachable from every state in a finite number of steps. The latter are those chains where it is possible to go from any state to any other state in a finite number of steps and are called *regular chains* when

$$(\exists n < \infty) : (\forall i, j \leq r)(\forall N > n)(p_{ij}^{(N)} > 0).$$

For regular chains, the  $ij$ th entry of  $\Pi^n$  becomes essentially independent of state  $i$  as  $n$  is larger. In the case of regular chains, we can define a stationary probability matrix [Snell, 1959]  $\Pi^\infty$  as:

$$\lim_{n \rightarrow \infty} \Pi^n = \Pi^\infty. \quad (3)$$

Note that for nonregular Markov processes this limit might not exist. For instance  $\Pi = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ .

The matrix  $\Pi^\infty$  consists of a row probability vector  $w$  which is repeated on each row. This vector  $w$  can be obtained as the only probability vector

satisfying  $w = w\Pi$  [Grinstead & Snell, 1952]. For the case of regular Markov processes obtained from random walks on graphs, this indicates that in the long run, the probability to be in a node is independent of the node where the process started.

## 2.2. Graph characterization

This section is devoted to the characterization of the underlying object over which we apply our algorithm of exploration, a graph. Beyond its main features, we discuss the consequences of connectedness in order to clearly define the frameworks over which the SRW algorithm can be defined. Finally, we briefly define the graph models studied numerically in Sec. 3.

Let us suppose that our Markov chain is defined by some graph topology. A graph  $\mathcal{G}$  is defined by a set of nodes,  $V \equiv \{v_1, \dots, v_n\}$ , and a set of links  $\Gamma \equiv \{\{v_i, v_j\}, \dots, \{v_k, v_l\}\}$ ,  $\Gamma$  being a subset of  $V \times V$ . In our approach, the graph is undirected and we avoid the possibility that a node contains auto-loops or that two links are connecting the same nodes. The size of the graph is  $|V|$ , i.e. the cardinal of the set of vertices. Its *average connectivity* is defined as:

$$\langle k \rangle \equiv \frac{2|\Gamma|}{|V|}. \quad (4)$$

The topology of our graph is completely described by a symmetrical,  $|V| \times |V|$  matrix,  $A(\mathcal{G}) = [a_{ij}]$ , the so-called *adjacency matrix*, whose elements are defined as:

$$a_{ij} = \begin{cases} 1 & \leftrightarrow \{v_i, v_j\} \in \Gamma \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

The connectivity of the node  $v_i$ ,  $k(v_i)$  is the number of links departing from  $v_i$  and it can be easily computed from the adjacency matrix as:

$$k(v_i) = \sum_{j \leq |V|} a_{ij}. \quad (6)$$

Following the characterization, we now define the degree distribution, which is understood as the probability that a randomly chosen node displays a given connectivity. In this way, we define the elements of such a probability distribution,  $\{p\}$  as:

$$p_i = \frac{|(v_j \in V) : (k(v_j) = i)|}{|V|}. \quad (7)$$

The above defined measures are the *identity card* of a given graph  $\mathcal{G}$ . One could think that it is enough because our main goal is to describe and characterize an exploration algorithm over  $\mathcal{G}$ . However, specially in the models of random graphs,

we cannot be directly sure that our adjacency matrix defines a fully connected graph, i.e. there exists, with probability 1 a path from another  $v_i$  to any node  $v_j$ . In deterministic graphs, we can solve this problem by assuming, *a priori*, that our combinatorial object is fully connected. Furthermore, we could agree that, when performing rewirings at random, we impose the condition of connectedness. The case of pure random graphs is a bit more complicated. Indeed, a random graph is obtained by a stochastic process of addition or removal of links [Bollobas, 2001]. Thus, we need a criteria to ensure that our graph is connected or, at least, to work over the most representative component of the obtained object. Full connectedness is hard to ensure in a pure random graph. Instead, what we can find is a *giant connected component*, GCC. Informally speaking, we can imagine an algorithm spreading at random links among a set of predefined nodes, the so-called Erdős–Rényi graph process. The growing graph displays, at the beginning, a myriad of small clusters of a few nodes and, when we overcome some threshold in the number of links we spread at random, a component much bigger than the others emerges, i.e. the GCC [Erdős & Rényi, 1960]. In this way, Molloy and Reed [1995] demonstrated that, given a random graph with degree distribution  $\{p\}$ , if

$$\sum_k k(k-2)p_k > 0 \quad (8)$$

then, there exists, with high probability, a giant connected component. The first condition we need to assume is thus, that the studied graphs satisfy inequality (8). Beyond this assumption, we impose the following criteria when studying our model networks:

- (1) In a deterministic graph (for example, a chain or a lattice) where we perform random rewirings, we do not allow rewirings that break the graph.
- (2) If a graph is the result of an stochastic process, the exploration algorithm is defined only over the GCC (this could imply the whole set of nodes).
- (3) The adjacency matrix is the adjacency matrix of the GCC. We *remove* the nodes that, in the beginning, participated in the process of construction of  $\mathcal{G}$  but fell outside the GCC.

All the model graphs studied in this work satisfy the above conditions.



In order to obtain a useful comparative analysis, we built different networks, all of them with  $|V| = 500$  nodes and  $|\Gamma| = 2000$  links. The results were averaged after 100 instances per network model (see Fig. 2). Let us briefly define the models to be studied with our exploration algorithm.

### 2.2.1. *Watts–Strogatz small-world network*

We built an annulus with 500 nodes in such a way that every node is connected to eight different nodes

(2000 undirected links) [Watts & Strogatz, 1998]. Once the annulus was constructed, every link suffered a random rewiring with connectivity  $p = 0.05$ .

### 2.2.2. *Erdős–Rényi graph*

Over a set of 500 nodes we spread at random 2000 links, avoiding duplication and self-interaction. It can be shown that the obtained graph displayed a binomial degree distribution [Erdős & Rényi, 1960]:

$$p_k = \binom{|V| - 1}{k} \pi^k (1 - \pi)^{|V| - k - 1}, \quad (9)$$

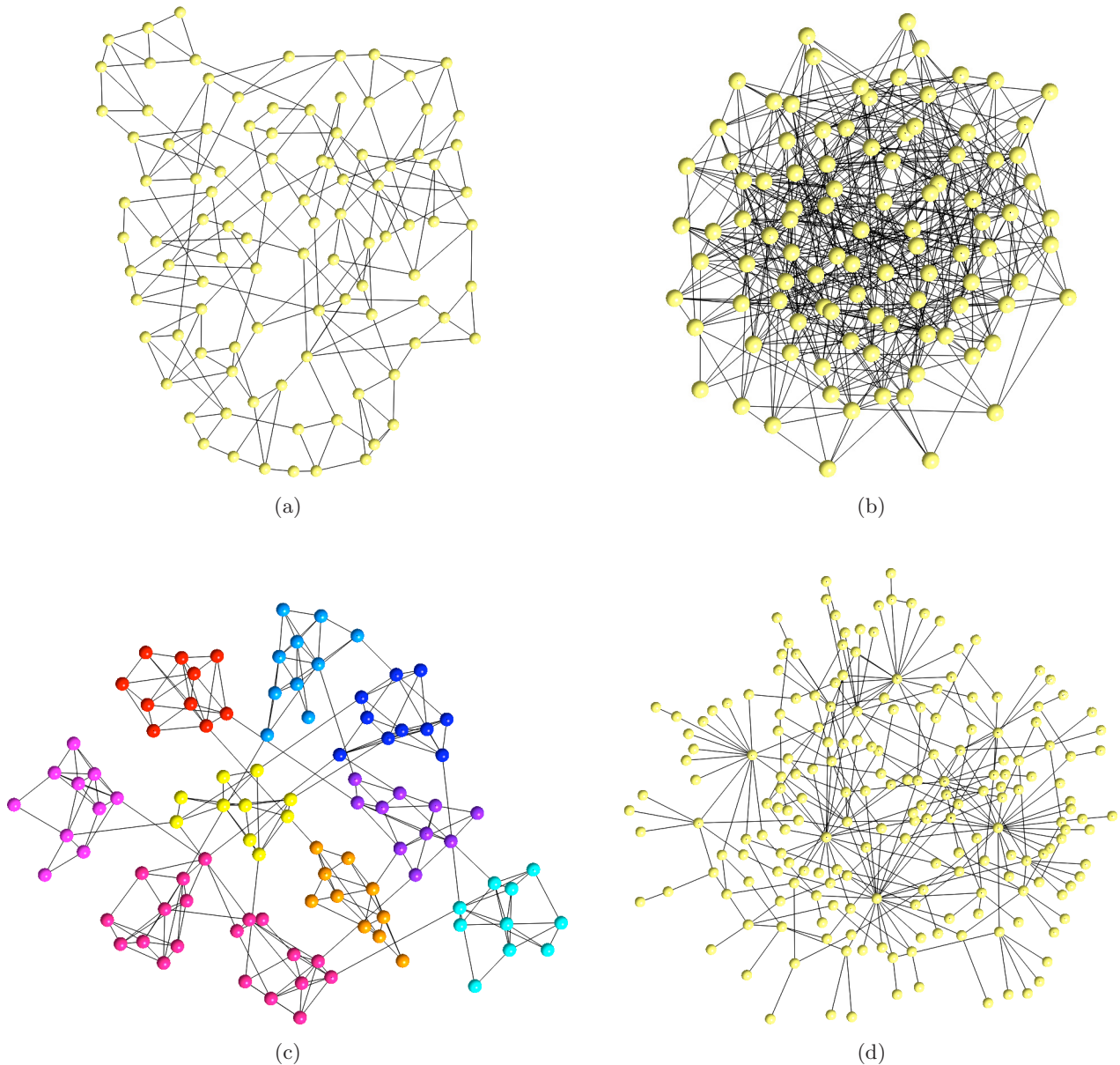


Fig. 2. Visualization of small examples ( $|V| = 100$ ) of the four network models analyzed here: (a) *Small-world network*. (b) *Random Erdős–Rényi network*. (c) *Random-modular network*: here a network is partitioned into ten modules, each one connecting to each other with a large probability, whereas a very small inter-module probability is used. (d) *Scale-free network* obtained by preferential attachment. See Sec. 2.2 for a detailed description of each network model.

$\pi$  being the probability of two nodes being connected. Its value corresponds to

$$\pi = |\Gamma| \binom{|V|}{2}^{-1} \quad (10)$$

### 2.2.3. Random-modular

We built ten different components of 50 nodes and 200 links, spread at random (as explained for Erdős–Rényi graphs) among the 50 nodes of every component. In this case, we ensure connectedness of such components. Once the ten components are constructed, every link suffers a random rewiring with a node either from the same component or not, with probability  $p = 0.05$ .

### 2.2.4. Preferential attachment

We provide a seed of nine connected nodes. Every new node was connected to eight of the existing nodes with probability proportional to the connectivity of the existing nodes, i.e. suppose that, at time  $t$  a new node  $v_i$  comes in to the graph. At this time step, the graph will display an adjacency matrix  $A(t)$ .

$$\mathbb{P}(a_{ij}(t) = 1) = \frac{k(v_j)(t-1)}{\sum_{v_k \in \mathcal{A}_t} k(v_k)(t-1)}, \quad (11)$$

where

$$\mathcal{A}_t = \{(v_k : \exists l) : (a_{kl}(t) > 0)\} \quad (12)$$

This operation is repeated in an iterative fashion (i.e. updating  $A$ ) eight times per node. It can be shown that, at the limit of a large number of nodes the outcome of this algorithm generates a graph whose degree distribution is a power law [Barabasi & Albert, 1999]:

$$p_k \propto k^{-\alpha}, \quad (13)$$

with  $\alpha = 3$ . It is worth noting that such an algorithm avoids the possibility of unconnected components.

### 2.3. Random walk over a graph as a Markov process

In this framework, the transition from node  $i$  to  $j$  is just the probability that a random-walker starts from some node  $i$  and reaches the node  $j$ , after some steps. Consistently, the probability of being in  $v_i$  as

we reach the node  $v_j$  in a single step (i.e.  $p_{ij}$ ) is:

$$p_{ij} = \frac{a_{ij}}{k(v_i)} \quad (14)$$

This is the general form for a Markov formalization of a random-walker within a graph defined by its adjacency matrix  $A$ . Throughout this work we assume that our graphs define *regular* Markov processes (see Sec. 2.1). Under the above definition of  $\Pi$ , regularity is assured if and only if the graph is not bipartite (i.e. it contains, at least, one loop containing an odd number of nodes). To see that bipartite graphs are not regular, it is enough to notice that for any pair of nodes  $(v_i, v_j)$  there are only either odd or even paths joining them, but not both. Hence if  $p_{ij}^{(n)} \neq 0$  then  $p_{ij}^{(n+1)} = 0$  and therefore the process cannot be regular.

Summarizing, despite the fact that connectedness ensures the process is ergodic,

$$(\forall v_i, v_j \in \mathcal{G})(\exists n : p_{ij}^{(n)} \neq 0)$$

it does not ensure regularity and therefore the  $\lim_{n \rightarrow \infty} \Pi^n$  might not exist. The existence of an odd loop breaks such parity problem and enables  $\Pi^n$  to stabilize to a specific matrix of stationary probabilities when  $n \rightarrow \infty$ . Thus, we must impose another assumption to our studied graphs: Our algorithm works over nonbipartite graphs which satisfy the criteria imposed in Sec. 2.2. It is straightforward to observe that, if the assumption of regularity holds, the above Markov process has a stationary state with associated probabilities proportional to the connectivity of the studied node [Noh & Rieger, 2004]:

$$p_{ij}^{(\infty)} = \frac{k(v_j)}{2|\Gamma|}. \quad (15)$$

From now on, we will refer to the transition matrix above defined as  $\Pi^{cl}$ , since it denotes the probabilities of the movements related to clustering.

### 2.4. Switcher-random-walker

In the retrieval model introduced here, the matrix of transition probabilities  $\Pi^{sw}$  is a linear combination of the switching transition probabilities  $\Pi^{sw}$  and the clustering transition probabilities  $\Pi^{cl}$ , as defined in the above section. The *Markov process* is a switcher-random-walker and the *states* represent the location of such walker in the network.

The matrix  $\Pi^{sw} = [p_{ij}^{sw}]$  is ergodic and regular since all entries are strictly greater than zero, and

has equal rows, i.e. constant columns. The reason is that the probability of reaching a node  $j$  through switching is independent of the source node  $i$ . In this way, we could consider that we define a scalar field  $\lambda$  over the nodes of the graph:

$$p_{ij}^{sw} = \lambda_j. \quad (16)$$

Consistently,

$$\sum_{j \leq |V|} \lambda_j = 1. \quad (17)$$

We can define this field in many different ways. As the more representative, we revise several scalar fields that can provide us interesting information about the process:

$$\lambda_j = \begin{cases} \frac{1}{|V|} \\ \frac{k(v_j)}{\sum_{i \leq |V|} k(v_i)} \\ \frac{K - k(v_j) + 1}{\sum_{i \leq |V|} k(v_i)} \end{cases} \quad (18)$$

In the first and most simple case, switching to any other node is a random uniform process, and we refer to this process as *uniformly distributed switching*. The second case corresponds to the situation where the probability to reach a given node through switching is proportional to its connectivity, which we call *positive degree gradient switching*. The last one assumes that  $K$  is  $\max\{k(v_i)\}$  and corresponds to the situation where the switcher jumps with more probability to weakly connected nodes, and we refer to it as *negative degree gradient switching*. These three variants of switching were studied when combined with a random-walker within the above graph topologies (see Fig. 3). They were denoted by  $\text{SRW}^-$ ,  $\text{SRW}^+$  and  $\text{SRW}^-$  respectively.

The matrix  $\Pi^{cl} = [p_{ij}^{cl}]$  defined in the above section is ergodic and regular but restricted to the transitions allowed by the adjacency matrix  $A$  of the network of study. We modeled as equi-probable the transitions among linked nodes of the network. Hence the probability of moving from a node  $v_i$  to a node  $v_j$  through clustering for a given graph  $\mathcal{G}$  with an adjacency matrix  $A_{\mathcal{G}} = [a_{ij}]$ , is

$$p_{ij}^{cl} = \frac{a_{ij}}{k(v_i)}. \quad (19)$$

Thus,  $\Pi^{srw} = [p_{ij}^{srw}]$  is defined as:

$$\Pi^{srw} = q\Pi^{sw} + (1 - q)\Pi^{cl} \quad (0 \leq q \leq 1), \quad (20)$$

where  $q$  is the probability of switching. Consistently, the entries of  $\Pi^{srw}$  are given by:

$$p_{ij}^{srw} = qp_{ij}^{sw} + (1 - q)p_{ij}^{cl}, \quad 0 \leq q \leq 1. \quad (21)$$

We observed that  $\Pi^{srw}$  is also ergodic and regular. This follows from the fact that  $\Pi^{sw}$  already has all entries strictly greater than zero, and thus  $\Pi^{srw}$  will have all entries greater than zero for any  $q > 0$ . For the case of  $q = 0$ ,  $\Pi^{srw}$  is just  $\Pi^{cl}$  which we assumed to be regular.

Among other interesting descriptive random variables that can be evaluated for regular chains, the matrix of the *mean first passage time* (MFPT) is a matrix  $\langle T \rangle = [\langle t_{ij} \rangle]$ , crucial for measuring the retrieval or exploratory performance of any stochastic strategy; the MFPT needed to go from a node  $i$  to a node  $j$  is denoted by  $\langle t_{ij} \rangle$  [Noh & Rieger, 2004] and represents the time (in step units) required to reach state  $j$  for the first time starting from state  $i$ . It is important to note that  $\langle t_{ij} \rangle$  is not necessarily equal to  $\langle t_{ji} \rangle$ , i.e. it might happen that the time required to go from state  $i$  to state  $j$  is different to the time required to go from state  $j$  to state  $i$ .

In order to obtain the analytical expression of MFPT, we must define first a fundamental matrix  $Z$  [Grinstead & Snell, 1952] which is given by

$$Z = (\mathbb{I} - \Pi^{srw} + \Pi_{srw}^{\infty})^{-1}, \quad (22)$$

where

$$\Pi_{srw}^{\infty} = \lim_{n \rightarrow \infty} (\Pi^{srw})^n, \quad (23)$$

and  $\mathbb{I}$  is the identity matrix of size  $|V| \times |V|$ .

In this case the entry  $z_{ij}$  of  $Z$  can be understood as a measure of the deviations of the  $ij$ th entry of  $(\Pi^{srw})^n$  from their limiting probabilities  $w$ , which, as commented in Sec. 2.1, is any of the equal rows of  $\Pi_{srw}^{\infty}$ . From  $Z$  and  $w$  we can obtain the analytical derivation of  $\langle T \rangle = [\langle t_{ij} \rangle]$  (for more details see [Grinstead & Snell, 1952]):

$$\langle t_{ij} \rangle = \frac{z_{jj} - z_{ij}}{w_j} \quad (24)$$

Finally, we denote as  $\langle \text{MFPT} \rangle_{\mathcal{G}}$  the averaged value of all entries  $\langle t_{ij} \rangle$  for a switcher random walker exploring a network  $\mathcal{G}$ . Since  $\langle T \rangle$  it is not necessarily symmetrical, we must take into account all the entries outside the main diagonal. The main diagonal was not taken into account, since it represents

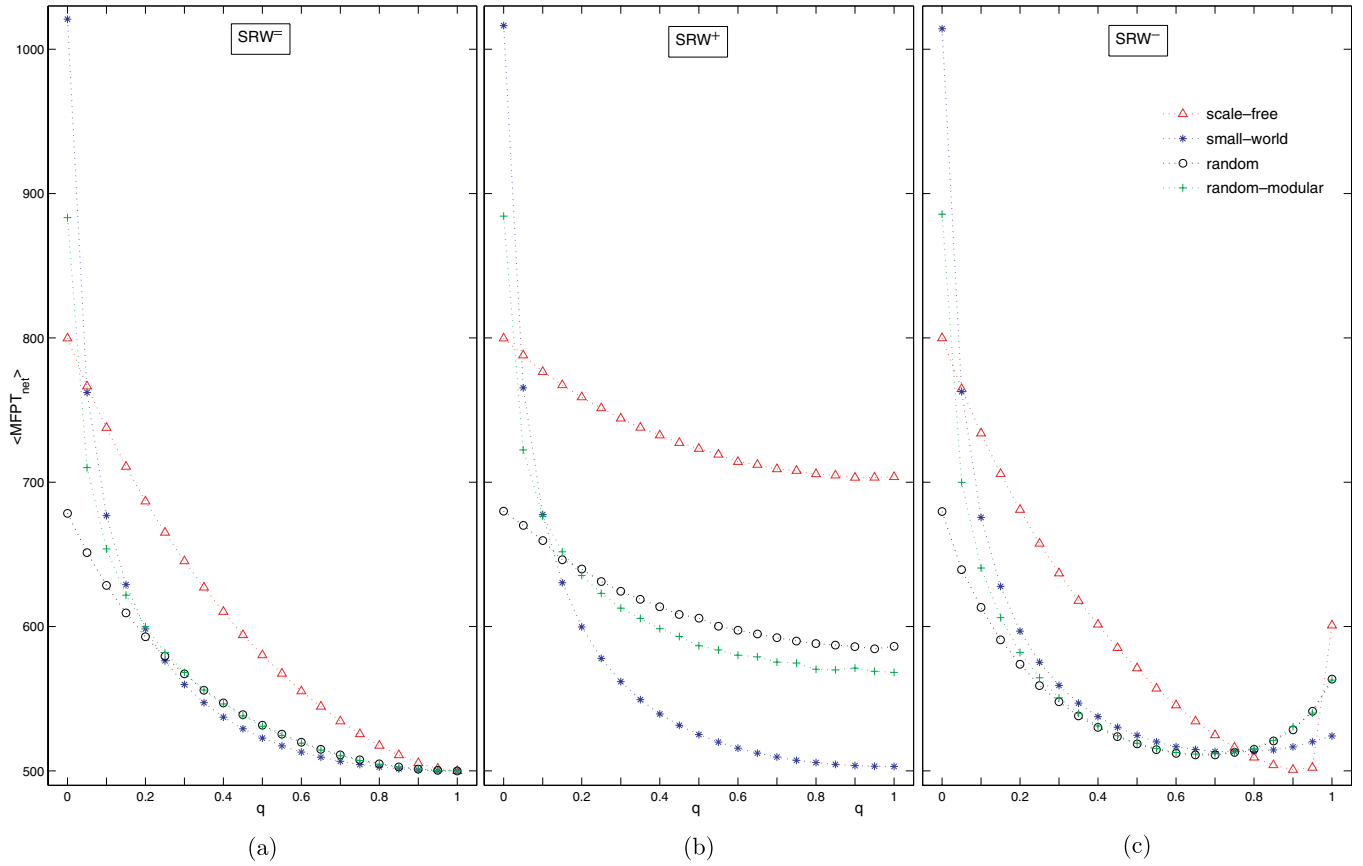


Fig. 3. Exploration performance based on the  $\langle \text{MFPT} \rangle_{\mathcal{G}}$  [see Eq. (25)] on four graph models for the three Markovian variants of SRW [see Eq. (18) for implementation details of each variant of switching]. Parameter  $q$  stands for probability of switching [see Eq. (20)]. (a)  $\text{SRW}^=$ , SRW that contains a uniformly distributed switching. (b)  $\text{SRW}^+$ , SRW that contains a switching with positive degree gradient. (c)  $\text{SRW}^-$ , SRW that contains a switching with negative degree gradient.

the returning time, which we do not consider as a part of the exploration of the net. Thus,

$$\langle \text{MFPT} \rangle_{\mathcal{G}} = \frac{1}{2 \binom{|V|}{2}} \sum_i \sum_{j \neq i} \langle t_{ij} \rangle. \quad (25)$$

This measure provides a general evaluation of how reachable is, on average, any node from any other node in a specific network using a switcher random-walker. It is interesting to notice that such measure has an upper bound which is precisely the size of the net. Indeed, let us suppose we have a clique of size  $m$ , i.e. a graph,  $\mathcal{G}(V, \Gamma)$ , where  $|V|$  equals  $m$  and every node  $v_i$  is connected to itself and to all  $m - 1$  remaining nodes. It corresponds to the case where the probability of switching is 1. Let  $X$  be a random variable whose outcomes are  $v_j$  such that,  $\forall v_j \in V$ :

$$\mathbb{P}(X = v_j) = \frac{1}{m}. \quad (26)$$

We define a stochastic process, namely, the realizations of  $X$  through different time steps,  $X(1), X(2), \dots, X(t)$ . Let us define another random variable,  $Y$ , namely the number of realizations of  $X$  needed to ensure that there has been one realization of  $X$  equal to  $v_j$ :

$$Y = \min_t \{X(t) = v_j\} \quad (27)$$

Clearly, and due to the symmetry of our experiment, all the nodes behave in the same way. Furthermore,

$$\langle Y \rangle = m \quad (28)$$

i.e. we need, on average  $m$  realizations of  $X$  in order to obtain, at least, one realization  $X = v_j, \forall v_j \in V$ . We observe that the above random experiment is exactly a random switching over a graph containing  $m$  nodes, and that  $\langle Y \rangle$  is the  $\langle \text{MFPT} \rangle$  of this process. Let us suppose we have a  $\langle \text{MFPT} \rangle < m$ .



This implies that, on average

$$(\forall v_j)\mathbb{P}(X = v_j) > \frac{1}{m} \quad (29)$$

which is a contradiction, since the graph has  $m$  nodes. Thus, for a given graph  $\mathcal{G}(V, \Gamma)$ :

$$\langle \text{MFPT} \rangle_{\mathcal{G}} \geq |V|. \quad (30)$$

This value represents a horizontal asymptote in the model of SRW as  $q$  increases, and it is clearly defined in our model experiments (see Fig. 3).

### 3. Results and Discussion

Our main result was that SRW exploration, a cognitive inspired strategy that combines random-walking with switching for random exploration of networks, decreased the  $\langle \text{MFPT} \rangle$  of all models for all SRW variants. This means that, on average, the number of steps needed to travel between every pair of nodes decreases and thus the overall exploration abilities of a SRW within the networks improve with respect to RW.

Regarding  $\text{SRW}^=$  [Fig. 3(a)], exploration performance of random-modular and small-world networks severely improves, overtaking scale-free at  $q = 0.1$ . Moreover, at  $q = 0.3$  all the networks but scale-free converged, leading to a remarkable scenario where modularity and high clustering coefficients are not topological handicaps for an efficient information retrieval.

Switching in  $\text{SRW}^+$  severely improves  $\langle \text{MFPT} \rangle$  in modular and small-world networks while hardly decreases in scale-free and random. The reason is that a random-walker on both kind of networks already shows a gradient to visit highly connected nodes [Noh & Rieger, 2004], and a positive-degree switching supported rather than compensated this effect due to redundancy on hubs Fig. 3(b).

In  $\text{SRW}^-$ , intermediate values of  $q$  (around 0.6 for all but scale-free models) showed optimal performance with a similar effect to the one produced by  $\text{SRW}^=$ . However, it only partially succeeded in compensating the already commented natural RW gradient for hubs [Fig. 3(c)]. Interestingly, those  $q$  values close to 1 produced an inverse situation where hubs are so unlikely to be reached that the overall exploration performance decreased for all the models but dramatically for scale-free model, where the degree heterogeneity is specially high. On the contrary, small-world model showed a very similar performance when explored by any of the three SRW variants. The reason is that in this model,

the degree distribution is very homogeneous, and thus different degree gradients of switching produced very little differences.

The approximate convergence of the exploration efficiency (for most of the topologies when using  $\text{SRW}^=$  or  $\text{SRW}^-$  with a moderate switching rate) allows a system to organize information or to evolve without compromising exploration and retrieval efficiency. In this sense, semantic memory might be organizing information in a strongly modular or locally clustered way without compromising retrieval performance of concepts. In a more general perspective, the addition of a switching mechanism and its interaction with random-walker dynamics opens a new framework to understand processes related to information storage and retrieval. Indeed, switching not only mitigates exploration deficits of certain network topologies but might also provide certain robustness to the system. For instance, the rewired links (known as short-cuts) in both small-world and random-modular models contribute to facilitate access to different regions of the network. Those short-cuts might compensate a switching impairment or dysfunction and vice versa, i.e. switching would ensure an accurate exploration of the network even though a targeted attack removed those short-cuts permanently.

Similar mechanisms to switching have been observed in the context of information networks. In particular, the iterative algorithm *PageRank* estimates a probability distribution used to represent the likelihood that a person randomly clicking on links will arrive at any particular page for a hyperlinked set of documents (e.g. the world-wide-web) [Brin & Page, 1998]. The user is supposed to be a *random-surfer* who begins at a random web page and keeps clicking on links but never hitting back. The *damping-factor* is an additional item that includes the fact that the user can get bored and start on another random page. The combination of these two processes is used by the Google Internet search engine to estimate the relevance of different links (PageRank values). Interestingly, while the objective (rank link targets) and the framework (hyperlinked documents, i.e. directed graphs) are not the same, the cognitive-inspired SRW described here and PageRank algorithm combine random-walks restricted to a topology with an extra-topological mechanism in order to evaluate tasks in complex networks.

The model proposed here could have implications in other systems that usually have a conflict

between organization and retrieval or spreading efficiency. It will be a topic of further studies in other phenomena unrelated to cognitive processes such as infection epidemiology, information spreading or energy landscapes.

## Acknowledgments

J. Goñi is a fellow of the Government of Navarra. I. Martincorena is a fellow of the Caja Madrid Foundation. This work was supported by James McDonnell Foundation to BCM, MEC of Spain BFM2006-03036 to SAT and the European Commission (NEST-Pathfinder: ComplexDis contract number: 043241) to PV. Thanks to Ricard V. Solé for his useful comments and support in the design of figures.

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