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The effects of asymmetric coupling in hyperchaotic attractors

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Abstract

Three hyperchaotic systems coupled asymmetrically between identical pairs are analized in order to look for general features in synchronization among them. Lyapunov Exponents (LE) has been calculated and plotted as a function of a coupling parameter and a symmetry parameter. By this way is possible to recognize some special regions where the complexity is reduced and some kind of synchronization exists.

Keywords: hyperchaos, synchronization, asymmetric coupling

1. Introduction

Hyperchaotic behavior appears in a nonlinear dynamical system when more than one Lyapunov exponent becomes positive and normally arises as a natural regime in extended space-time systems, delayed systems or in situations where many oscillators are coupled (a normal situation in complex networks).

In all of these cases, it is usually very difficult to understand what is happening physically inside the system for different reasons, like the presence of spatial symmetries restricting the possible solutions, or delays transforming the system into an infinite-dimensional one. But sometimes, when the attractors presents some kind of symmetry properties it is easier to take a clear point of view about the system's dynamics [1].

In a recent paper [2] we reported an analysis of a several low dimensional systems on which windows of synchronization appear suppressing the chaotic oscillations. In this work we analyze the effects of the asymmetric coupling in the hyperchaotic extension for some chaotic attractors as Chen or Lü and another attractor with some symmetry properties [3]. The effects of the coupling varies from the kind of synchronization achieved to the chaos suppression. Also some general conclusions will be reported.

2. Chen Attractor

Chen's System [4] is a very important reference in the field of the hyperchaotic attractors. In this work we used the equation set that appears in [5]. Introducing some differences in order to couple two identical attractors between them. Instead of use the coupling matrix and then look for the Master Stability Function as it was proposed in [6] we introduce two parameters for coupling them. One related to the symmetry (θ) and the other to the coupling strength (ε).

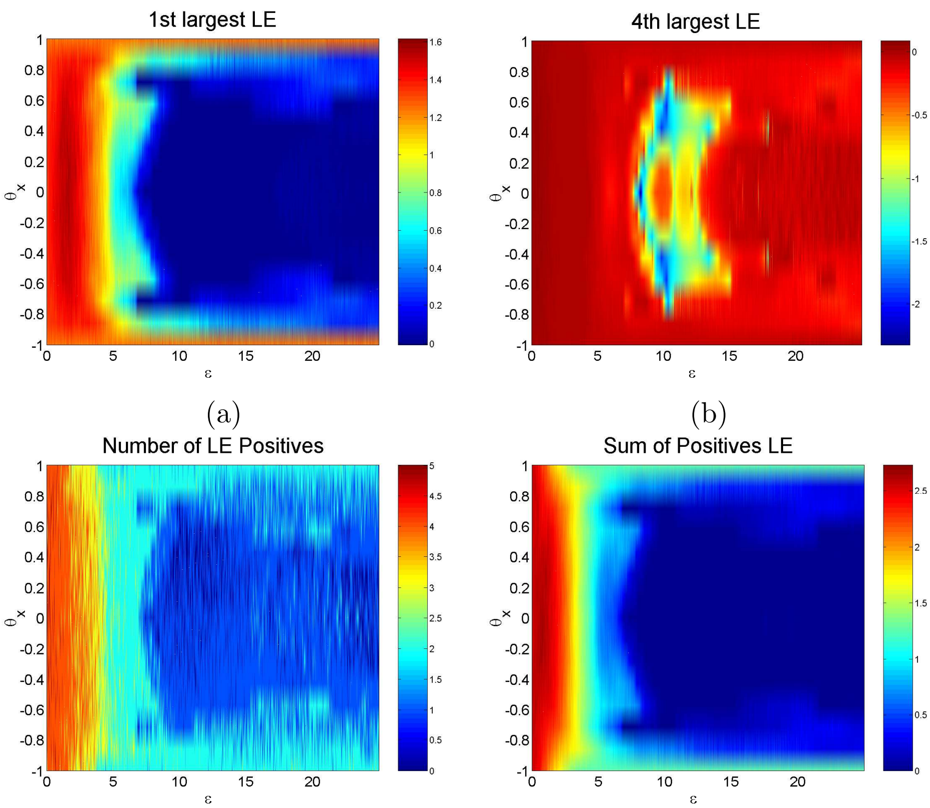
The θ parameter selects which is the driver or the driven in the coupled system. Note that the Master-Slave configuration is one specific case, that occurs when θ is equal -1 or 1. Also it is possible a symmetric coupling when $\theta = 0$. So all the cases are constrained to be in $\theta \in [-1, 1]$.

On the other side, ε parameter is used for setting the coupling strength between two variables. The interval of values of ε is $[0, \infty)$, the "effective" range depends on the maximum amplitude of the signal variable. Then ε can not be normalized easily because the maximum amplitude changes with ε. So, a new normalization must be done for each value of ε, and this requires a lot of time for data processing.

Once the considerations above have been done, it is possible to write the equation set as follows:

$$\begin{aligned} \dot{x}_{1,2} &= a(y_{1,2} - x_{1,2}) + \frac{\varepsilon_x}{2}(1 \pm \theta_x)(x_{2,1} - x_{1,2}) \\ \dot{y}_{1,2} &= -dx_{1,2} - x_{1,2}z_{1,2} - cy_{1,2} - w_{1,2} \\ \dot{z}_{1,2} &= x_{1,2}y_{1,2} - bz_{1,2} \\ \dot{w}_{1,2} &= x_{1,2} + k \end{aligned} \qquad (1)$$

The asymmetric coupling is done through x variables. The parameter values are $a = 36, b = 3, c = 28, d = 16$ and being k a fixed time delay. In this work this delay is set for obtaining an hyperchaotic attractor [7]. For further information, there is an study of the Chen's dynamics using k as control parameter [8].



(a) (b) (c) (d)

Figure 1: In this figure the first and the fourth largest Lyapunov Exponents are shown in (a) and (b). The value of the exponent is represented by the color and the corresponding values is shown in each colorbar. The coupling strength is represented in the abscissa and the symmetry of the coupling in the ordinate.

Figure 2. shows some colormaps where the Lyapunov Exponents appear as a function of the coupling strength (abscissa axis) and the symmetry parameter (ordinate axis), the most red coloured belongs to the largest value for LE. The 1^{st} and the 4^{th} largest LE are shown in (a) and (b) respectively. In (c) is show the number of positive LE, and the sum of them is shown in (d) in order to provide an upper boundary for the entropy of the system.

These results show a similar behavior reported in [2], which corresponds to an "island" where the Lyapunov Exponents decrease. In this case, the island is observed clearly in the 4th largest Lyapunov Exponent instead of the largest Lyapunov Exponent. Note that the effect of the island affects at other LE also, but in both cases in the closest-to-zero positive exponent there is a transition among positive and negative values. Due to the LE's are crossing through cero hyperchaos suppression occurs within the islands.

The chaotic Chen's basin of attraction shows the riddled property [5] for an specific region of the attractor. In these numerical simulations this behavior is extended to more regions inside the coupled attractor because a bubbling transition [9] arises and the system becomes more unstable. These effects can be checked in the Lyapunov Exponent map as some "spikes" that appear inside. An interesting property of this riddled basin is that different initial conditions involve different dynamic behaviors from chaos suppression, as occurs in [2] to a decrease in the autocorrelation signal. This property was not cited in [5].

3. Lü Attractor

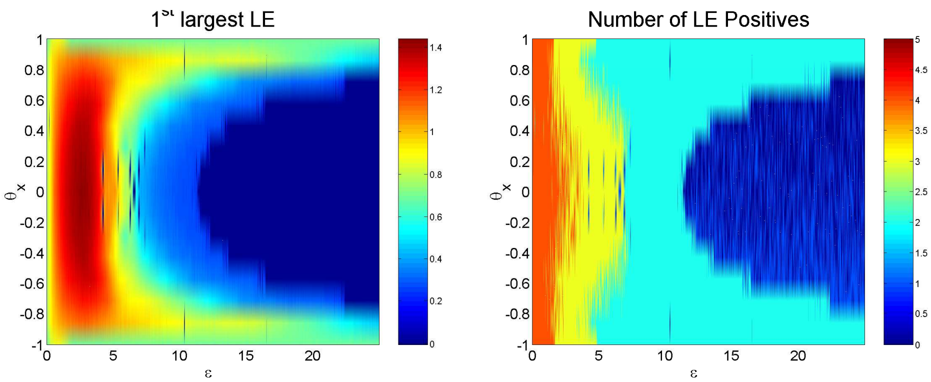
Another famous chaotic attractor is the Lü attractor which is the transition model between the chaotic Chen attractor and Lorenz attractor [10]. But this work is focused on the hyperchaotic attractors, so the model used is an extended version to hyperchaos generated via state feedback control [11]. The coupling model is the same that it was used with the Chen attractor, so the coupled equation set is written as follow:

$$\begin{aligned} \dot{x}_{1,2} &= a(y_{1,2} - x_{1,2}) + w_{1,2} + \frac{\varepsilon_x}{2}(1 \pm \theta_x)(x_{2,1} - x_{1,2}) \\ \dot{y}_{1,2} &= -x_{1,2}z_{1,2} - cy_{1,2} \\ \dot{z}_{1,2} &= x_{1,2}y_{1,2} - bz_{1,2} \\ \dot{w}_{1,2} &= x_{1,2}z_{1,2} + dw_{1,2} \end{aligned} \quad (2)$$

The parameters do not vary too much respect the Chen's values due to in both cases the Lorenz was the reference. There are several articles where the bifurcation diagrams and the dynamics changing the parameter values were studied [8, 12, 13], but in this work we set the parameter values as $a = 36$, $b = 3$, $c = 20$ and $d = 1$, in order to obtain an hyperchaotic attractor with 2 positive LE [11].

The largest Lyapunov Exponent against the coupling and the symmetry parameter is shown in the figure 3. (a). Note that some islands appears at coordinates $\varepsilon_x \approx 3$ and $\theta_x \approx \pm 0.2$. These islands are not an effect of the riddled basin [9], they arise from the coupled attractor's equation system.

Also the number of the positive LE is shown in the figure 3. (b). Note that in these islands hyperchaos suppression is achieved by the asymmetric coupling because the number of positive LE decays to zero.



(a) (b)

Figure 2: The largest LE is represented in (a) as a function of the coupling strength (abscissa) and the symmetry (ordenate) for the asymmetrically coupled Lü identical attractors. The value of the exponent is represented as a color of the colorbar. The number of positive LE is shown in (b)
.

4. A Takens-Bogdanov Attractor

The last attractor studied in this work is a mathematical model used in time-dependent pattern formation processes [3]. The hyperchaotic oscillator was studied in [14]. The following equation set was used:

$$\begin{aligned} x'_{1,2} &= y_{1,2} + \frac{\varepsilon_x}{2}(1+\theta_x)(x_{2,1}-x_{1,2}) \\ y'_{1,2} &= \mu x_{1,2} + x_{1,2}(a(x^2_{1,2}+z^2_{1,2}) + bz^2_{1,2}) \\ z'_{1,2} &= w_{1,2} \\ w'_{1,2} &= \mu z_{1,2} + z_{1,2}(a(x^2_{1,2}+z^2_{1,2}) + bx^2_{1,2}) \end{aligned} \tag{3}$$

Applying the same method for numerical simulations, some of the Lyapunov Exponents maps obtained are shown in figure 4..

Some discussions about the synchronization of two identical symmetrically coupled attractors was done in [15], and we report here some results considering asymmetric coupling.

As it can be seen in the figure 4., this system has a very "sensitive" riddled basin. Also the two largest LE are not modified in the same way of the other ones, due to the symmetry and the hyperchaotic behavior .

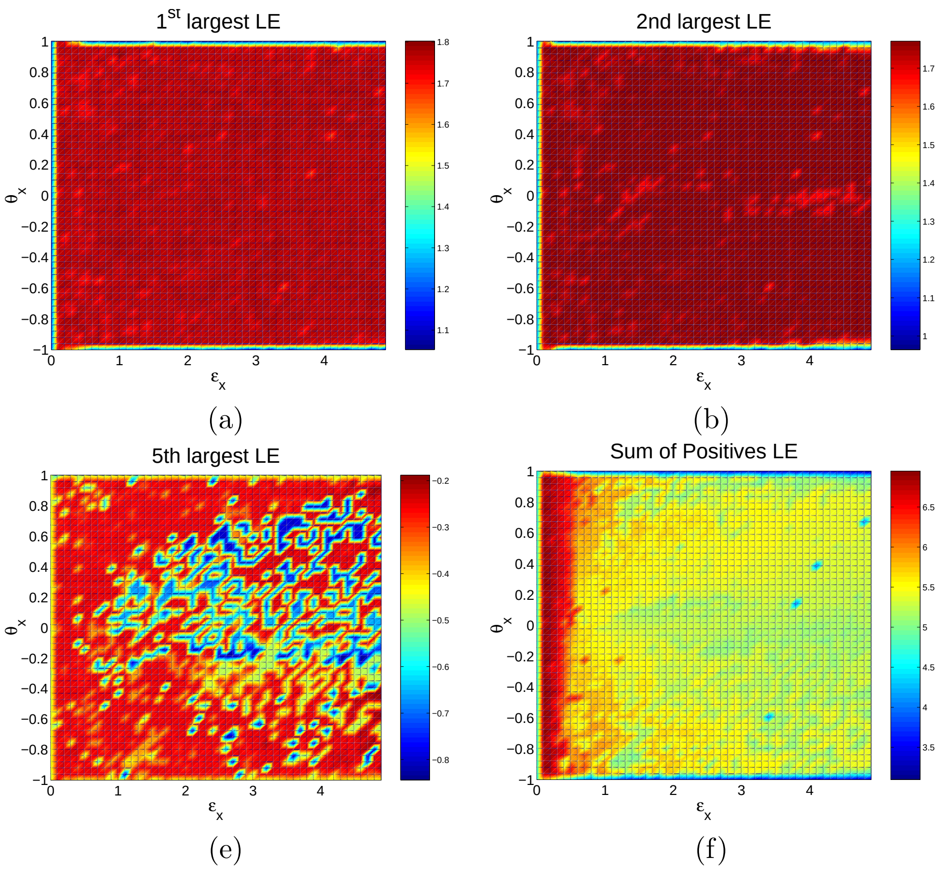


Figure 3: The first, second and fifth largest LE are shown in (a),(b) and (c). The value of the exponent is represented in colorbar for each plot. The abscissa is the coupling strength and the symmetry factor is the ordinate. The sum of all the positive LE is shown in (d).

5. Conclusions

Calculations of the LE of two asymmetrically coupled identical hyperchaotic attractors as a function of θ, the coupling symmetry, and ε, the coupling strength parameter.

Each hyperchaotic system presents a “fingerprint” in the LE map. In this region some of the positive LE decrease suddenly. The complexity of the system is reduced and some kind of synchronization appears.

From the analysis performed we conclude also that an increase in the coupling strength does not necessarily imply a decrease in the complexity of the system or in a better synchronization. The master-slave configuration (unidirectional coupling) it is not the best option to reduce the complexity of the coupled system as in can be seen in the figures 2. and 3..

On the other side, the asymmetrical coupling allow to change the dynamical properties of a system, and in some cases permits hyperchaos suppression as occurs in Lü and Chen systems. This hyperchaos suppression does not imply complete synchronization, as it was exposed in [15]. As example, the hyperchaotic Chen attractor the complete synchronization is achieved by hyperchaos suppression. Instead of the hyperchaotic Takens-Bogdanov and Lü attractors where the complete synchronization is achieved, but in different regions of the LE map related with the decrease of LE.

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References

[1] R. Gilmore, C. Letellier, *The Symmetry of Chaos*, Oxford University Press, 2007.

[2] J. Bragard, G. Vidal, H.L. Mancini, C. Mendoza, S. Boccaletti, *Chaos suppression through asymmetric coupling*, Chaos, 17(4), 2007.

[3] T. Ondarçuhu, G.B. Mindlin, H.L. Mancini, C. Pérez-García, *Dynamical Patterns in Bénard-Marangoni Convection in a Square Container*, Phys. Rev. Lett. 70(25), 3892-3895, 1993.

[4] T. Zhou; Y. Tang; G. Chen, *Chen's attractor exists*, Int. J. Bif. Chaos (2004), 14(9).

[5] Z. Liu; Y.Li; G.Chen, *The basin of attraction of the Chen attractor*, Chaos, Solitons & Fractals (2007) 34.

[6] L.M. Pecora; T.L. Carroll, *Master Stability Function for Synchronized Coupled Systems*, Phys. Rev. Lett. 80(10), 2109-2112, 1998.

[7] T. Gao; Q. Gu; Z. Chen, *Analysis of the hyper-chaos generated from Chen's system*, 5657 10.1016/j.chaos.2007.06.125 S0960-0779(07)00422-5.

[8] T. Gao; Z. Chen; Q. Gu; Z. Yuan, *A new hyper-chaos generated from generalized Lorenz system via non-linear feedback*, Chaos, Solitons & Fractals 35 (2008) 390-397.

[9] S.C. Venkataramani; B.R. Hunt; E. Ott *Bubbling Transition* Phys. Rev. E 54(2), 1346-1360, 1996.

[10] J. Lü; G. Chen, *A new chaotic attractor coined*, Int. J. Bif. and Chaos, 12(3), (2002) 659-661.

[11] A. Chen; J. Lu; J. Lü; S. Yu, *Generating hyperchaotic Lü via state feedback control* Physica A 364, (2006), 103-110.

[12] S. Chen; J. Hu; C. Wang; J. Lü, *Adaptive synchronization of uncertain Rössler hyperchaotic system based on parameter identification*, Phys. Lett. A, 321 (2004) 50-55.

[13] G. Wang; X. Zhang; Y. Zheng; Y. Li, *A new modified hyperchaotic Lü system*, Phys. Lett. A, 371 (2006) 260-272.

[14] G. Vidal; H. Mancini, *Complete Synchronization Between Hyperchaotic Attractors*, Proceedings Net-Works 2007, Aranjuez (Madrid), edited by R. Criado, M. Romance.

[15] G. Vidal; H. Mancini, *Hyperchaotic synchronization under square symmetry*, IJBC (Submitted)