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COMPARISON OF DATA FROM BÉNARD–MARANGONI CONVECTION IN A SQUARE CONTAINER WITH A MODEL BASED ON SYMMETRY ARGUMENTS

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We report the results of a Bénard–Marangoni convection experiment. The container has a square symmetry and a small aspect ratio (i.e. the quotient between a typical horizontal length and the fluid depth). Based on a few dynamical considerations and the symmetries of the problem we improve on a previously proposed model, building a new one which successfully predicts the existence and parameter space location of qualitatively new solutions.

1. Introduction

Pattern formation and spatiotemporal complexity are the center of attention of a fast growing number of scientists. Among the most studied systems stand the convection experiments. The lack of a qualitative theory of partial differential equations is in part responsible for a seemingly never-ending succession of surprises even for "simple" experiments. For example, the Feigenbaum cascade and other routes to chaos were observed in the convection of a fluid enclosed in a rectangular container [Libchaber & Maurer, 1983; Dubois *et al.*, 1983] (Rayleigh–Bénard problem). In this case, the instability mechanism is provided by buoyancy. A variant of this problem is the so-called Bénard–Marangoni problem where the fluid has an open surface, which provides a new instability mechanism (surface tension variation with temperature). Recently some attention has been drawn to this instability in small aspect ratio vessels [Rosenblat *et al.*, 1982; Ondarçuhu *et al.*, 1993a]. This system displays a dynamics by no means trivial, even for parameter values close to those of the onset of the convection. To explain such a dynamics constitutes a challenge to the theoretical study because the small aspect ratio condition enhances the role played by the boundary conditions, which are always tricky to deal with. In this work we report the results on a small aspect ratio convective experiment. The set-up consists of a square container heated from below with an upper free surface, the details being provided in Sec. 1. The parameters of our system (depth of the fluid, temperature) are such that the system organizes itself into four

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internal convective cells. In some cases the four cells are quadrilateral, in others two are quadrilateral and two pentagonal. Moreover, in some cases there is a complicated dynamics that consists in alternation between those states.

In a previous work [Ondarçuhu *et al.*, 1993b] we reported preliminary results from this experiment. The choice of an appropriate scalar variable allowed us to register time series data files for different values of the parameters, and a model was constructed to summarize the results. The two ingredients used to build such a model were (a) the symmetry of the set-up and (b) the observation that the original convective pattern, which shared the symmetry of the container, could break into either a pair of symmetrically conjugated patterns or an oscillatory one, slightly changing the "route" in parameter space. This last observation suggested that the system was operating at parameter values close to those for which Takens–Bogdanov bifurcation occurred [Guckenheimer & Holmes, 1983; Arrowsmith & Place, 1990], and therefore provided the structure of the linear part of our model. Typically, when more than one mode are going unstable, the dynamics observed is more complicated than what one might expect from the superposition of the individual modes due to the presence of nonlinearities. We introduced in our model nonlinearities that preserved reflection symmetry and obtained a dynamics qualitatively similar to the one observed experimentally. This is reviewed in Sec. 1.

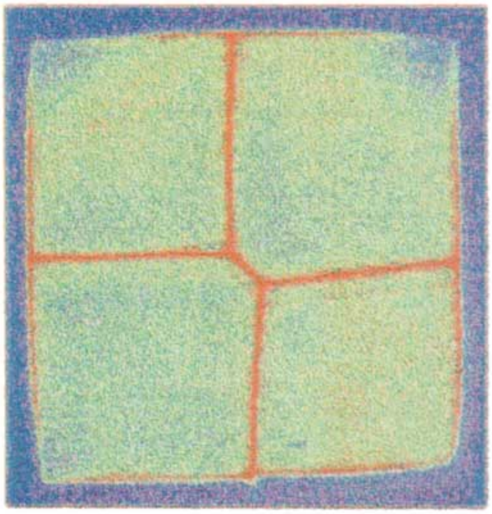
From the theoretical point of view, there was a puzzling question. The only symmetry introduced in the model was reflection symmetry, while the boundary conditions impose a larger symmetry: the symmetry of a square. Was it possible to construct a model in which the linear part would reflect the fact that symmetry breaking phenomena compete with oscillatory phenomena but are now equivariant under the full symmetry of the system? Would such a model predict new dynamics? The answer to both questions was positive. The new model had actually been studied by Armbruster [1990], [Armbruster *et al.*, 1989], and moreover for wide ranges of parameter space the model displays chaotic behavior. These theoretical questions lead us to repeating our experiment, now paying attention to certain regions of parameter space in which (according to the theory) chaotic behavior could be possible. The results of those experiments suggest that the new theory is more complete than the one proposed by us before, as the predictions of the theory have been verified experimentally.

This paper is organized as follows. In Sec. 2 we review our first results [Ondarçuhu *et al.*, 1993b] and the model constructed from the time series data. Section 3 contains a discussion on the shortcomings of the model derived in Sec. 2, and a new model is proposed which includes the previous one as a particular case, and enables us to predict new qualitatively different behavior. In Sec. 4 we report new experimental data files from the regions of parameter space that according to our new model could show chaos, and comparison with the theory is performed. Section 5 summarizes our results and proposes new questions.

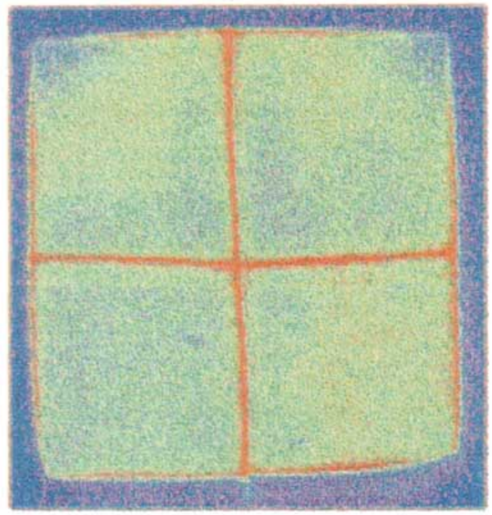
2. A First Look at the Data

As we described briefly in the introduction, the system under study consisted of a fluid (350 cSt silicone oil) contained in a square vessel of small aspect ratio heated from below [Ondarçuhu *et al.*, 1993b]. The actual dimensions are reported in the caption of Fig. 1. Using a shadowgraph technique we obtained the images displayed in that figure, where the red lines correspond to the cold parts of the pattern, i.e. regions in which the flow is descending. Figures 1(a)–1(c) display the three qualitatively different patterns that constitute the "skeleton" of the dynamics of our system. The first convective pattern that is observed as the system is heated from below is shown in Fig. 1(b). The four cells are quadrilateral, and therefore the pattern and the boundary conditions share the same symmetry. If the temperature is further increased, the system bifurcates to either a pattern like the one presented in Fig. 1(a) or a pattern like the one presented in Fig. 1(c). The patterns are symmetric conjugates by reflection symmetry. As reported by Ondarçuhu *et al.* [1993b], when the temperature is further increased the patterns begin to oscillate. The oscillations consist of (seemingly) periodic modulations of the length of the link between the square cells [see Figs. 1(a), 1(c)]. For higher values of the temperature, the system begins to oscillate in a qualitatively different way, namely an alternation between a pattern as the one in Fig. 1(a) and another as the one in Fig. 1(c), passing through the symmetric pattern 1(b). The order of magnitude of a typical oscillation time is one minute.

In order to make a quantitative description of the phenomena, we must construct a relevant



(a)



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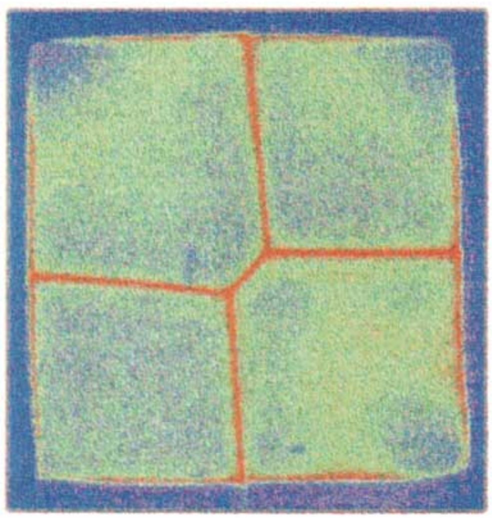


(c)

Fig. 1. False-color shadowgraph images of the convective patterns in a container of aspect ratio $\Gamma = 4.46$ (size of the cell, 68×68 mm^2) filled with silicone oil of viscosity 350 cSt. Red lines correspond to minima of the temperature field where the motion of the fluid is downwards. (b) corresponds to the symmetric pattern appearing at threshold and (a) and (c) to the asymmetric ones. They are born from the symmetric pattern in a pitchfork bifurcation.

observable quantity (i.e. a number both measurable and informative). The important features that such a quantity has to reflect are (a) length of the link and (b) its inclination (left or right). Let us define

$$x = d\cos(\alpha)\,, \qquad (1)$$

with $\alpha \in (0, \pi)$ as shown in Fig. 2. Notice that the patterns of Fig. 1 preserve a symmetry, reflection with respect to the diagonals of the square. In terms of our model this constrains α to actually only two values ($\alpha = \pi/4$ or $\alpha = 3\pi/4$). It is interesting that the first symmetry breaking stationary bifurcation kept part of the original symmetry. In the framework of the theory of bifurcations in the presence of symmetries, this result is not surprising (see discussion on the equivariant branching lemma by Golubitsky *et al.* [1985]).

Our first approach to the analysis was to record a time series data of the scalar x for each of the qualitatively different behaviors that we had

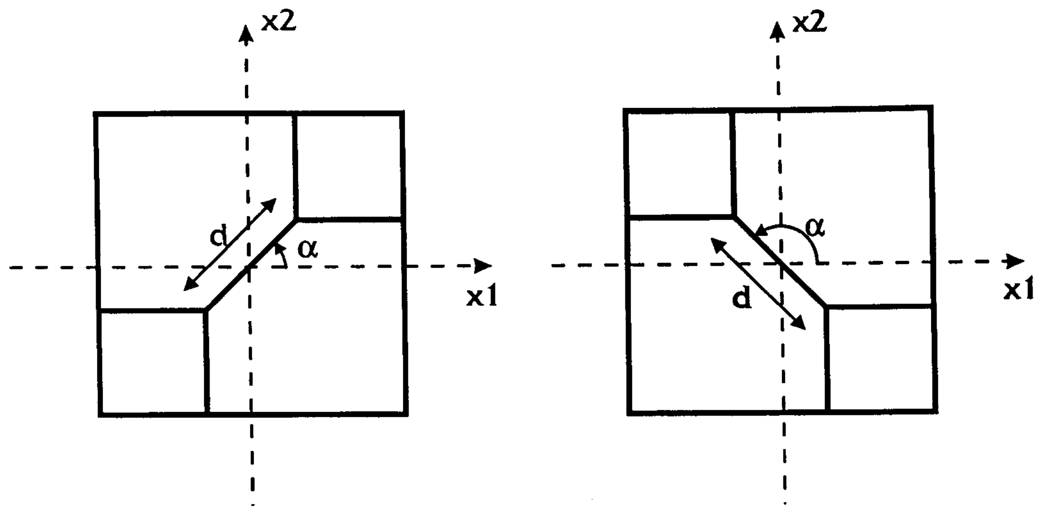


Fig. 2. Schematic representation of the asymmetric patterns where d stands for the length of the diagonal segment and α is the angle between the segment and the X_1 axis.

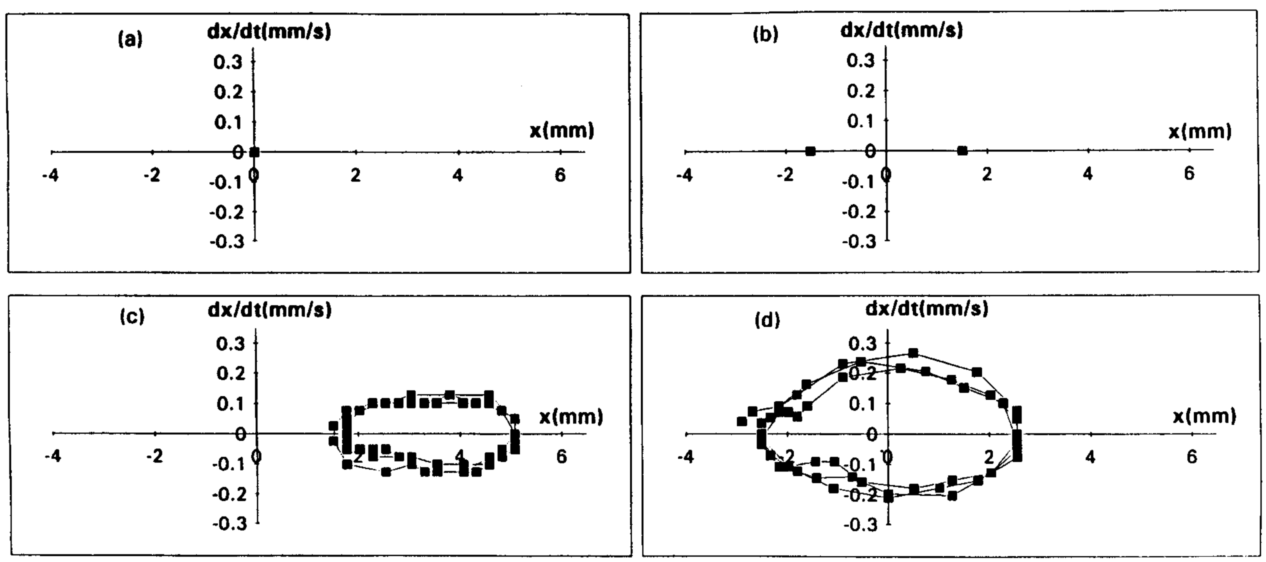


Fig. 3. Reconstructed phase space (x, x') for different time series data. (a) displays a fixed point corresponding to the symmetric pattern. (b) shows the fixed points associated with the asymmetric patterns. (c) displays a limit cycle (an asymmetric oscillation) and (d) a limit cycle corresponding to the symmetric oscillation.

identified. Is it possible to construct a dynamical model for our system directly from the time series data? We began by embedding the data in the (x, x') space (where x' stands for the time derivative of x). Notice Fig. 3 shows that is a good embedding (in the sense that no self intersections of the flow are observed within the experimental precision), and therefore one can attempt to write a system of equations to model the dynamics using these variables. Such a system reads as follows,

$$x' = y\,, \tag{2}$$

$$y' = f(x, y)\,. \tag{3}$$

This system can be equivariant under reflection symmetry broken in the first stationary bifurcation $(x \to -x)$ provided that $f(x, y)$ is properly chosen. According to Eq. (2), the action of the reflection on the variable y has to be $y \to -y$; therefore the equivariance of the system will be guaranteed if $f(x, y) = -f(-x, -y)$. Performing a Taylor expansion around $(x, x') = (0, 0)$ [the mathematical representation of the stationary pattern of Fig. 1(b)],

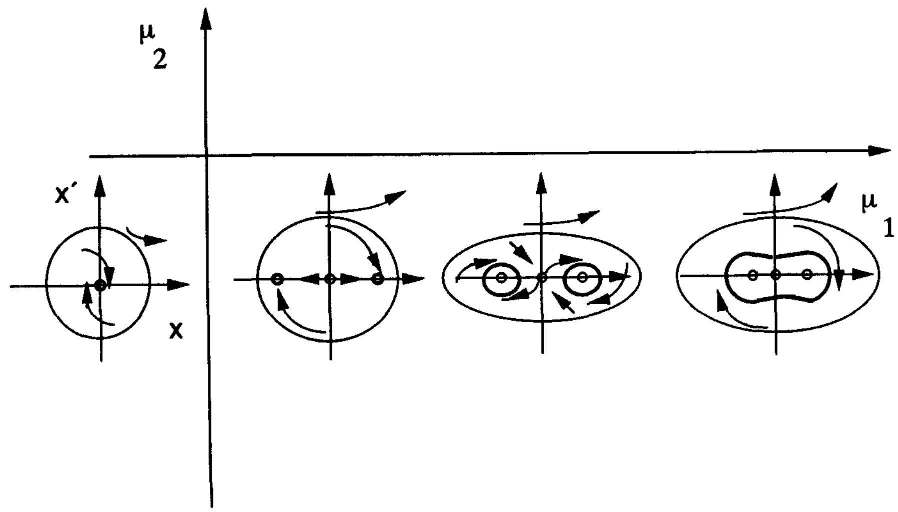


Fig. 4. Unfolding of the Z_2 equivariant Takens–Bogdanov bifurcation.

it is possible to write

$$f(x,\, y) = \mu_1 x + \mu_2 y + a_1 x^3 + a_2 x^2 y + a_3 x y^2 + a_4 y^3 \,. \tag{4}$$

We still will have some information extracted from the experiment to "prune" this model. If Hopf and symmetry breaking bifurcations occurred closely in parameter space, the Jacobian of the model

$$\begin{pmatrix} 0 & 1 \\ \mu_1 & \mu_2 \end{pmatrix}$$

has to have two eigenvalues close to zero, i.e. μ_1 and μ_2 are themselves close to zero. Perturbations of the singular case ($\mu_1 = \mu_2 = 0$) were studied by Takens and Bogdanov [Guckenheimer & Holmes, 1983], who reported that interesting dynamics such as periodic and homoclinic orbits can occur. Moreover a normal form reduction shows that the study of the whole family of vector fields can be reduced to a study of only two systems, namely

$$x' = y \,, \tag{5}$$

$$y' = \mu_1 x + \mu_2 y \mp x^3 + x^2 y \,. \tag{6}$$

The solutions of this system for the case with minus sign are displayed in Fig. 4. Notice the qualitative agreement between the experimentally obtained phase space portraits (Fig. 3) and the theoretical ones.

This theoretical approach is in some sense "nonconventional." Typically one begins with the Navier–Stokes equations and tries to identify a small number of modes which are active for realistic parameter values by means, for example, of a Galerkin expansion. Then, after truncation at some order in the nonlinearities, one proposes a finite-dimensional dynamical system which provides, through numerical simulations, theoretical solutions to be compared with the experimental ones. In our approach, the starting point was dynamical observation (symmetry breaking bifurcations competing with Hopf bifurcations) plus geometrical consideration (the boundary conditions having square symmetry).

3. A Puzzling Theoretical Aspect

There is a puzzling element in the theoretical analysis performed in the previous section. The only symmetry that was used in the construction of the model was a reflection symmetry with respect to an axis perpendicular to two of the sides (X_2 in Fig. 2), while symmetries of the square are generated not only by the reflection with respect to that axis but also by reflection with respect to one of the diagonals of the square. In principle there is no reason to worry. It is true that a model both equivariant under D_4 (symmetries of the square) and with a linear part compatible with the collision between a Hopf and a pitchfork bifurcation will require a larger phase space. But it is also true that the D_4 symmetry will force the existence of subspaces invariant under its subgroups (as we will show later). In other words, the fact that only a subgroup of the symmetry was used so far leaves open the possibility that a more general model (now equivariant under the action of the full group) could predict new dynamical behaviors for unexplored regions of the parameter space. In this section we explore that possibility.

In order to build a system that is both equivariant under the D_4 group and has a linear part reflecting the collision between Hopf and symmetry breaking bifurcations, one has to enlarge the phase space. Armbruster [1989, 1990] addressed this problem. He studied vector fields that are equivariant with respect to D_4 acting on R^4 and have a linear part

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The most general vector field with this linear part and equivariant under D_4 has been shown to be, up to cubic order, reducible to the following normal form:

$$x' = y + \varepsilon^2 f z(zy - wx)\,, \tag{7}$$

$$y' = \mu x + x(a(x^2+z^2)+bz^2) + \varepsilon(\nu y + y(c(x^2+z^2) + ez^2 + dx(xy+zw))) + \varepsilon^2 f w(zy-wx)\,, \tag{8}$$

$$z' = w - \varepsilon^2 f x(zy - wx)\,, \tag{9}$$

$$w' = \mu z + z(a(x^2+z^2)+bx^2) + \varepsilon(\nu w + w(c(x^2+z^2) + ex^2 + dz(xy+zw))) - \varepsilon^2 f y(zy-wx)\,, \tag{10}$$

where the D_4 group is generated in this representation by the two reflections

$$\tau(x, y, z, w) = (z, w, x, y)\,, \tag{11}$$

$$\rho(x, y, z, w) = (-x, -y, z, w)\,. \tag{12}$$

Notice that the D_4 symmetry forces the existence of two-dimensional invariant subspaces given by $x = y = 0$, $z = w = 0$ and $x = \pm z$, $y = \pm w$. Within these subspaces, the system is equivalent to a $Z(2)$ (reflection) equivariant Takens–Bogdanov bifurcation problem, exactly as the one we proposed in the previous section.

So far, our new dynamical model is mathematically more appropriate than that proposed in the previous section, and contains it as a subcase (the dynamics being restricted to a Z_2 invariant subspace). But the real challenge for this model is the following; can it predict new solutions? There is an even more challenging question. We did not arrive at the conclusion that this model was necessary from a study of the Navier–Stokes equations with realistic parameters, but from purely dynamical considerations. Can we somehow predict in which regions of our parameter space the new solutions could be found? In order to answer these questions we will explore the dynamics of our new model. To a large extent, this will be a review of some of the work reported by Armbruster [1990]. A systematic study of Eqs. (7)–(10) is beyond the scope of this paper, as there are nine parameters to take into account. Despite this difficulty, we have the preliminary experimental results reported in Sec. 2 as a guide; our new model might predict new dynamics, but it must fit what we already observed. That will help us "prune" the parameter space.

In Armbruster [1990] some features of the solutions of Eqs. (7)–(10) were described. It is natural to expect a large variety of solutions. But let us concentrate our description of the system to parameter values close to those in which the dynamics is constrained to a reflection invariant subspace. We will follow Armbruster [1990] closely. As we have already pointed out, the dynamics in such subspaces is equivalent to one corresponding to a $Z(2)$ equivariant Takens–Bogdanov problem. It is well known that for such a problem a homoclinic bifurcation takes place where two symmetrically conjugated limit cycles coalesce into one symmetric limit cycle (see Fig. 4). But these homoclinic orbits are unstable in the four dimensional model as the origin is a saddle. Therefore, at least for a region of the parameters close to those in which the dynamics in the invariant subspaces is an homoclinic solution, the system will explore a wider region of the phase space, which will generically be chaotic. The conclusion is that even if we choose parameters such that the dynamics is largely restricted to a $Z(2)$ subspace (as we need in order to explain the results described in Sec. 2), we should find an edge of parameters for which the dynamics can be chaotic. In Fig. 5 we show numerical integrations of Eqs. (7)–(10). The parameters were chosen so that the dynamics would be largely the dynamics of a $Z(2)$ Takens–Bogdanov problem. Notice the complexification of the periodic solution as we approach the saddle in Fig. 5(d). If the parameter μ_2 is further increased, two symmetric conjugate strange attractors collide in a crisis, after which a symmetric chaotic solution emerges.

The structure of the chaotic solutions of system (7)–(10) itself deserves a detailed study. Several inner crisis that enlarge the antisymmetric strange attractors occur as the parameters are varied. In Fig. 6(a) we display a segment of time series of the variable x. Notice the alternation between period four looking segments with segments in which the dynamics consists of modulated oscillations. That time series was taken after a crisis between a period doubling strange attractor and an unstable

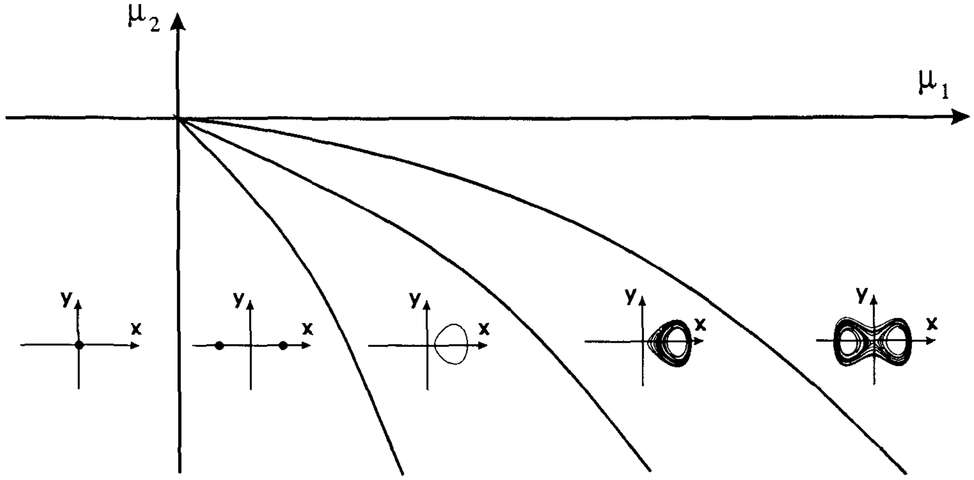


Fig. 5. Unfolding of the D_4 equivariant Takens–Bogdanov bifurcation [Eqs. (7)–(10) in the text] for parameter values chosen so that many features of the Z_2 equivariant problem are present (i.e. the dynamics is for wide regions with the unfolding parameters restricted to a Z_2 invariant subspace). For the simulations reported in this figure we used $a = -1$, $b = 1.1$, $c = 1.5$, $d = -0.5$, $e = -1.5$, $f = 0$.

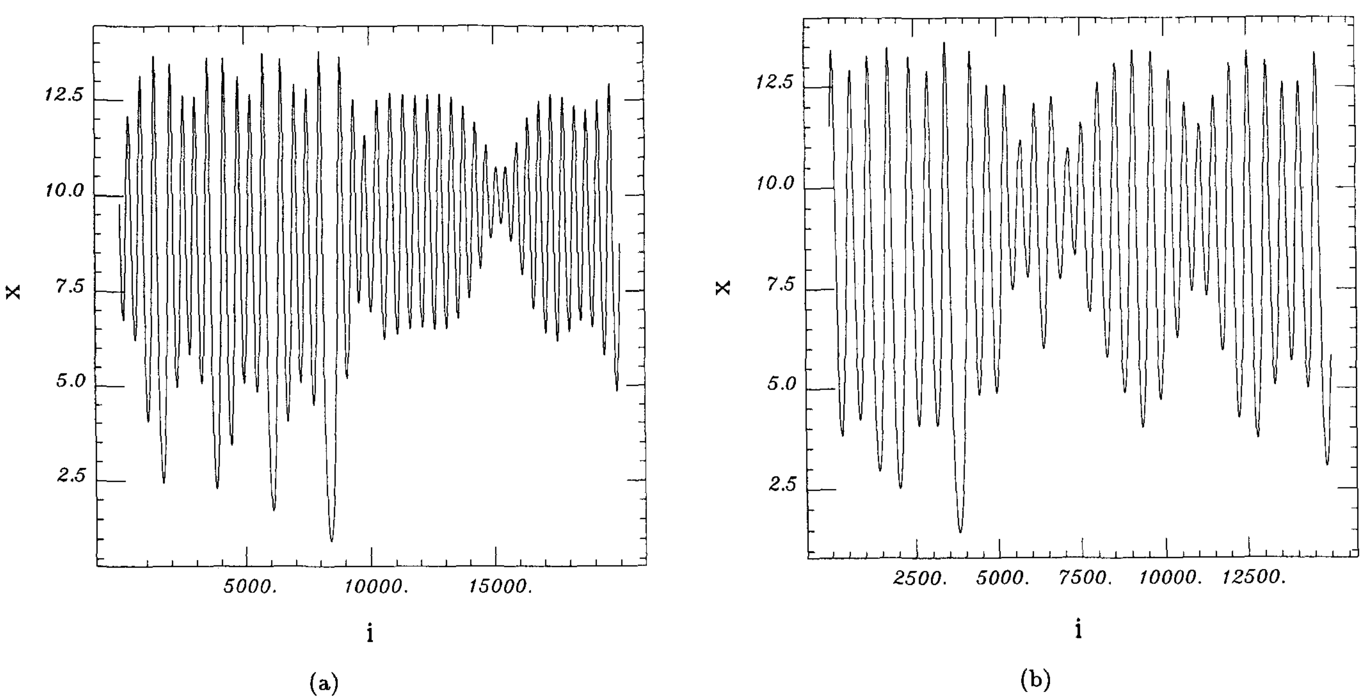
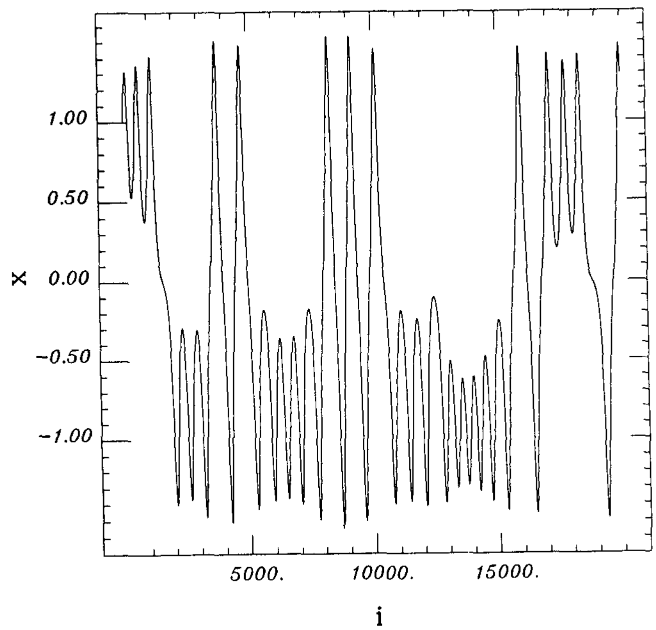


Fig. 6. Three times series data segments of the solutions of Eqs. (7)–(10) in the text for the values of a, b, c, d, e, f reported in the caption of Fig. 5. $\mu_2 = -1$ and $\mu_1 = 0.88$ (a), $\mu_1 = 0.91$ (b) and $\mu_1 = 0.93$ (c). (a) displays a segment of the x component of an asymmetric chaotic solution in which period-4 looking segments alternate with oscillations. In (b), a segment is shown in which modulated oscillations are dominant. (c) displays a segment of a symmetric strange attractor.



(c)

Fig. 6. (*Continued*)

orbit. Figure 6(b) presents a time series segment in which modulated oscillations are dominant. Figure 6(c) shows a segment of time series data of a set-symmetric strange attractor which arises after two symmetrically conjugated strange attractors collide.

These theoretical considerations suggested the possibility of new and richer dynamics in regions of parameter space that we had previously disregarded. According to the numerical simulations of the model, Eqs. (6) and (7), it is possible to find chaotic solutions. That would occur between the temperatures for which symmetric oscillations exist and the ones for which the solutions are antisymmetric oscillations. In the following section our new experimental findings are reported.

4. A Closer Look at the Data

According to the theoretical considerations discussed in the previous section, there were hints that new dynamics could occur in regions of the parameter space which we had over-looked [Ondarçuhu *et al.*, 1993b]. In this section we will report data from new observations, and analyze the time series data in order to identify the nature of the dynamics encountered.

The theory of dynamical systems provides many new tools for analysis of time series data. Some of these are merely theoretical games in the sense that they disregard the experimental fact that no matter how skillfully the experiment is run or recorded, there is always noise. Somewhere between the inspiring comments of Poincaré as to the necessity to look for qualitative behavior and the new sophisticated tests for chaotic data reported in the last years, that fact seems to have been overlooked many times. Our approach to the analysis of these time series data files is topological in nature. We will look for the presence of determinism by trying to identify unstable periodic orbits embedded in the data, and we will compare some qualitative features of these with those expected from theory.

As we have done before, we register the time series data of the scalar x (defined in Sec. 2). This time, we took more points for each "typical oscillation". This was done automatically using image processing techniques [Ekstrom, 1984] which allowed us to sample two measurements per second (more than one hundred points per typical oscillation). The method used to identify the link between the two square cells accumulates an error of approximately two pixels (while the typical amplitude of the oscillation is of the order of 30 pixels). A plot of a segment of x versus t is shown in Fig. 7, along with a smoothed time series obtained after applying a simple high frequency filtering process.

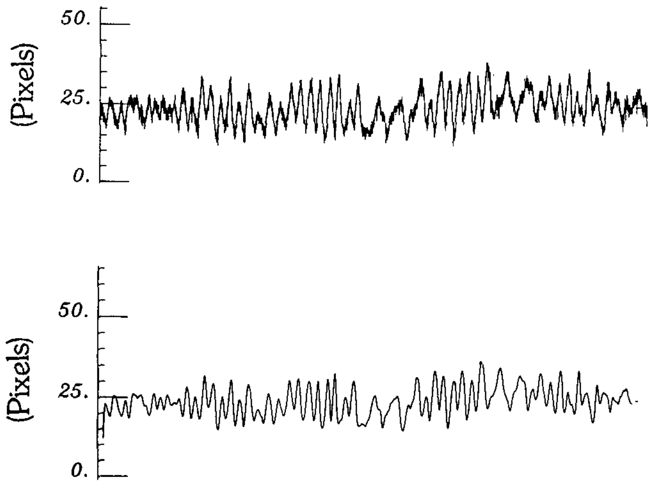


Fig. 7. Top: Experimental time series segment of the x variable. The units of x are pixels (one pixel corresponds to 0.25 mm) and the units of the time are $\frac{1}{10}$ sec. Bottom: $x(i)$ versus i after smoothing, where i stands for the position of x in the data file. The aspect ratio was $\Gamma = 4.49$ and the temperature was $T = 60.1C$.

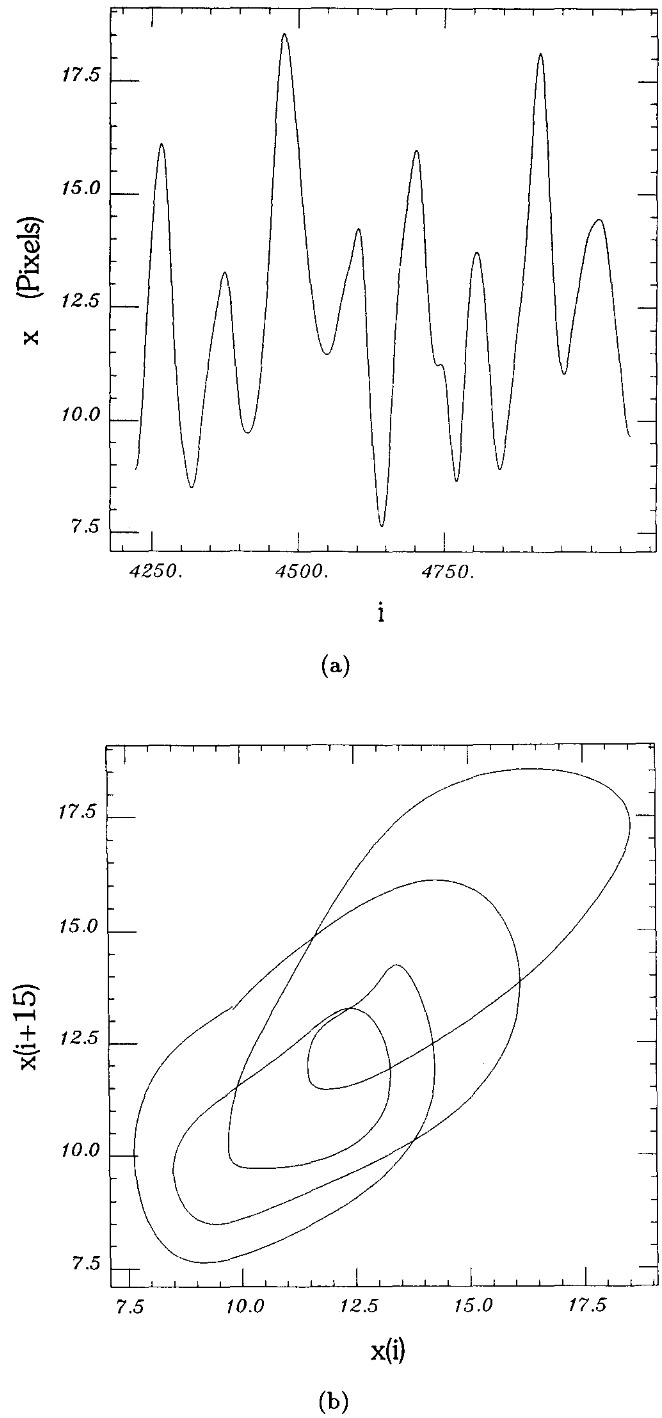
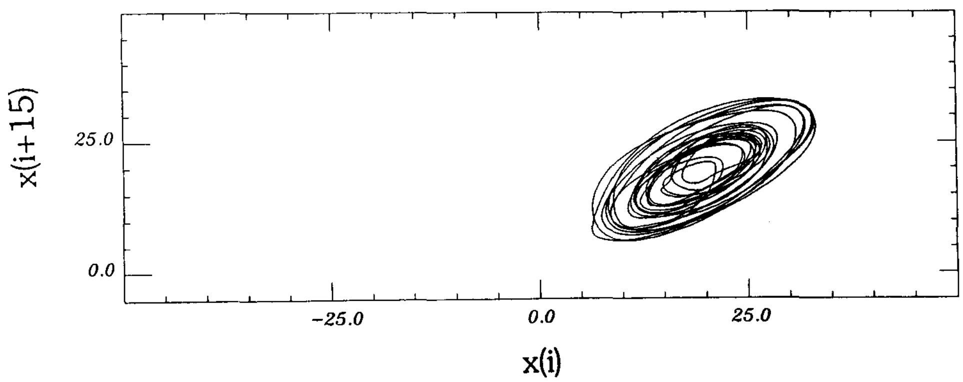


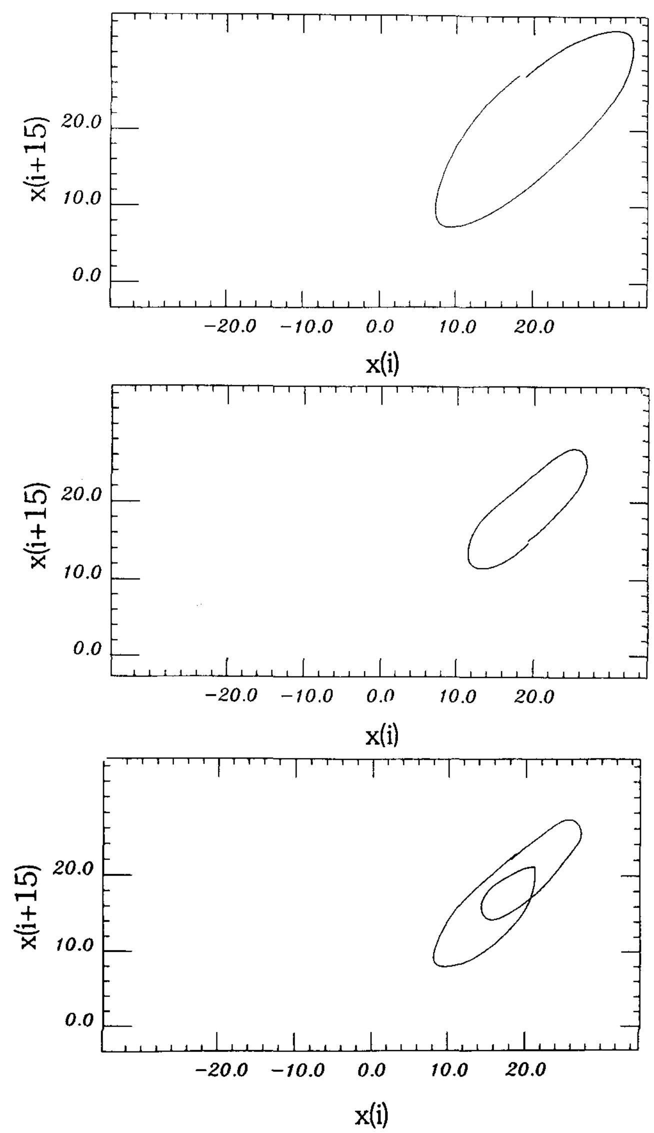
Fig. 8. Segments of the data file that provide evidence of the presence of unstable periodic orbits. (a) shows a good approximation of a period-4 segment, and (b) a time delay embedding $(x(i), x(i+15))$ of the same segment. The aspect ratio was $\Gamma = 4.49$ and the temperature was $T = 62.6C$.

The time series data shown in Fig. 7 presents a feature common in many of the data files. First, it is clear that there is a dynamics more complicated than simple oscillations. Also noticeable is the alternation between modulated oscillations and a qualitatively different regime. In general it is a delicate issue to state that a complicated data set obtained experimentally is chaotic. A strong topological signature is the presence of segments of the data files that behave almost as periodic orbits. That is due to the fact that embedded in a strange attractor there are unstable periodic orbits. Therefore, if a point in the attractor is near an unstable periodic orbit with relatively low positive eigenvalues, it can evolve in the neighborhood of that orbit and return to an epsilon neighborhood of the starting point. Clearly this will not be the case if the periodicity of the unstable orbit is not low enough. From now on, we will measure periods in units of the period associated to the dominant frequency in the data (which we have been denoting as the "typical oscillation time").

This method of detection of unstable periodic orbits has been implemented in Tufillaro *et al.* [1990], Mindlin & Gilmore [1992] and Mindlin *et al.* [1991]. Selecting pieces of the data file such that $(x(i) - x(i+p)) < \varepsilon$ for at least p points (the solution staying close to an orbit for at least twice its period), we obtained candidates for unstable periodic orbits constituting the skeleton of the solution. Figure 8(a) shows a period-4 segment and a two-dimensional time delay embedding. Notice that it is hard to identify the fact that a segment is not a periodic orbit. Figure 9(a) displays a



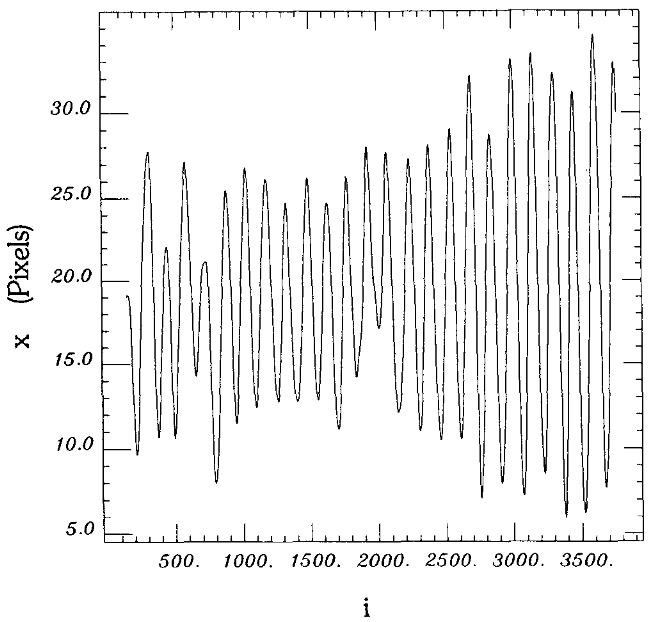
(a)



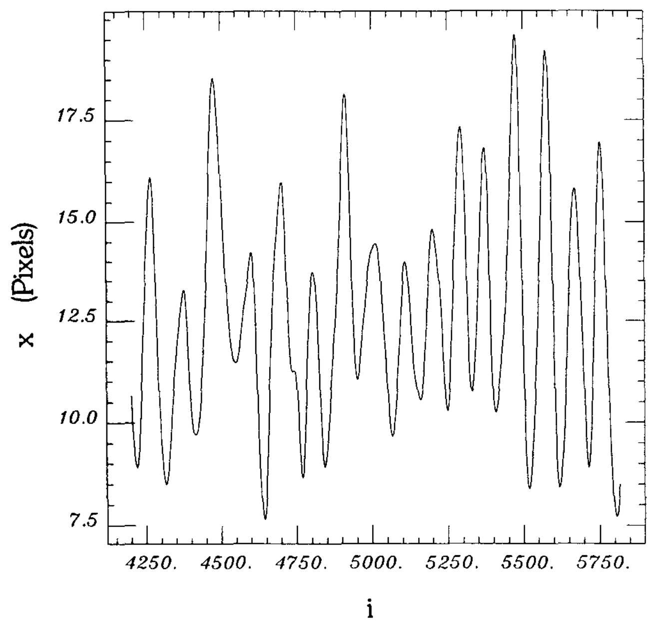
(b)

Fig. 9. (a) shows a time delay embedding ($x(i)$, $x(i+15)$) for a complicated experimental solution. (b) shows segments of the same file which are good approximations of period-1 unstable orbits (top, middle) and period-2 orbits (bottom). The aspect ratio was $\Gamma = 4.49$ and the temperature was $T = 63.5C$.

time delay embedding of a complicated experimental solution. In Fig. 9(b) we show segments of the same file which are good approximations of orbits of periods 1 and 2. The topological features of these solutions might help us to understand the mechanism involved in the creation of this complicated solution as parameters were changed [Mindlin *et al.*, 1990]. Notice that the two period-1 orbits do not link each other, while the period-2 orbit links one of the period-1 orbit once. This is compatible with a horseshoe mechanism of stretching and folding the phase space. According to this



(a)



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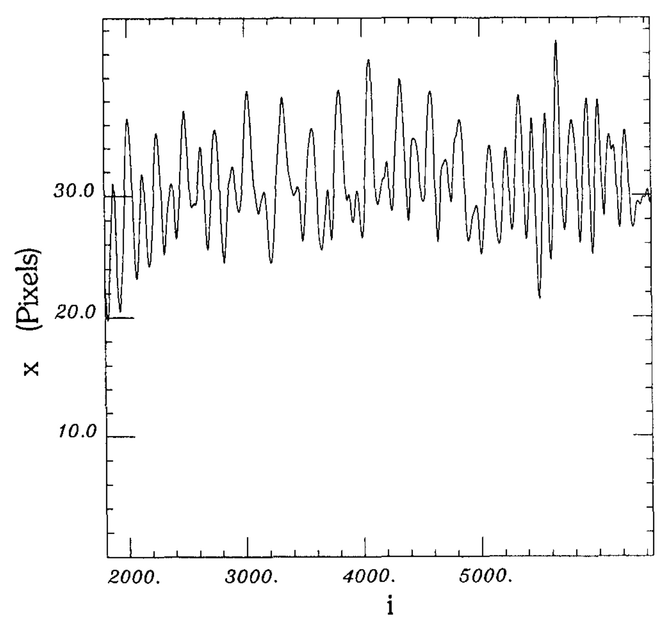

(c)

Fig. 10. (a)[(b)] shows a time series segment which presents an approximation of a period-2 (4) repeated twice followed by a modulated oscillation. In (c) alternations between subharmonic structures and modulations can be seen. The aspect ratio was $\Gamma = 4.49$ and the temperatures were $T = 63.5C$ (a), $T = 62.6C$ (b), $T = 60.1C$ (c).

observation, the period-4 should be knotted as a torus knot $T_{(2,3)}$, and perform a three-dimensional phase space embedding of the reconstructed period-4 unstable orbits, which is the case [Mindlin *et al.*, 1991]. It is worthwhile to remark that horseshoe-like behavior is present in the theoretical model as well.

Another typical feature of the complex solutions that we observed is the alternation between segments that present a subharmonic structure (segments that look like period-4, 2, etc) with segments that look quasiperiodic. The segments shown in Fig. 10 illustrate that point. In Fig. 10(a) [10(b)] we can see an approximation of a period-2 (4) that is repeated twice, followed by a modulated oscillation. Larger time series data presenting the alternation between modulations and subharmonic-like structures are displayed in Fig. 10(c).

If the temperature is increased further, we observe a dynamics that consists of sporadic alternations between positive and negative irregular oscillations. Figure 11 displays one of such data files. In terms of our theoretical model, this corresponds to a symmetric chaotic solution that emerges after a crisis in which two symmetrically conjugated strange attractors collide. As the temperature is increased, the time spent in each of the antisymmetric subregimes is smaller, and the system performs more and more regular symmetric oscillations.

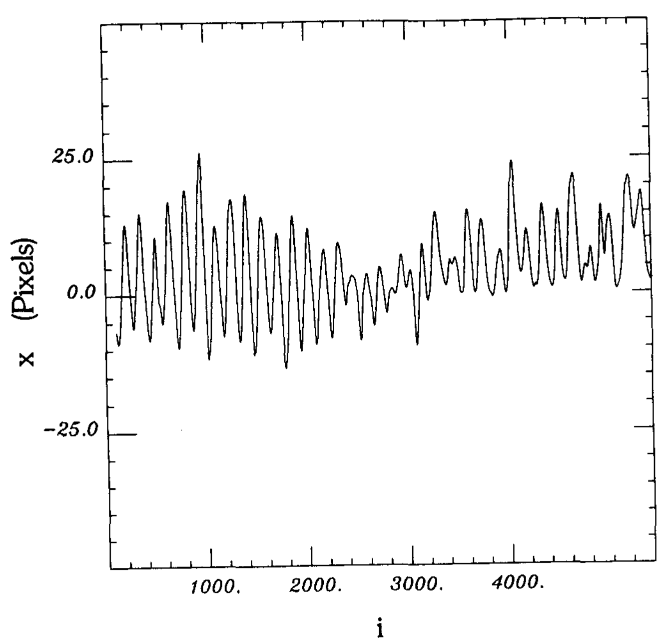

Fig. 11. A time series segment corresponding to a chaotic set-symmetric oscillation. The aspect ratio was $\Gamma = 4.49$ and the temperature was $T = 67.3C$.

The results previously described allow us to build some evidence in favor of the model described in Sec. 2. The theory predicts an edge of parameters for which the system would behave chaotically and irregular behavior was found. This irregular behavior showed clear signatures of determinism (as the presence of long segments that approximate periodic solutions) and some features present in the numerical simulations as modulations alternating with subharmonic-like segments. Also in the theory a crisis gives rise to a symmetric strange attractor as the parameter μ_1 is increased, which is experimentally found to be the case when the temperature is increased (we have already pointed out that agreement between our first model and the experiment required identification between μ_1 and a growing function of the temperature [Ondarçuhu *et al.*, 1993b]).

5. Conclusions

We report a simple experiment (a free upper surface Bénard system) in which the boundary conditions imposed a square symmetry. The solutions show a symmetry breaking process followed by the appearance of oscillations. We show that these simple observations allow us to build a theoretical dynamical model which reflects many of the features present in the experiment.

A conventional approach to the theoretical study of these phenomena consists in reducing the Navier–Stokes equations to a finite-dimensional dynamical system. This procedure is in general cumbersome. Our approach is dynamical in nature; from a scalar time series data and symmetry considerations we build a model. A major contribution to the understanding of our experiment came from the theory of equivariant vector fields, which suggested that a four-dimensional model (not a two-dimensional one as proposed by us in a previous paper) should be considered. That new model predicted the existence of a region of the parameter space in which the dynamics should be chaotic, and new observations corroborated the predictions of the theory.

Small aspect ratio systems constitute a challenge to theoreticians. To deal with boundary conditions in extended nonlinear systems is by no means trivial, but we hope this work will encourage further studies as it shows that even this simple experiment displays a rich variety of complex solutions.

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