Multi-task Online Learning for Probabilistic Load Forecasting

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Machine learning algorithms for load forecasting

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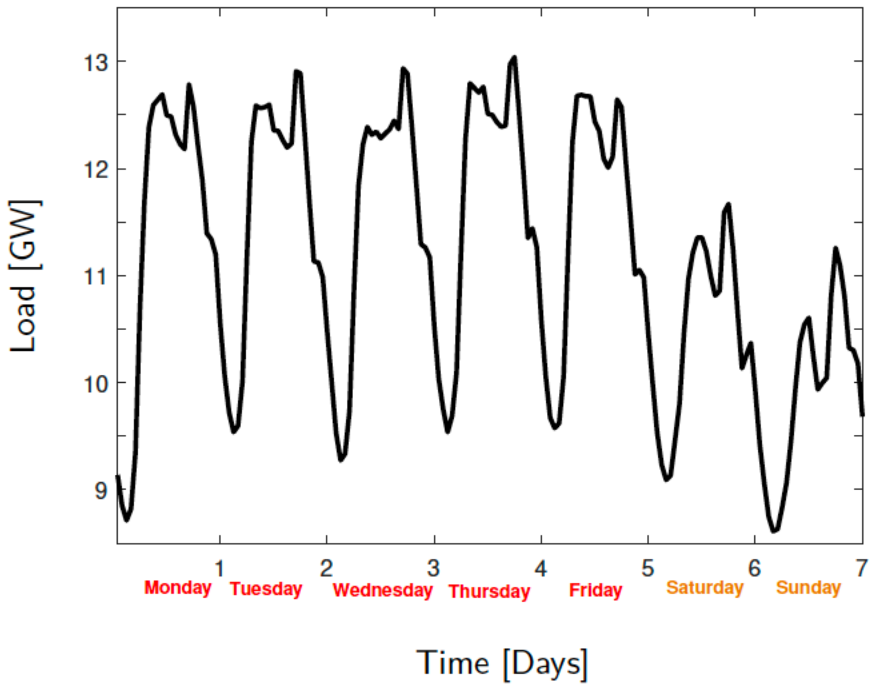
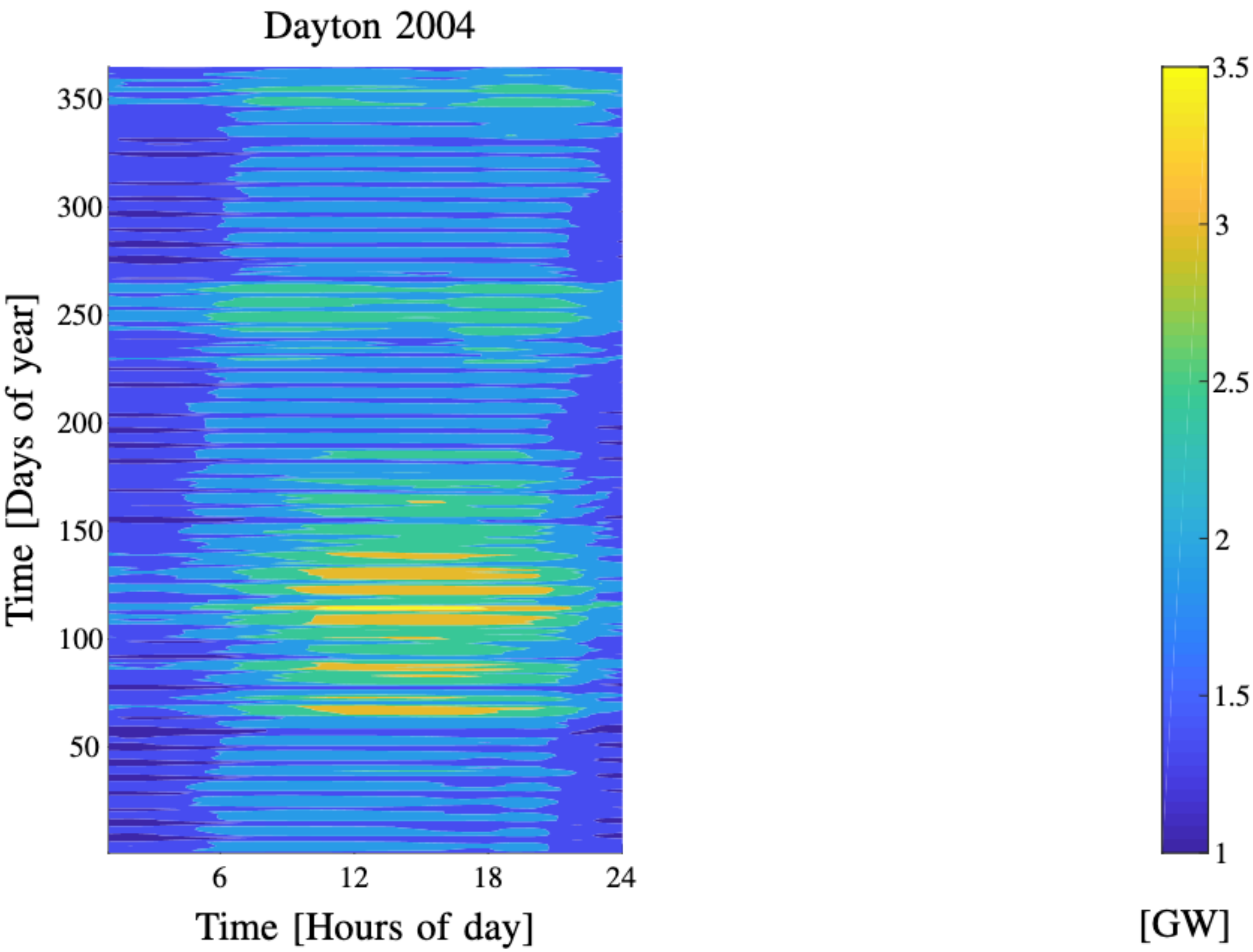
Load forecasting

- Load forecasting is important for energy management tasks
- Forecasting methods exploit consumption patterns influenced by multiple factors such as hours of the day, holidays, or weather conditions
- Energy demand is uncertain due to consumer behaviors or unexpected events



Load consumption patterns change over time



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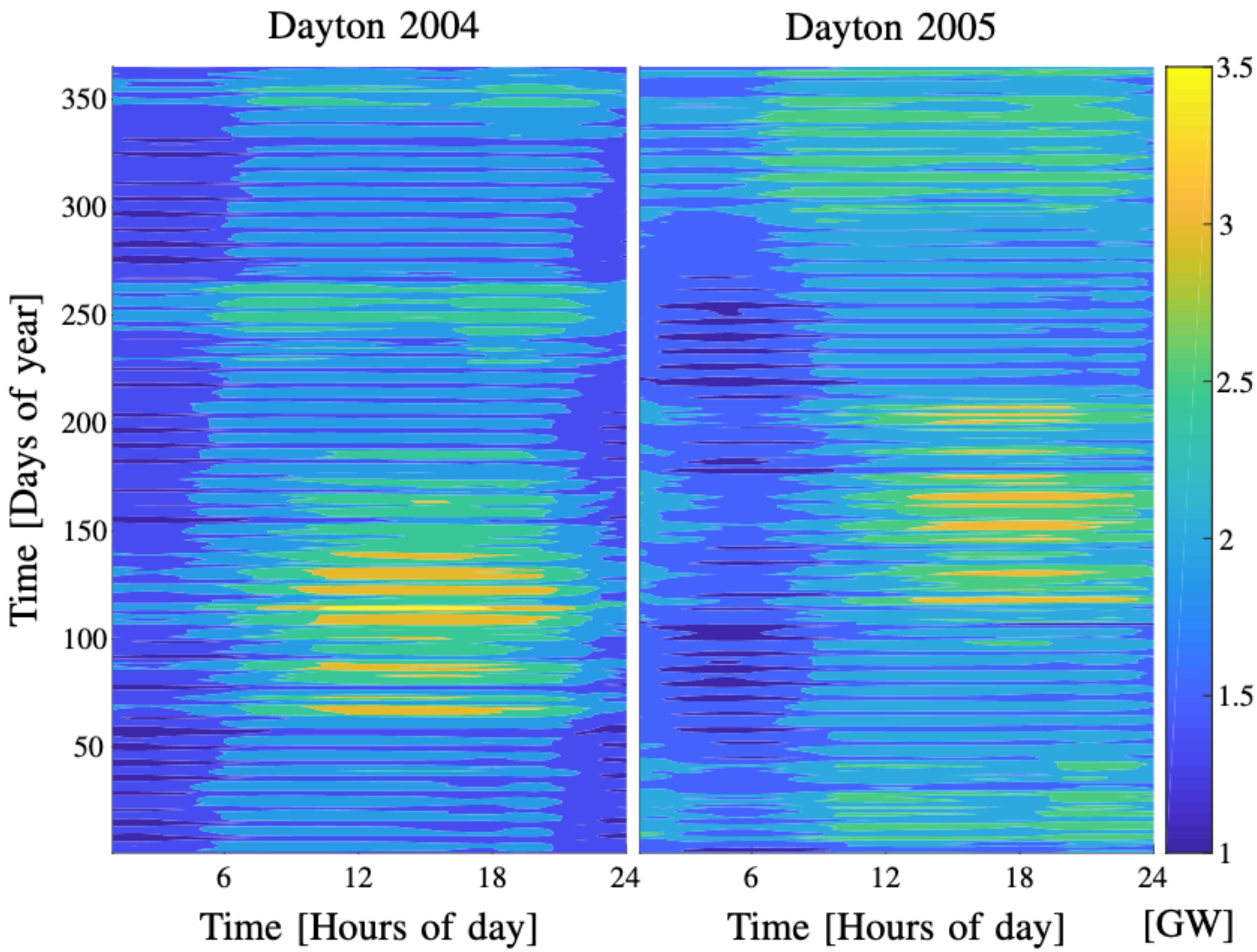
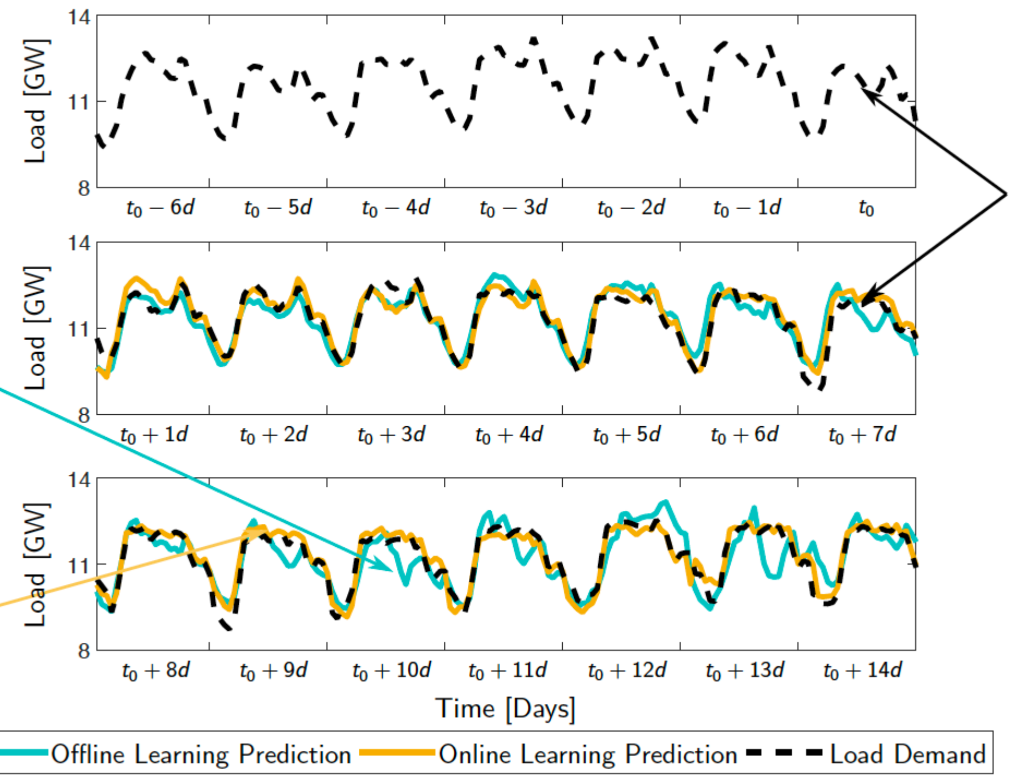


Online learning



Consumptions change from a two-peak pattern to a flatter pattern

Methods based on offline learning cannot accommodate to such changes

Methods based on online learning correctly adapt models after few days

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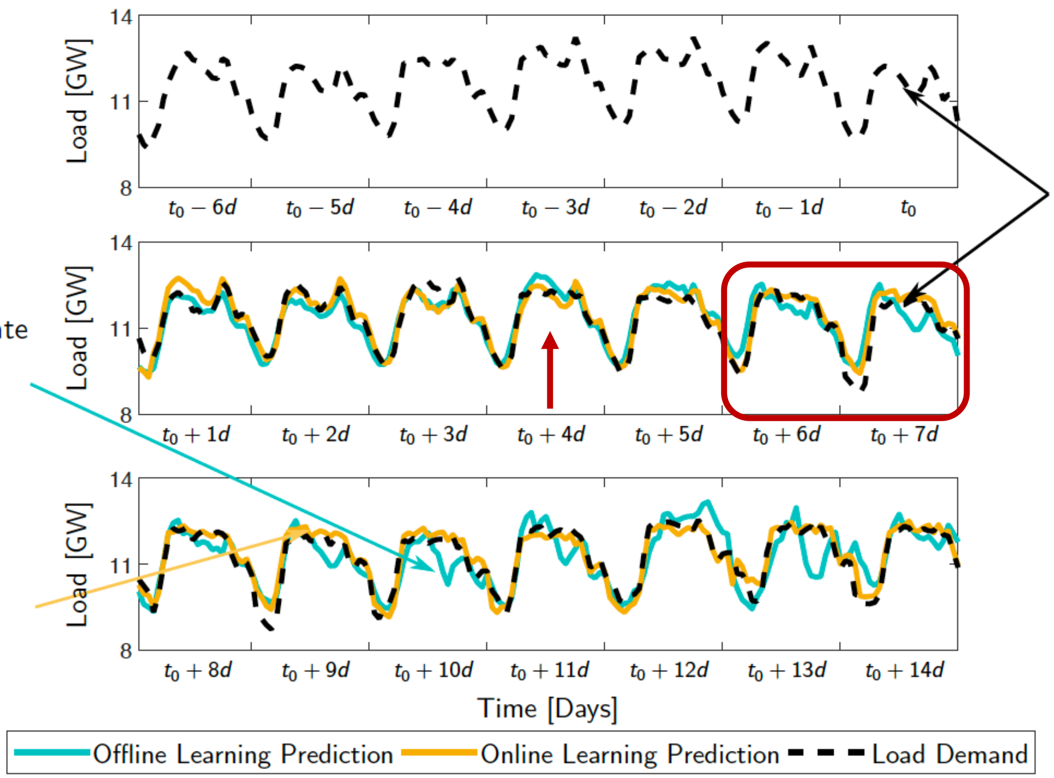
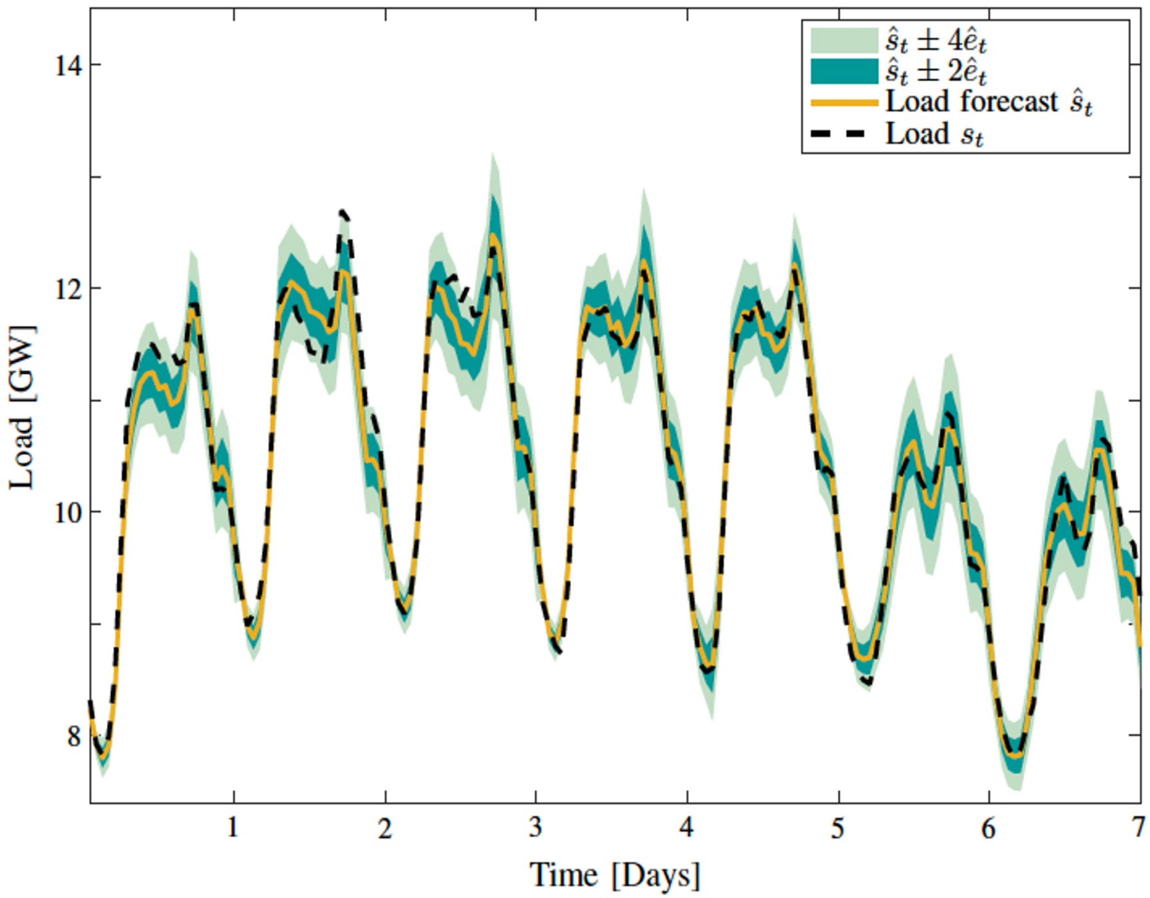


Consumptions change from a two-peak pattern to a flatter pattern

Probabilistic forecasting techniques



Outline

- Introduction and motivation
- Problem formulation
- Models and theoretical results
- Extension to high-dimensional models
- Experimental results
- Conclusions

Outline

- Introduction and motivation
- Problem formulation

Problem formulation

Forecasting algorithms obtain a vector of L forecasts corresponding with future loads

$$\hat{\mathbf{y}} = [\widehat{s}_{t+1}, \widehat{s}_{t+2}, ..., \widehat{s}_{t+L}]^\top$$

given an instance vector of past loads and observations related to future loads

$$\mathbf{x} = \left[s_{t-m}, ..., s_t, \mathbf{r}_{t+1}^\top, ..., \mathbf{r}_{t+L}^\top\right]^\top, m < t$$

where t is the time when the load is measured.

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The calendar information

$$c\left(t\right) \in \{1, 2, ..., C\}$$

includes details such as hour of day and holidays.

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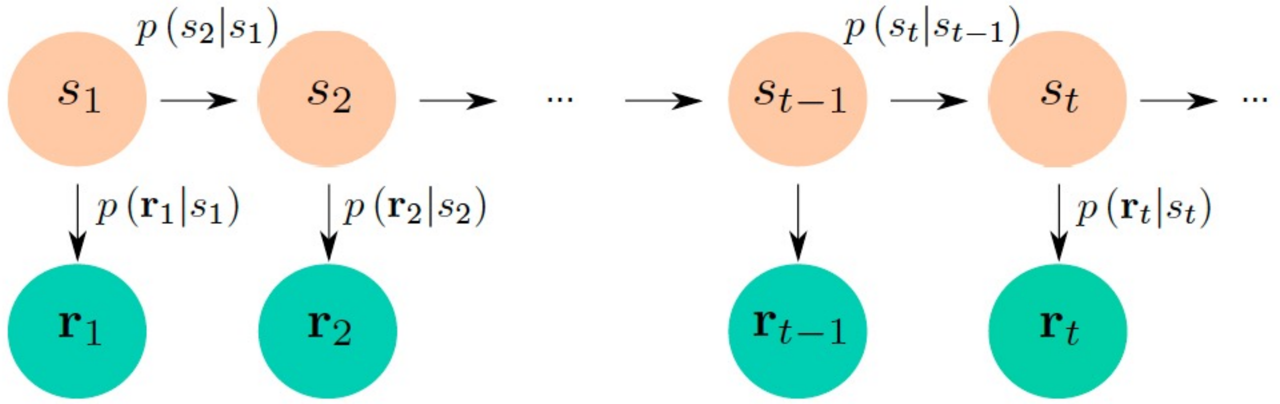
Probabilistic Load Forecasting Based on Adaptive Online Learning



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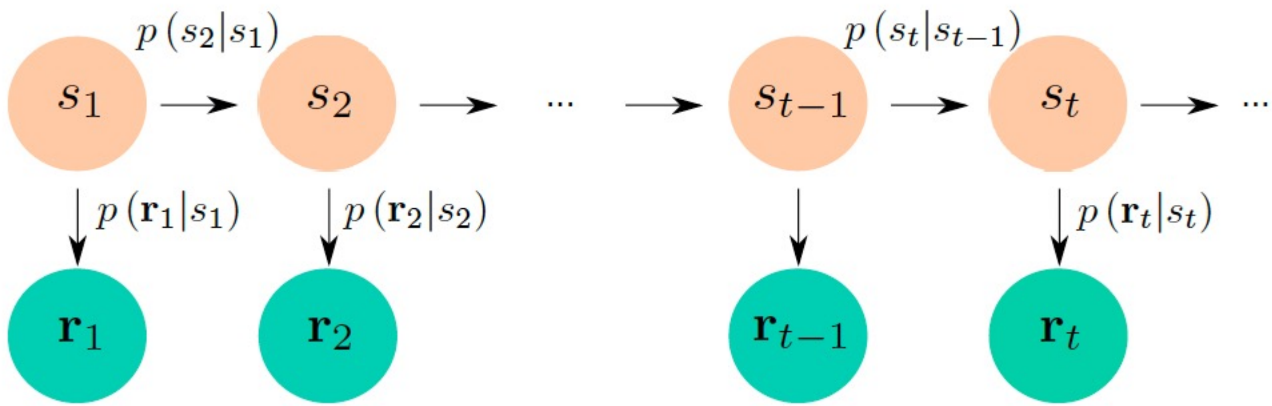

Model relationship consecutive loads

$$p(s_t|s_{t-1}) = N\left(s_t; \mathbf{u}_s^{\mathrm{T}} \boldsymbol{\eta}_{s,c}, \sigma_{s,c}\right)$$

Model relationship loads and observations

$$p(\mathbf{r}_t|s_t) \propto p(s_t|\mathbf{r}_t) = N\left(s_t; \mathbf{u}_r^{\mathrm{T}} \boldsymbol{\eta}_{r,c}, \sigma_{r,c}\right)$$

Learning HMM parameters

The HMM describing is characterized by parameters

$$\Theta = \{\boldsymbol{\eta}_{s,c}, \sigma_{s,c}, \boldsymbol{\eta}_{r,c}, \sigma_{r,c} : c = 1, 2, ..., C\}$$

Parameters Θ can be obtained by maximizing the weighted log-likelihood given by

$$L_i\left(\boldsymbol{\eta}, \sigma\right) = \sum_{j=1}^{i} \lambda^{i-j} \log N(s_{t_j}; \boldsymbol{u}_{t_j}^{\top} \boldsymbol{\eta}, \sigma)$$

with $i \leq n$, $\lambda \in (0,1)$ and $s_{t_1}, s_{t_2}, ..., s_{t_n}$ and $\boldsymbol{u}_{t_1}, \boldsymbol{u}_{t_2}, ..., \boldsymbol{u}_{t_n}$ sequences of loads and corresponding feature vectors with the same calendar type.

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Online learning of HMM parameters

Theorem

Let $\boldsymbol{\eta}_i^* \in \mathbb{R}^K$, $\sigma_i^* \in \mathbb{R}$ parameters that maximize the weighted log-likelihood.
If parameters $\boldsymbol{\eta}_i \in \mathbb{R}^K$, $\sigma_i \in \mathbb{R}$ are given by recursions

$$\boldsymbol{\eta}_i = \boldsymbol{\eta}_{i-1} + \frac{\mathbf{P}_{i-1}\mathbf{u}_{t_i}}{\lambda + \mathbf{u}_{t_i}^\mathsf{T}\mathbf{P}_{i-1}\mathbf{u}_{t_i}}(s_{t_i} - \mathbf{u}_{t_i}^\mathsf{T}\boldsymbol{\eta}_{i-1}),$$

$$\sigma_i = \sqrt{\sigma_{i-1}^2 - \frac{1}{\gamma_i}\left(\sigma_{i-1}^2 - \frac{\lambda\left(s_{t_i} - \mathbf{u}_{t_i}^\mathsf{T}\boldsymbol{\eta}_{i-1}\right)^2}{\lambda + \mathbf{u}_{t_i}^\mathsf{T}\mathbf{P}_{i-1}\mathbf{u}_{t_i}}\right)},$$

with

$$\mathbf{P}_i = \frac{1}{\lambda}\left(\mathbf{P}_{i-1} - \frac{\mathbf{P}_{i-1}\mathbf{u}_{t_i}\mathbf{u}_{t_i}^\mathsf{T}\mathbf{P}_{i-1}}{\lambda + \mathbf{u}_{t_i}^\mathsf{T}\mathbf{P}_{i-1}\mathbf{u}_{t_i}}\right), \qquad \gamma_i = 1 + \lambda\gamma_{i-1},$$

for any $i \leq n$ such that the matrix $\sum_{j=1}^{i}\lambda^{i-j}\mathbf{u}_{t_j}\mathbf{u}_{t_j}^\mathsf{T}$ is not singular.

Probabilistic forecasts

Theorem

If $\{s_t, \mathbf{r}_t\}_{t\geq 1}$ is a HMM characterized by parameters Θ. Then, for $i = 1, 2, \ldots, L$

$$p\left(s_{t+i} | s_t, \mathbf{r}_{t+1}, \ldots, \mathbf{r}_{t+i}\right) = N\left(s_{t+i}; \hat{s}_{t+i}, \hat{e}_{t+i}\right)$$

where $\hat{s}_{t+i}$ and $\hat{e}_{t+i}$ can be computed by the following recursions

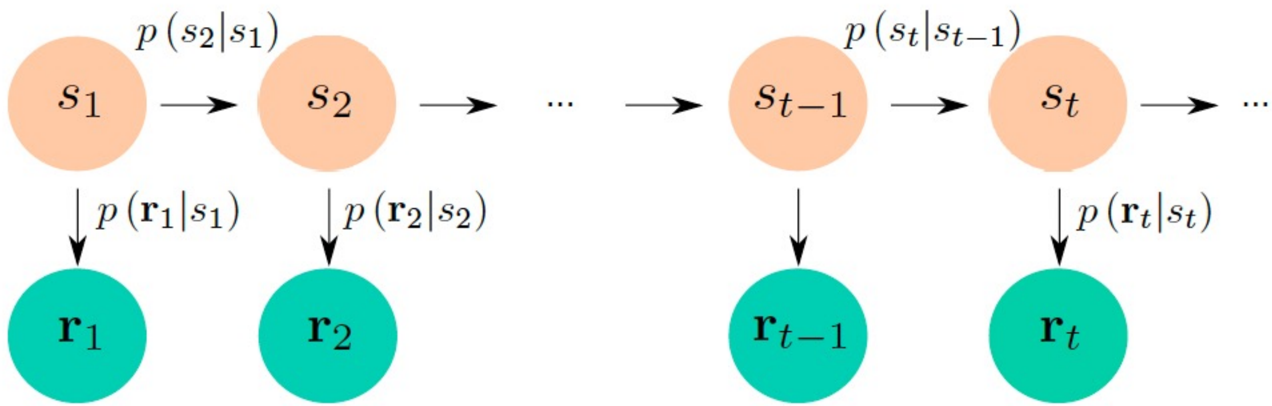
$$c = c\left(t+i\right), \hat{\mathbf{u}}_s = \left[1, \hat{s}_{t+i-1}\right]^{\mathrm{T}}, \mathbf{u}_r = u_r\left(\mathbf{r}_{t+i}\right)$$

$$\hat{s}_{t+i} = \frac{\hat{\mathbf{u}}_s^{\mathrm{T}} \boldsymbol{\eta}_{s,c} \sigma_{r,c}^2 + \mathbf{u}_r^{\mathrm{T}} \boldsymbol{\eta}_{r,c} \left(\sigma_{s,c}^2 + \left([0,1]\,\boldsymbol{\eta}_{s,c}\right)^2 \hat{e}_{t+i-1}^2\right)}{\sigma_{r,c}^2 + \sigma_{s,c}^2 + \left([0,1]\,\boldsymbol{\eta}_{s,c}\right)^2 \hat{e}_{t+i-1}^2}$$

$$\hat{e}_{t+i} = \sqrt{\frac{\left(\sigma_{s,c}^2 + \left([0,1]\,\boldsymbol{\eta}_{s,c}\right)^2 \hat{e}_{t+i-1}^2\right) \sigma_{r,c}^2}{\sigma_{r,c}^2 + \sigma_{s,c}^2 + \left([0,1]\,\boldsymbol{\eta}_{s,c}\right)^2 \hat{e}_{t+i-1}^2}}$$

for $i = 1, 2, \ldots, L$, and $\hat{s}_t = s_t$, $\hat{e}_t = 0$.

Probabilistic Load Forecasting Based on Adaptive Online Learning



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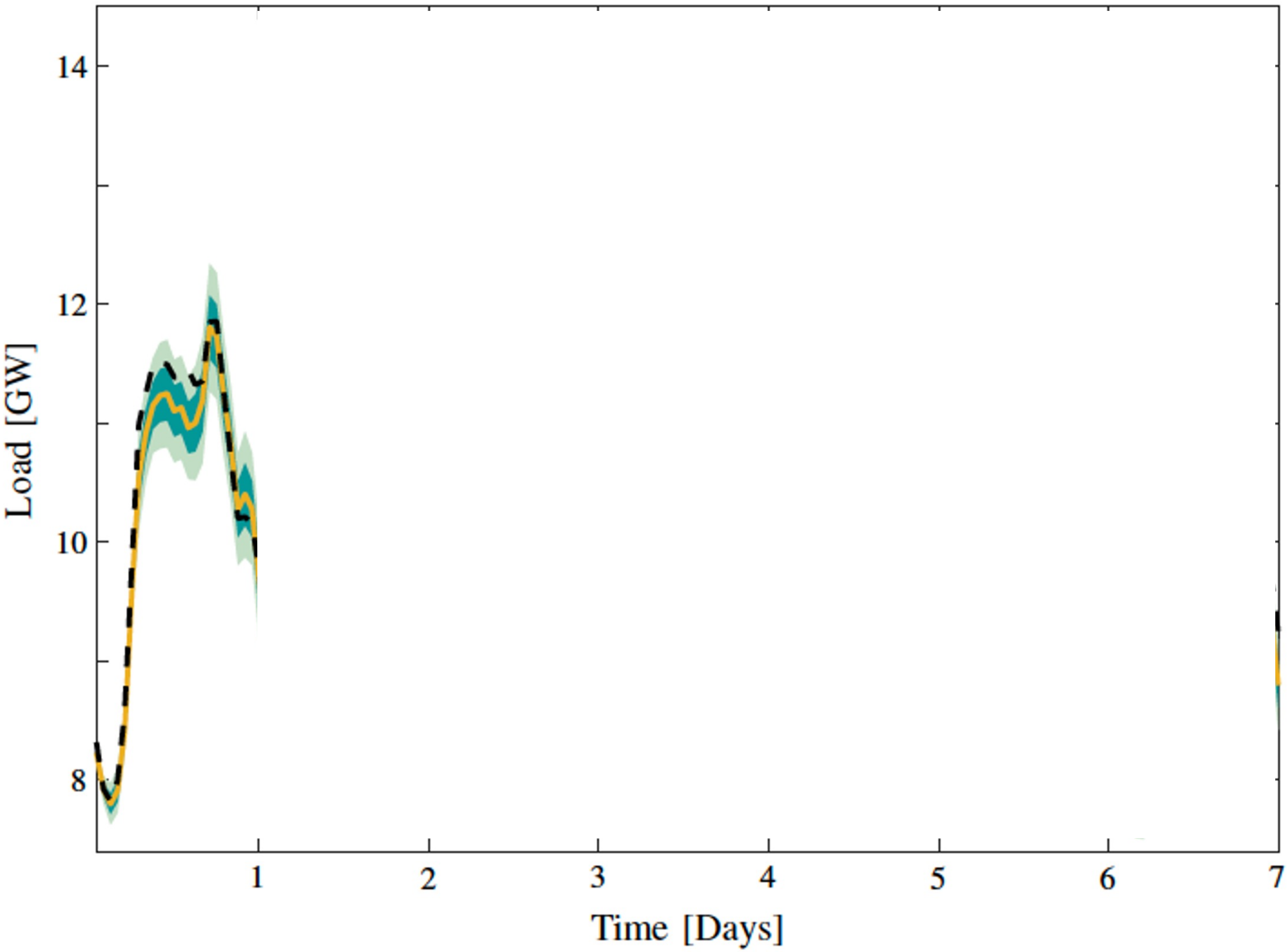
Online learning

$$\boxed{\boldsymbol{\eta}_i} = \boxed{\boldsymbol{\eta}_{i-1}} + \frac{\mathbf{P}_{i-1}\mathbf{u}_{t_i}}{\lambda + \mathbf{u}_{t_i}^{\mathrm{T}} \mathbf{P}_{i-1} \mathbf{u}_{t_i}} \left(s_{t_i} - \mathbf{u}_{t_i}^{\mathrm{T}} \boldsymbol{\eta}_{i-1}\right)$$

Probabilistic forecasts

$$p(s_{t+i}|s_t, \mathbf{r}_{t+1}, \ldots, \mathbf{r}_{t+i}) = \boxed{N\left(s_{t+i}; \hat{s}_{t+i}, \hat{e}_{t+i}\right)}$$

Load forecasting in real-time



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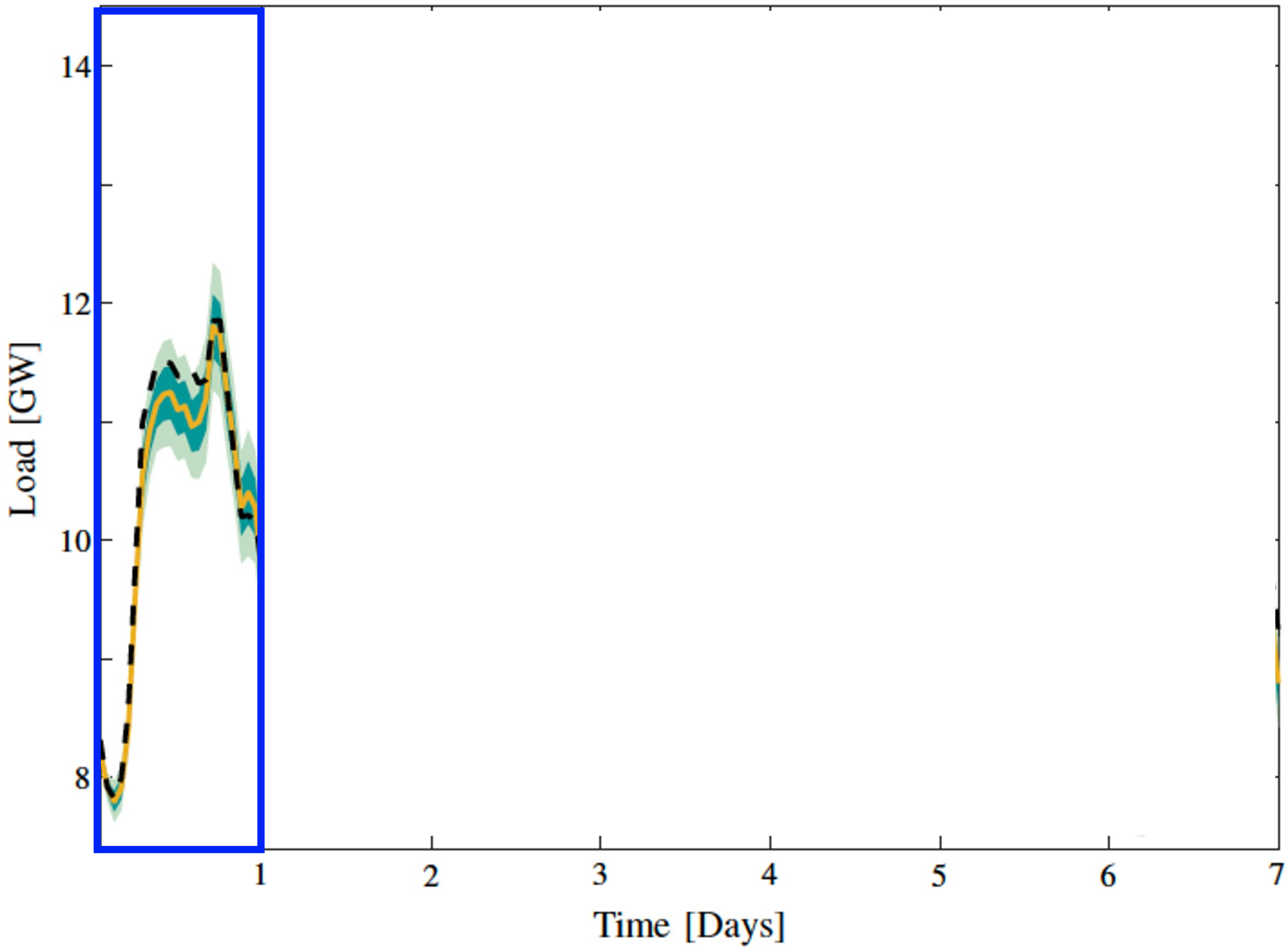

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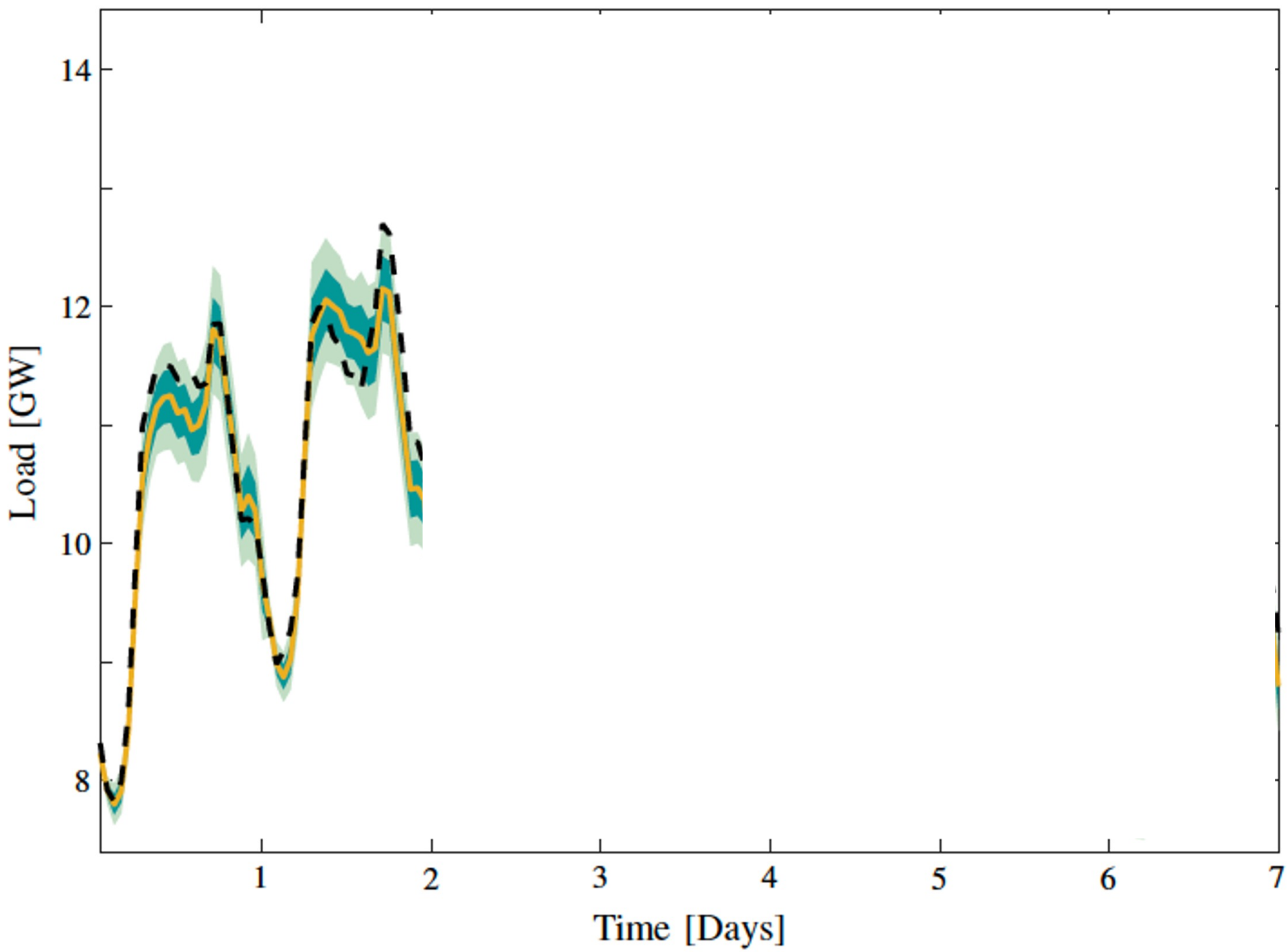

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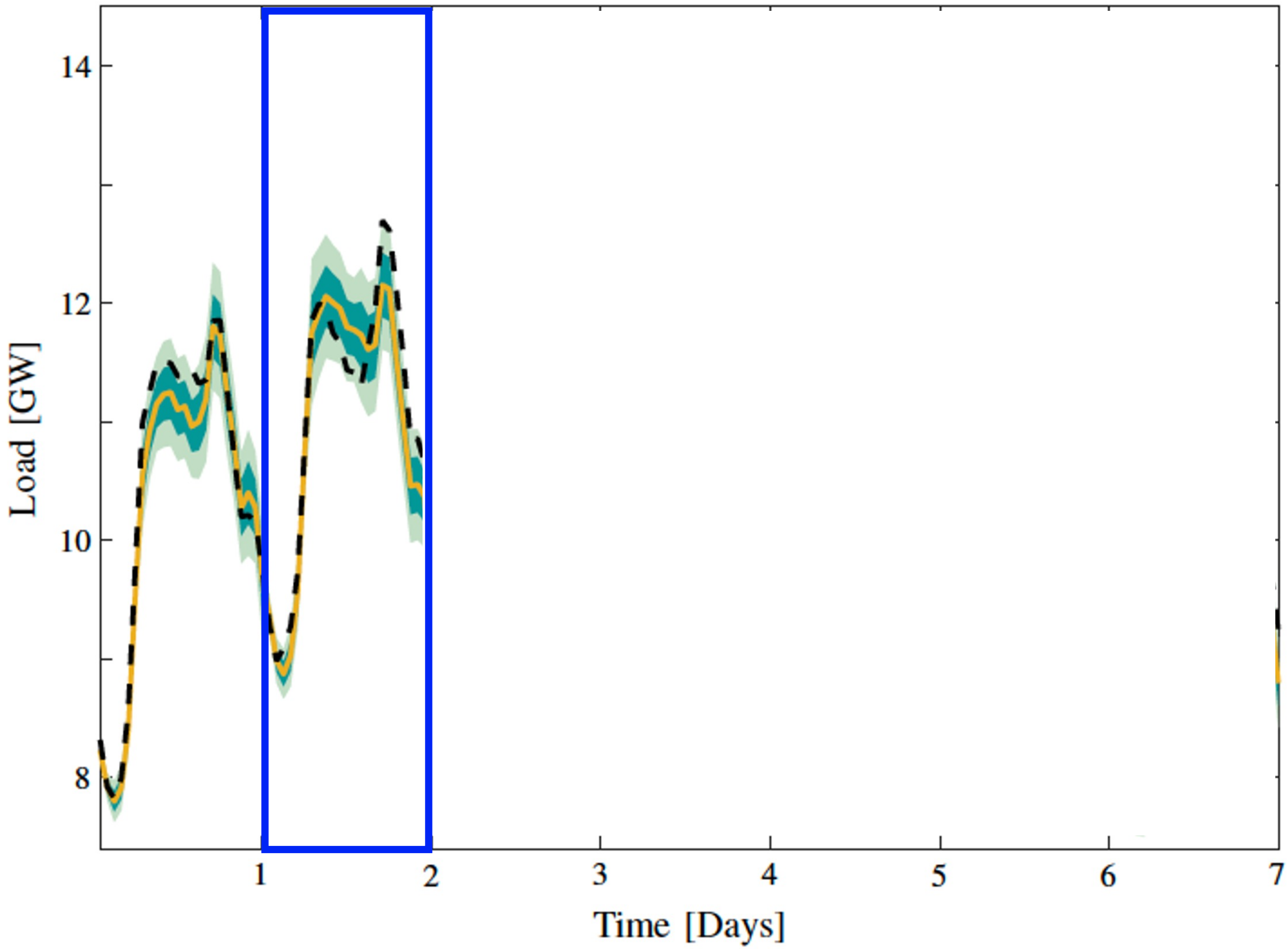

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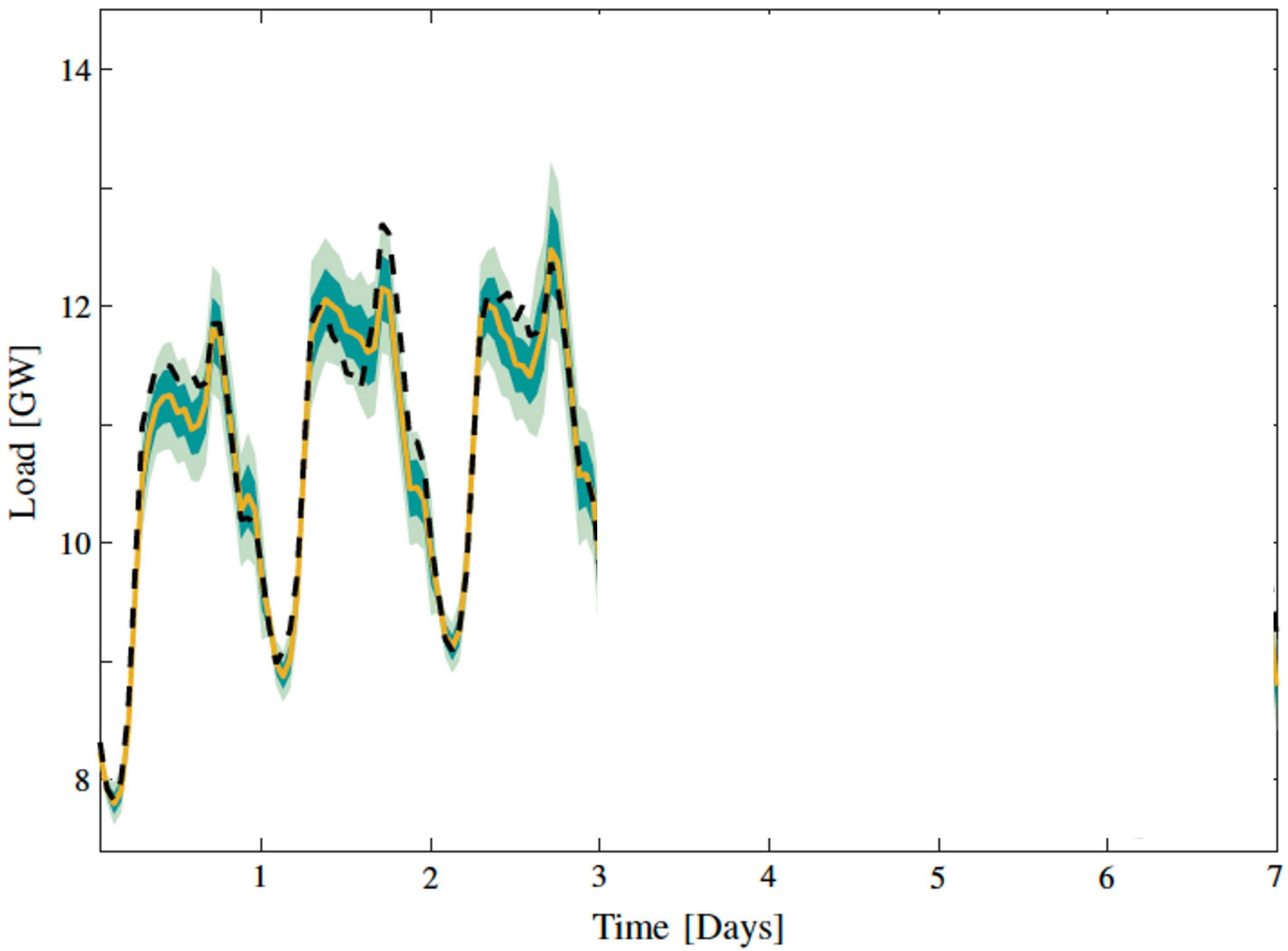

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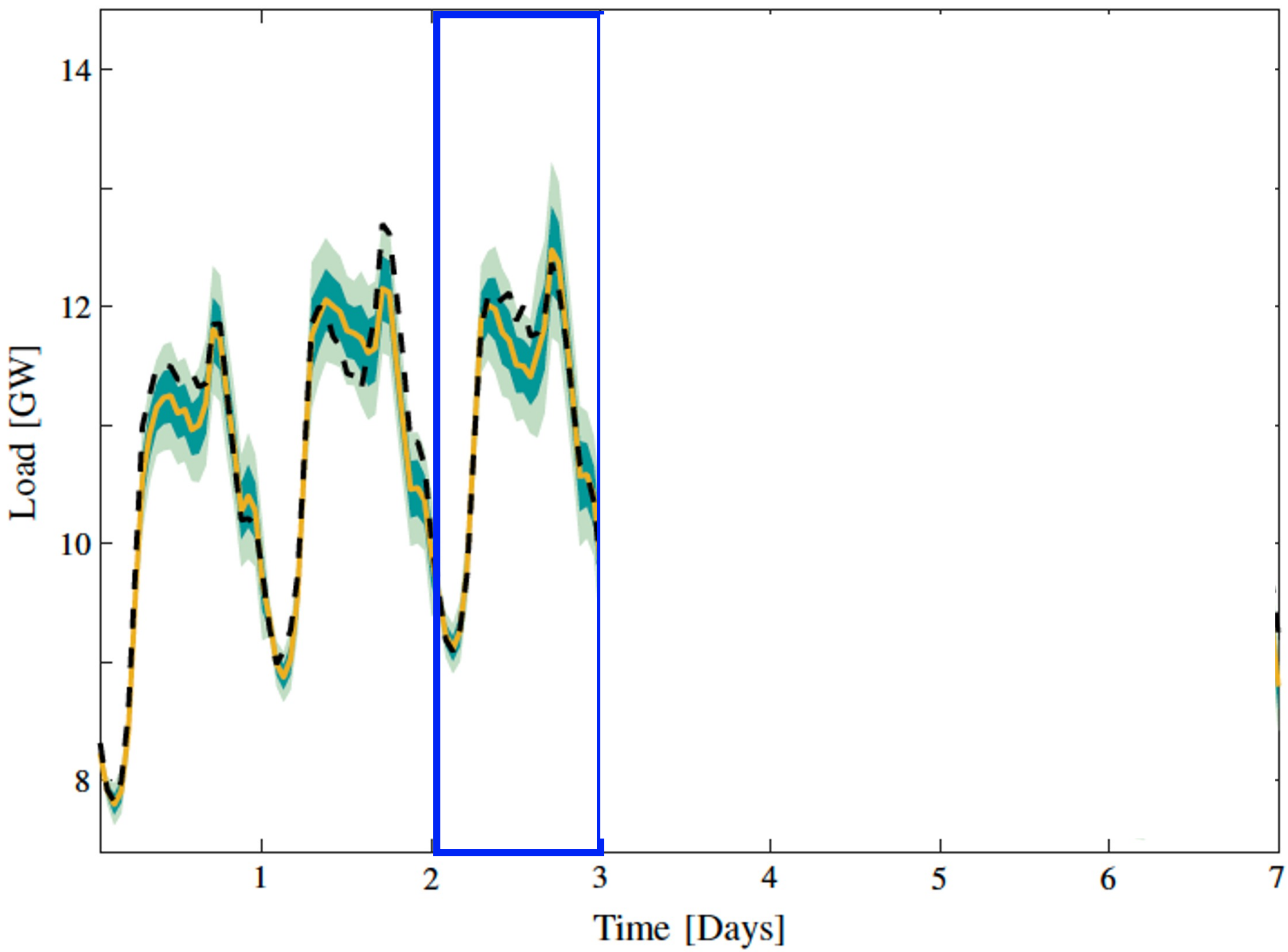
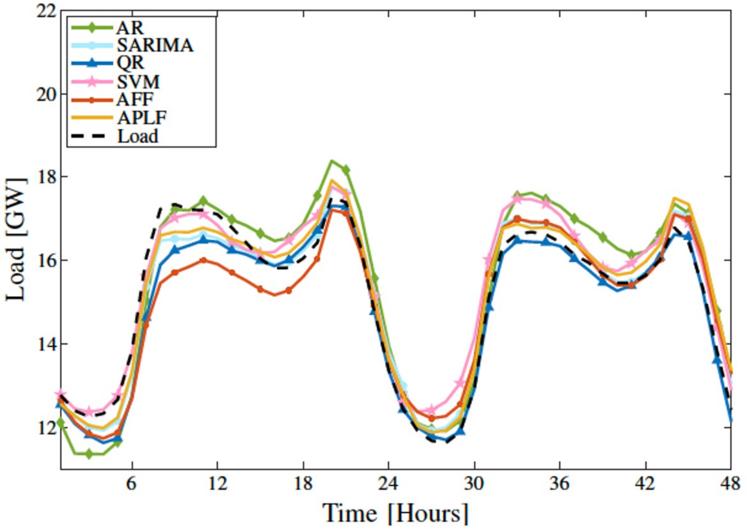
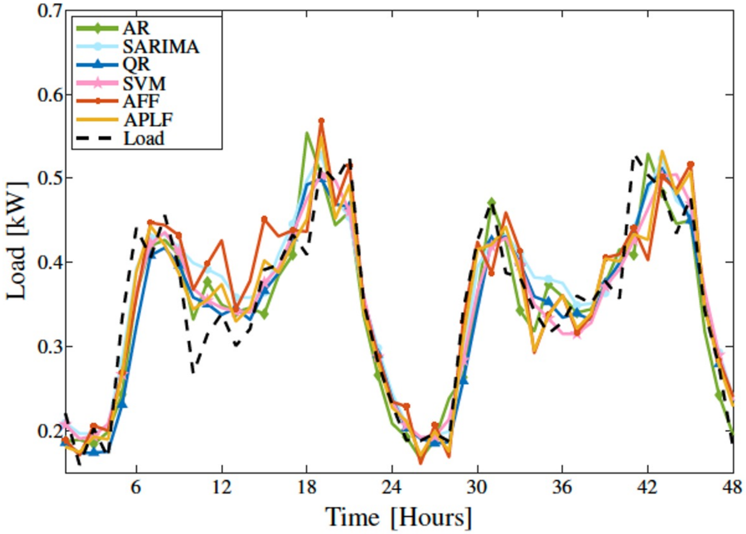
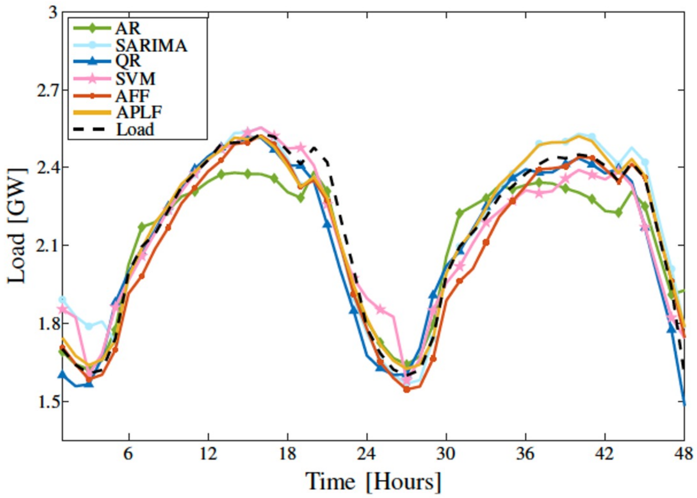


Forecasting in different regions





New England



100 buildings

Dayton

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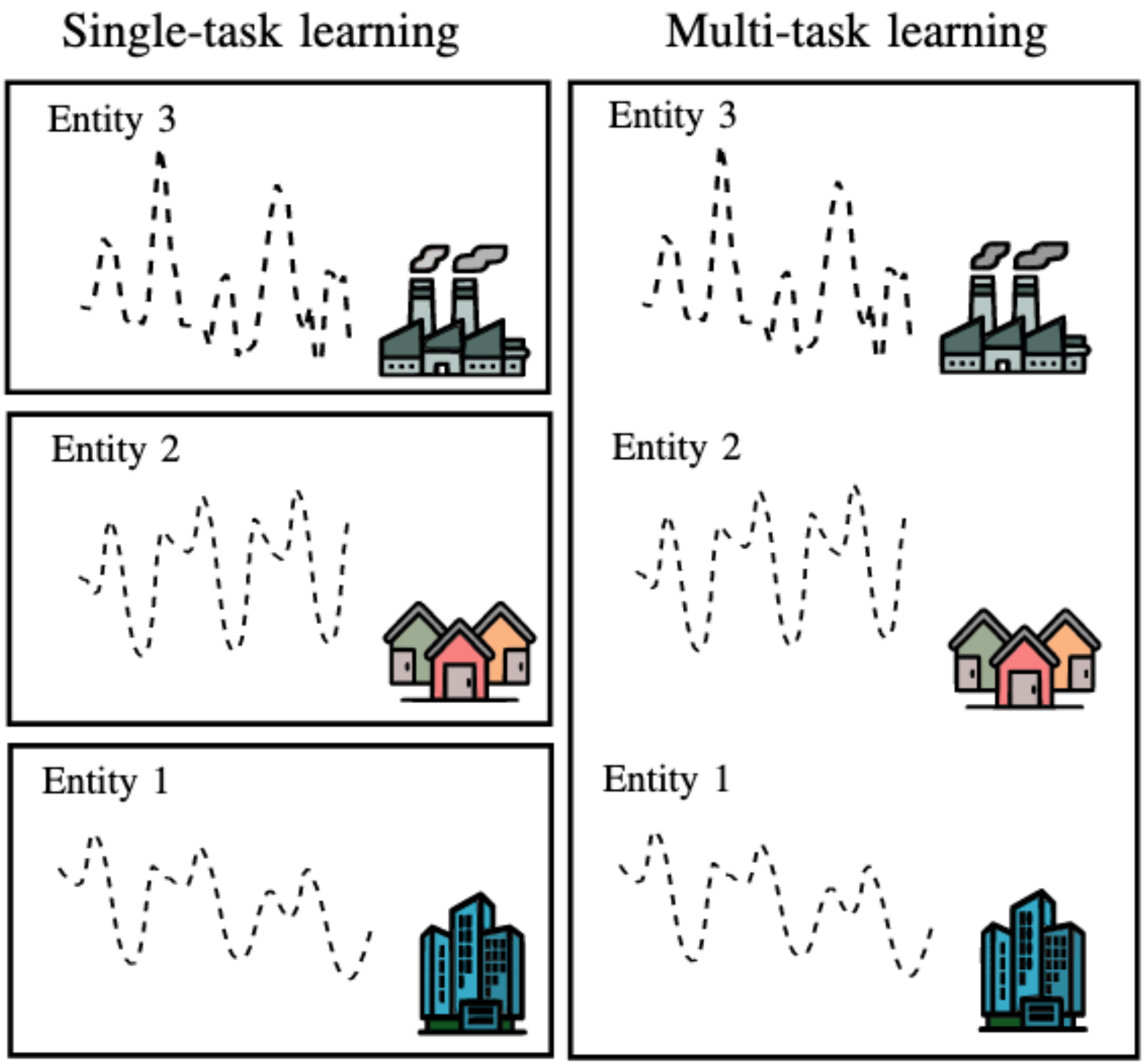
Multi-task learning techniques

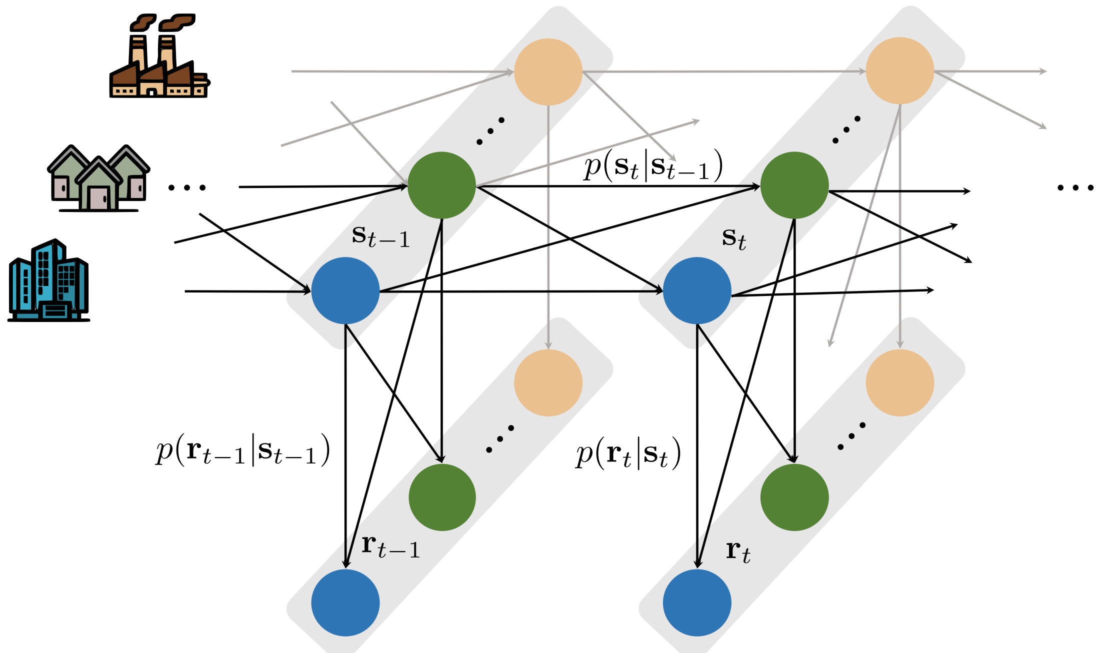


Multi-task Hidden Markov model

Hidden Markov Model characterized by the following Gaussian distributions:

$$p(\boldsymbol{s}_t|\boldsymbol{s}_{t-1}) = \mathcal{N}(\boldsymbol{s}_t; \boldsymbol{M}_{s,c}\boldsymbol{u}_s, \boldsymbol{\Sigma}_{s,c}) \qquad \text{Loads}$$

$$p(\boldsymbol{r}_t|\boldsymbol{s}_t) \propto p(\boldsymbol{s}_t|\boldsymbol{r}_t) = \mathcal{N}(\boldsymbol{s}_t; \boldsymbol{M}_{r,c}\boldsymbol{u}_r, \boldsymbol{\Sigma}_{r,c}) \qquad \text{Observations}$$



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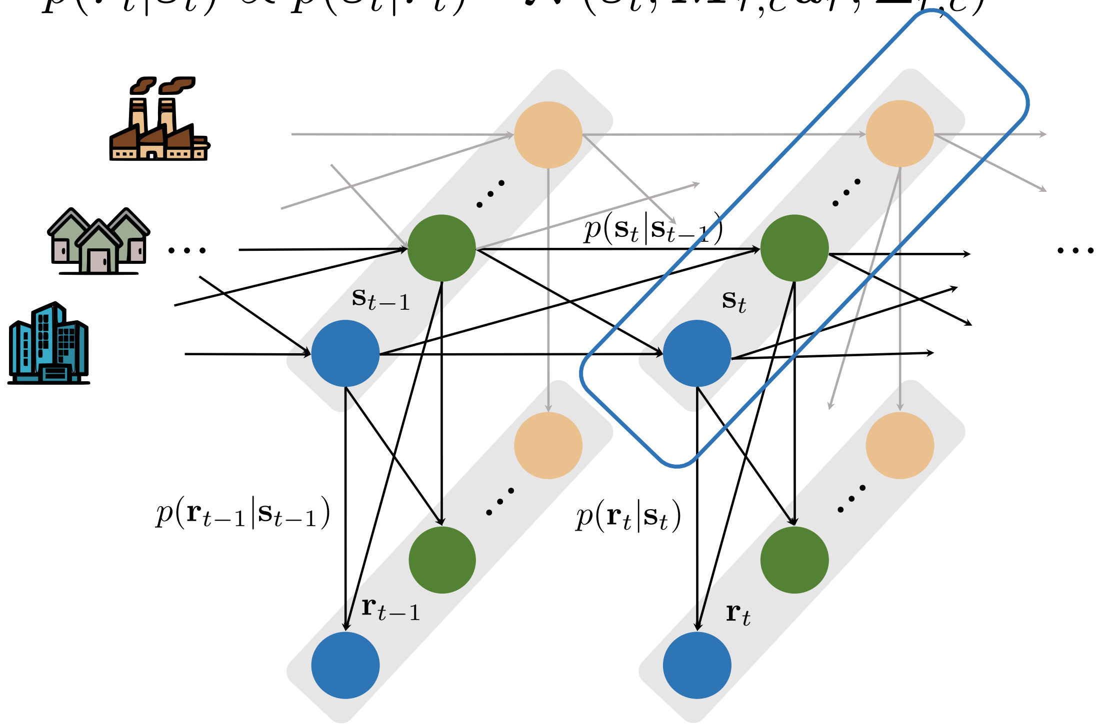

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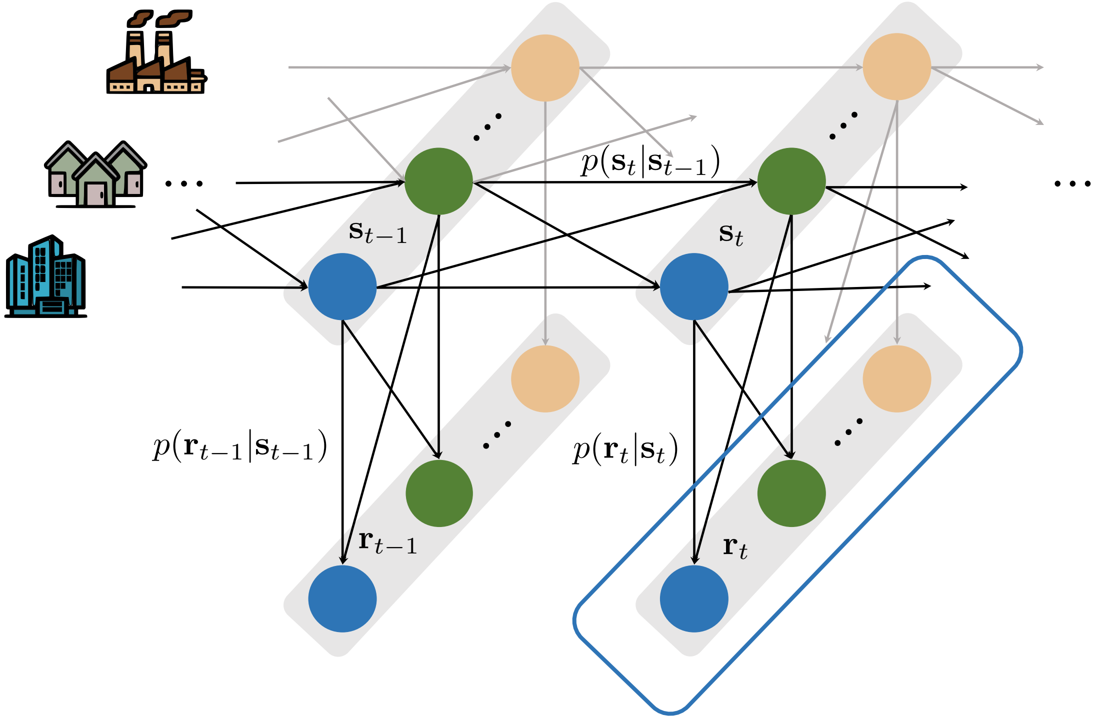

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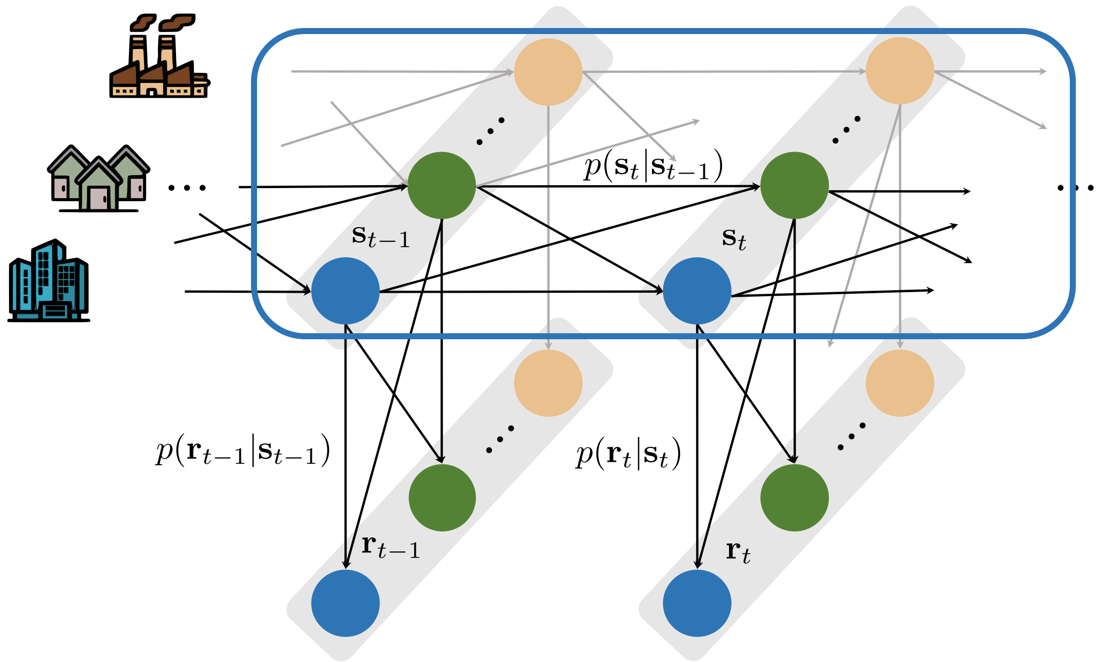

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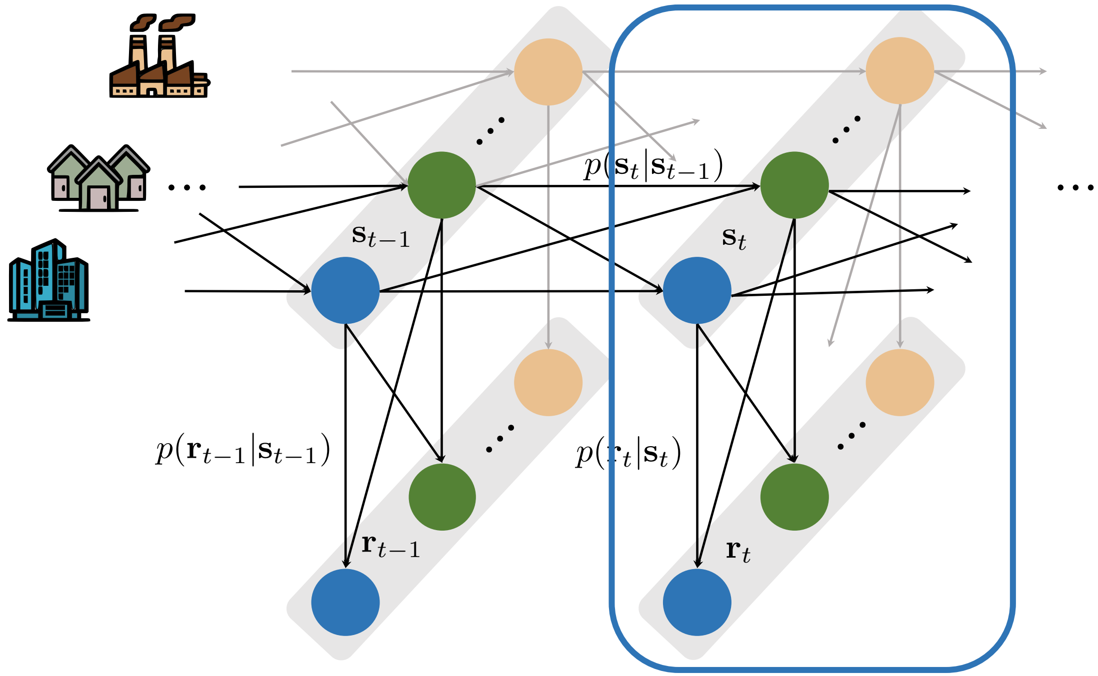

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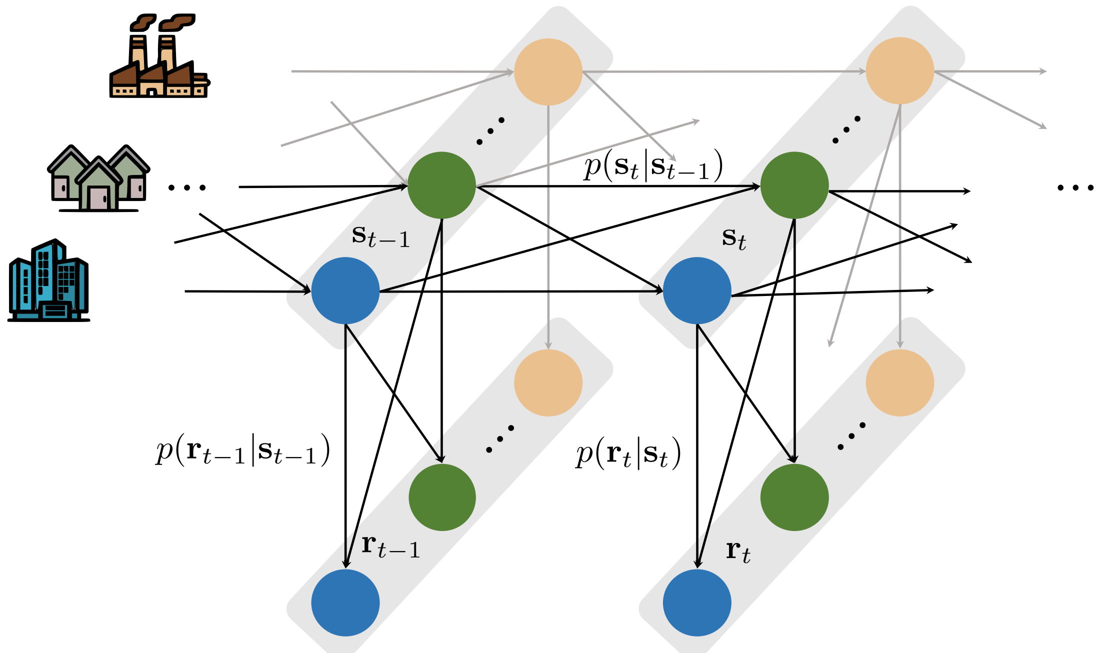

The calendar variable describes time factors that may affect load demand

$$c(t) \in \{1, ..., C\}$$

Online learning of multi-task HMM parameters

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Parameters $\boldsymbol{M}_{s,c}$, $\boldsymbol{\Sigma}_{s,c}$, $\boldsymbol{M}_{r,c}$ and $\boldsymbol{\Sigma}_{r,c}$ are given by recursions

$$\boldsymbol{M}_i = \boldsymbol{M}_{i-1} + \frac{(\boldsymbol{s}_{t_i} - \boldsymbol{M}_{i-1}\boldsymbol{u}_{t_i})\boldsymbol{u}_{t_i}^\top \boldsymbol{P}_{i-1}}{\lambda + \boldsymbol{u}_{t_i}^\top \boldsymbol{P}_{i-1}\boldsymbol{u}_{t_i}}$$

$$\boldsymbol{\Sigma}_i = \boldsymbol{\Sigma}_{i-1} - \frac{1}{\gamma_i}\left(\boldsymbol{\Sigma}_{i-1} - \frac{\lambda^2(\boldsymbol{s}_{t_i} - \boldsymbol{M}_{i-1}\boldsymbol{u}_{t_i})(\boldsymbol{s}_{t_i} - \boldsymbol{M}_{i-1}\boldsymbol{u}_{t_i})^\top}{(\lambda + \boldsymbol{u}_{t_i}^\top \boldsymbol{P}_{i-1}\boldsymbol{u}_{t_i})^2}\right)$$

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Multi-task probabilistic forecasting

Using the most recent learned parameters of the HMM we can provide probabilistic forecasts

$$p(\boldsymbol{s}_{t+i}|\boldsymbol{s}_t, \boldsymbol{r}_{t+1:t+i}) = \mathcal{N}(\boldsymbol{s}_{t+i}; \widehat{\boldsymbol{s}}_{t+i}, \widehat{\boldsymbol{E}}_{t+i}) \quad i = 1, 2, ..., L$$

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$$\widehat{\boldsymbol{s}}_{t+i} = \boldsymbol{W}_1(\boldsymbol{W}_1 + \boldsymbol{W}_2)^{-1}\boxed{\boldsymbol{M}_{r,c}\boldsymbol{u}_r} + \boldsymbol{W}_2(\boldsymbol{W}_1 + \boldsymbol{W}_2)^{-1}\boxed{\boldsymbol{M}_{s,c}\widehat{\boldsymbol{u}}_s}$$

$$\widehat{\boldsymbol{E}}_{t+i} = \boldsymbol{W}_2(\boldsymbol{W}_1 + \boldsymbol{W}_2)^{-1}\boldsymbol{W}_1$$

with

$$\boldsymbol{W}_1 = \boldsymbol{\Sigma}_{s,c} + \boldsymbol{M}_{s,c}\boldsymbol{N}\widehat{\boldsymbol{E}}_{t+i-1}(\boldsymbol{M}_{s,c}\boldsymbol{N})^\top$$

$$\boldsymbol{W}_2 = \boldsymbol{\Sigma}_{r,c}$$

$$c = c(t+i), \widehat{\boldsymbol{u}}_s = [1, \widehat{\boldsymbol{s}}_{t+i-1}]^\top, \boldsymbol{u}_r = u_r(\boldsymbol{r}_{t+i})$$

$$\widehat{\boldsymbol{s}}_t = \boldsymbol{s}_t, \widehat{\boldsymbol{E}}_t = \boldsymbol{0}$$

Outline

- Introduction and motivation
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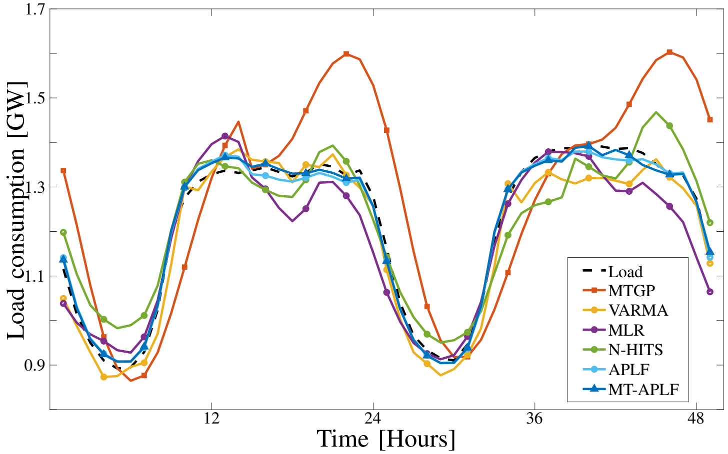
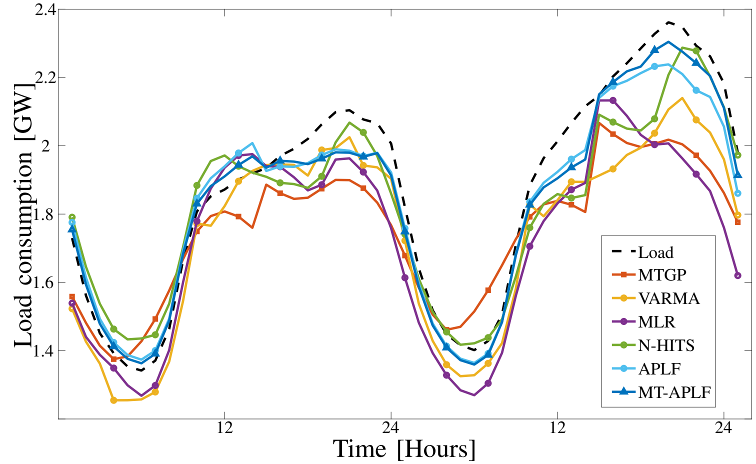
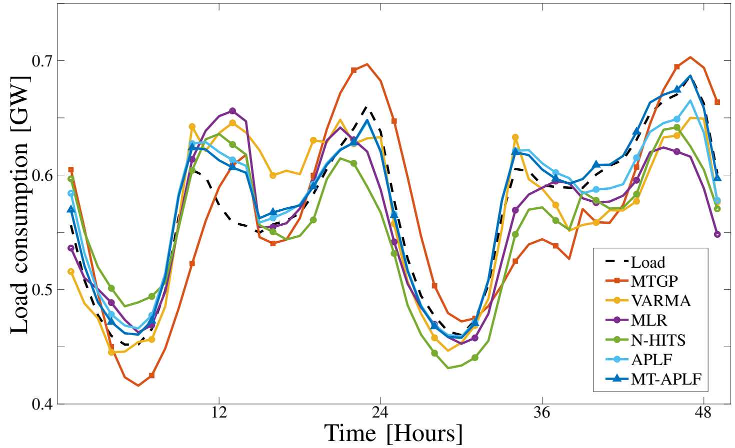
Prediction accuracy in 3 regions of New England



Maine



Vermont



Western Massachusetts

Results

		APLF		MTGP		VAR		MLR		N-HITS		MT-APLF	
		MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE
GEFCom2017	Entity 1	5.04	0.25	6.33	0.28	7.07	0.34	11.01	0.62	7.52	0.35	**4.57**	**0.22**
	Entity 2	3.18	**0.05**	4.55	0.07	5.28	0.09	6.87	0.13	4.77	0.07	**3.04**	**0.05**
	Entity 3	4.53	0.19	6.15	0.24	6.98	0.29	8.92	0.40	6.72	0.25	**3.90**	**0.16**
	Entity 4	4.29	0.08	5.95	0.0 1	6.97	0.13	9.83	0.21	6.52	0.11	**3.73**	**0.07**
	Entity 5	4.41	0.06	5.96	0.07	6.57	0.09	10.99	0.16	6.83	0.08	**4.04**	**0.05**
	Entity 6	5.15	0.12	6.34	0.14	7.03	0.17	12.75	0.35	7.16	0.16	**4.43**	**0.10**
	Entity 7	4.64	**0.03**	5.64	0.04	6.48	0.05	8.66	0.07	6.05	0.04	**4.37**	**0.03**
	Entity 8	4.86	0.13	6.43	0.16	7.18	0.19	9.63	0.29	6.29	0.16	**4.06**	**0.10**
	TOTAL	4.51	0.11	5.92	0.14	6.70	0.17	9.83	0.28	6.48	0.15	**4.02**	**0.10**
NewEngland	Entity 1	9.80	0.13	11.34	0.14	12.91	0.16	16.15	0.23	10.47	0.14	**9.29**	**0.12**
	Entity 2	5.58	0.09	6.81	0.10	7.44	0.11	11.50	0.20	7.32	0.11	**5.11**	**0.08**
	Entity 3	11.67	0.07	14.01	0.08	19.68	0.10	31.51	0.15	16.30	0.09	**11.12**	**0.06**
	Entity 4	5.52	0.23	6.95	0.29	7.63	0.32	12.72	0.61	7.33	0.31	5.34	**0.22**
	Entity 5	7.52	0.08	9.07	0.09	9.62	0.11	15.79	0.20	9.54	0.10	**7.00**	**0.07**
	Entity 6	6.16	0.12	7.63	0.15	8.85	0.17	14.81	0.34	8.36	0.16	**6.00**	**0.12**
	Entity 7	8.11	0.22	9.21	0.23	10.32	0.26	18.31	0.50	8.95	0.23	**7.36**	**0.19**
	Entity 8	4.71	0.16	6.03	0.21	6.93	0.24	11.41	0.42	6.42	0.22	4.47	**0.15**
	TOTAL	7.38	0.14	8.88	0.16	10.42	0.18	16.53	0.33	9.34	0.17	**6.96**	**0.13**
PJM	Entity 1	4.08	0.82	6.39	1.13	7.94	1.42	6.16	1.11	5.52	1.04	**3.78**	**0.74**
	Entity 2	5.25	0.95	8.47	1.24	9.20	1.15	7.21	0.97	6.41	0.99	**4.48**	**0.69**
	Entity 3	5.20	0.14	10.05	0.25	9.46	0.22	7.16	0.17	6.94	0.18	**4.65**	**0.11**
	Entity 4	5.10	0.20	7.44	0.27	9.20	0.34	7.32	0.27	6.79	0.27	**4.72**	**0.18**
	Entity 5	5.16	1.04	8.16	1.41	6.46	1.15	6.36	1.15	6.19	1.18	**4.51**	**0.83**
	Entity 6	4.26	0.09	5.43	0.11	6.44	0.12	6.41	0.13	5.48	0.11	**4.12**	**0.08**
	Entity 7	8.78	0.18	12.79	0.23	24.64	0.48	22.51	0.44	10.45	0.20	**8.34**	**0.17**
	Entity 8	4.01	0.44	10.37	1.00	6.62	0.63	5.80	0.59	5.34	0.54	**3.66**	**0.37**
	TOTAL	5.23	0.48	8.64	0.70	9.99	0.69	8.62	0.60	6.64	0.55	**4.78**	**0.40**

Github with the code and documentation

README License

Probabilistic Load Forecasting based on Adaptive Online Learning (APLF)

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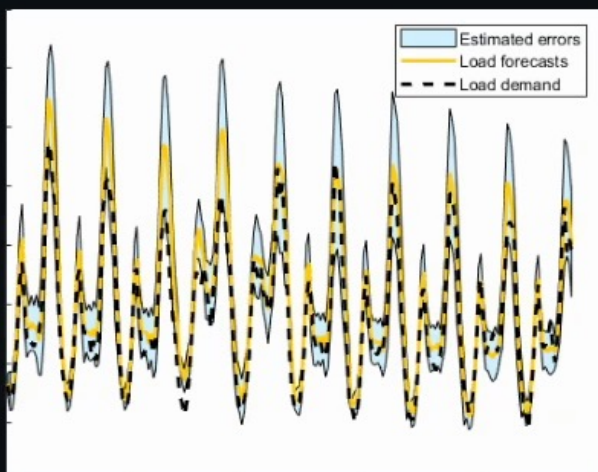
This repository contains code for the paper Probabilistic Load Forecasting based on Adaptive Online Learning [1]. We use the implementation details described in the paper.



Implementation of the method

APLF folder contains the Python file, a Jupyter notebook and a Matlab folder that contains all the Matlab scripts required to execute the method:

Outline

- Introduction and motivation
- Problem formulation
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- Extension to high-dimensional models
- Experimental results
- Conclusions

Conclusions

- Proposed multi-task learning method can **learn the relationship** among multiple entities, **adapt to changes** in consumption patterns and **asses load uncertainties**.
- Parameters of the model are updated using a simple **recursive algorithm**, and the method provides **probabilistic predictions** with the most recent parameters
- Experimental results demonstrate that the proposed method achieves **higher performance** compared to existing multi-task learning techniques for load forecasting.